Internal Management Directive

The Use of Significant Figures and Rounding Conventions in Water Quality Permitting

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Department of Environmental Quality

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Document Development

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1. Purpose

The purpose of this internal management directive is to explain the conventions for significant figures, rounding and precision for DEQ's Water Quality Permitting Program. This Directive is authorized by Oregon Revised Statutes 468.035, 468B.035 and 468B.050.

2. Applicability

This Directive sets forth the conventions that DEQ water quality permitting staff should use when developing permit limits and determining compliance with permit limits. Adherence to these conventions will ensure clarity and consistency in permit limit development and compliance determinations.

Because many of the permits issued by DEQ were developed prior to the development of this IMD, there will be instances in which permit writers and permit holders may agree to follow conventions established when those permits were written.

As new permits are written, permit writers should revise permit limits and associated reporting requirements to be consistent with the contents of this IMD.

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3. Background

The process of developing and demonstrating compliance with water quality permits involves the analysis and interpretation of environmental data. This data is collected by a variety of public and private organizations employing a variety of sampling, analytic and data management practices that have varying levels of precision. The challenge for DEQ permit writers and compliance staff is how to interpret and use this data in a manner that is statistically relevant and consistent.

4. Conventions

There are 3 categories of conventions described in this document: those pertaining to the determination of the correct number of significant figures, those pertaining to rounding and those pertaining to precision. In some cases, the conventions vary with the context in which they are being applied: measurements or calculations. Where there are differences, these are noted.

4.1 Significant Figures

Regardless of the measuring device, there is always some uncertainty in a measurement. Significant figures include all of the digits in a measurement that are known with certainty as well as the last digit, which is an approximation. This has implications both for permit limit development and for establishing compliance with a permit limit.

Table 1 below lists the conventions in use at DEQ regarding significant figures.

Table 1: DEQ's Conventions for Significant Figures

Conventions	Example	No. of Significant Figures
1. All non-zero digits (1-9) are to be counted as significant.	23	2
	231	3
2. All zeros between non-zero digits are always significant.	4308	4
	40.05	4
3. For numbers that do ***not*** contain decimal points, the trailing zeros may or may not be significant. In this situation, the number of significant figures is ambiguous.	470,000	2 to 6
4. For numbers that ***do*** contain decimal points, the trailing zeros are significant.	0.360	3
	4.00	3
5. If a number is less than 1, zeros that follow the decimal point and are before a non-zero digit are not significant.	0.00253	3
	0.0670	3

As indicated in the third convention above, numbers that contain trailing zeros but that do not contain decimal points can be problematic. For example, "10" could be either one or two significant figures. There is no way to know what was intended unless it is written in terms of scientific notation or there is a note that explicitly states how many significant figures there are.

Replacing "10" with "10." is not a robust solution to this problem since Excel replaces "10." with "10" and the information that the user intended to provide is lost.

The problem with how to interpret the number 10 (or 20 or 30, etc.) is pervasive enough that EPA recently changed the MCL for arsenic in drinking water from 10 ppb to 0.010 ppm in order to clarify the number of significant figures associated with the MCL.

The number of significant figures associated with various conventional and toxic parameters may be found in Appendix B of this document. For QL information on toxic parameters, refer to Appendix C of DEQ's Internal Management Directive on the Reasonable Potential Analysis Process. This document may be found at:
http://www.deq.state.or.us/wq/pubs/imds/rpaIMD.pdf

4.2 Rounding

In reporting results and in calculating permit limits or mass loads, it is necessary to round off the results to the correct number of significant figures. There are different rounding conventions in use, and DEQ has adopted a hybrid approach in which the rounding convention used depends on the context. The difference between the conventions involves the treatment of the number 5. In reporting measured values, 5 is rounded to the nearest even number. In the case of calculated values, the number 5 is rounded up.

The two sets of conventions are listed below.

Table 2: DEQ's Conventions for Rounding for Calculated and Measured Values

Conventions for Rounding	Examples	
	Rounding Off *Calculated* Values	Rounding Off *Measured* Values
1. If the digit being dropped is 1, 2, 3 or 4, leave the preceding number as-is.	1.11 → 1.1 1.12 → 1.1 1.13 → 1.1 1.14 → 1.1	Same
2. **For calculations:** If the digit being dropped is 5, round the preceding digit up.	1.15 → 1.2	1.15 → 1.2
3. **For measurements:** If the digit being dropped is 5, round off the preceding digit to the nearest even number (0 is considered an even number when rounding off).	1.25 → 1.3	**1.25 → 1.2**
4. If the digit being dropped is 6, 7, 8 or 9, increase the preceding digit by one.	1.16 → 1.2 1.17 → 1.2 1.18 → 1.2 1.19 → 1.2	Same

The conventions shown above for measured results are consistent with the rules for rounding found in **Standard Methods for the Examination of Water and Wastewater**, **Part 1050 B**. These are the rules used by the DEQ lab in reporting laboratory results.

For calculated results, the digit 5 is handled consistent with the convention used by Excel software, and is rounded up. This is explained in the DEQ Lab Quality Manual for DEQ's Laboratory and Environmental Assessment Division (LEAD) as follows:

> *Where commercial software packages and spreadsheets employ a different rounding routine (e.g., rounds up in all cases), the analyst shall NOT change the results generated by the software.*

The DEQ document may be found at: www.deq.state.or.us/lab/techrpts/docs/DEQ91LAB0006QMP.pdf

If a permit holder chooses to use the same convention for calculated values as for measured values, the permit holder is allowed to do so, provided the permit holder is willing to commit to doing so on a consistent basis. This decision may necessitate the purchase of special software.

A shorthand version of the information presented in this section is as follows:

> **Measured values – the digit 5 should be rounded to the nearest even number.**
> **Calculated values – the digit 5 should be rounded up, unless the permit holder has chosen to follow the convention for measured values. The permit holder must do so on a consistent basis.**

4.3 Significant Figures, Decimal Places, and Reporting

There are two types of permit limits: those for which compliance will be determined based on the results of a laboratory or field measurement and those for which compliance will be based on the results of a calculation.

If compliance will be established based on a laboratory or field measurement, the number of significant figures in the permit limit should be the same as the number of significant figures associated with the laboratory or field measurement. **Appendix B** contains a table with this information for conventional and toxic parameters. Appendix C of the Reasonable Potential Analysis IMD contains a table of QLs for toxic parameters. This IMD available at: http://deq05/wq/wqpermits/PCGuidance.htm#IMD

If compliance with a permit effluent limit will be determined based on the results of a calculation, the number of significant figures in the permit limit should be determined in a manner that is consistent with DEQ's rules for the determination of the number of figures to report as listed below.

Table 3: DEQ's Conventions Determining the Number of Figures to Report

Convention	Example
1. **For addition or subtraction**, the number of decimal places* in the result is equal to the number of decimal places in the least precise value used in the calculation. *The number of decimal places is equal to the number of digits to the right of the decimal point.	13.681 – 0.5 = 13.181 becomes 13.2 0.5 is reported to only one decimal place so the final answer has one decimal place. Note that the number of digits in the answer is determined by the number of decimal places in the least precise measurement, and not on the number of significant figures.
2. **For multiplication or division**, the number of significant figures in the final result is equal to the smallest number of significant figures of the values used in the calculation.	2.5 x 3.42 = 8.55 becomes 8.6 2.5 has the fewest significant figures (2) so the final result has 2 significant figures.
3. **When a calculation involves multiple arithmetic operations,** the number of significant figures is determined by both of the above rules with arithmetic operations performed in the following order: a. Operation(s) in parentheses b. Multiplication c. Division d. Addition e. Subtraction In a situation with multiple operations it is important not to round answers after each intermediate step. Instead keep track of the right most digit that would be retained based on rules 1 and 2 above (shown in the example on the left by an underline). The order of operations is seldom an issue in permitting. This information is included for completeness.	(2.5 x 3.42) + 13.681 – 0.5 = 21.731 becomes 21.7 **1) First do the operation in parenthesis** (in this case multiplication – Rule 2 above) = 8.55 + 13.681 – 0.5 **2) Next perform addition -Rule 1 above** = 22.231 – 0.5 **3) Then subtraction** – Rule 1 above = 21.731 all digits carried through = 21.7 final rounding In step 1, (based on rule 2), 8.55 would only be reported to 2 figures (retaining one figure to the right of the decimal). In this case, one place to the right of the decimal is the limiting digit for steps 2 and 3 and therefore the final result is reported to one decimal place.
4. Values that are not considered. Values that are considered "exact" numbers are not included in the determination of the final number of significant figures. Here are some	**Example 1**: For a POTW with a design flow of 1.5 MGD, the mass load of a pollutant measured at 5.25 mg/L is calculated as follows:

examples of exact values: a. **Design flow** of a treatment facility. By contrast, the *measured* flow at a facility is not an exact number and *does* affect the number of significant figures in a calculation. Measured flows at treatment plants typically have 2 significant figures. b. **Conversion factors**. Note: these should be selected so that the number of digits is at least that associated with measured values used in a calculation. c. **Values below the Quantitation Limit**. Where the permittee is allowed to treat <QL as zero when averaging, the zero is not considered when determining the final number of significant figures. d. **Counted values** such as: i. Bacteria measurements ii. The number of samples iii. Values denoting time (days, months, etc.)	5.25 mg/L x 1.5 MGD flow x 8.34 = 219 lbs The result contains 3 significant figures because the concentration of 5.25 contains 3 significant figures. The other numbers in the calculation, 1.5 MGD (design flow) and 8.34 (conversion factor), have no effect on the number of significant figures in the result. Note that if the MGD of the facility were *measured* at the plant rather than being supplied by the design engineer, the number of significant figures associated with the flow would matter. Flow measurements typically have 2 significant figures. **Example 2:** What is the average of the following 3 concentrations: 4.6 mg/L, 2.3 mg/L and <QL Answer: (4.6 + 2.3 + 0)/3 = 2.3 mg/L The number of decimal places is equal to the number of decimal places for the detected concentrations. The 3 in the denominator is a counted value and does not affect the determination of the number of significant figures or decimal places in the final rounding.

4.4 Permit Examples

Here are some examples of how these rules may apply when developing mass load limits or when determining compliance with monthly mass load limits.

Example 1: Calculate a permit limit for the average daily mass load of ammonia.

Facility Info:

Average Dry Weather Design flow = 1.25 MGD
Permit limit for ammonia (Total Ammonia as N) = 5.0 mg/L
Conversion factor from MGD and mg/L to pounds per day = 8.34

The allowable mass load for ammonia from this facility is calculated as follows:
1.25 MGD x 5.0 mg/L x 8.34 = 52.13 lbs/day → 52 lbs/day

Comments
The resulting permit limit has been rounded off to 2 significant figures because that is how many significant figures are associated with the ammonia concentration. The number of significant figures in the permit limit is unaffected by the number of digits in the design flow or the conversion factor (see Conventions 4.a and b. in Table 3).

If the calculated result had been 52.5 lbs/day instead of 52.13 lbs/day, the permit limit would have been rounded up to 53 lbs/day (see Rounding Convention 3 in Table 3).

Note that if the allowable ammonia concentration was greater than 10, the permit limit would contain 3 significant figures instead of 2. This is because if the ammonia concentration is greater than 10, it has 3 significant figures instead of 2 (see Appendix B for more information).

Example 2: Calculate the 7-day average concentration for ammonia.

Permit limit = 4.5 mg/L, sampled 4 times a week

Measured concentrations = 0.5, 2.5, 12.7 mg/L and <0.1 mg/L

(0.6 + 2.5 + 12.7 + 0)/4 = 3.95 mg/L → 4.0 mg/L

Comments
The result has been rounded off to 2 significant figures because the permit limit contains 2 significant figures and it is rounded up (see Rounding Convention 3 in Table 3).

0.6 mg/L only has one place to the right of the decimal so the result is reported to one decimal place (see Convention 1, Table 3)

Note that the lab result 12.7 contains more significant figures than the permit limit. This value is consistent with the information provided in Appendix B for ammonia. Lab results for ammonia have 2 significant figures when the values are below 10 mg/L and 3 when they are above 10 mg/L; however, it doesn't affect the final answer of 4.0 mg/L.

Note that the nondetect is treated as zero and it does not affect the number of significant figures in the final result (see convention 4.c. in Table 3). The value of 4 in the denominator also has no affect because it is a counted number (see Convention 4.d. in Table 3).

Example 3: Determine if the following facility is in compliance with their permit limit for average daily mass load of ammonia of 38 lbs/day.

Facility Info:
- Average daily flow = 0.85 MGD
- Average daily concentration of ammonia (measured as Total Ammonia as N) = 5.0 mg/L
- Conversion factor from MGD and mg/L to pounds per day = 8.34

The allowable mass load for ammonia from this facility is calculated as follows:
0.85 MGD x 5.0 mg/L x 8.34 = 35.5 lbs/day → 36 lbs/day

Comments
The result has been rounded off to 2 significant figures because that is how many significant figures are associated with the ammonia concentration and with the average daily flow from the facility. The conversion factor has no affect on the number of significant figures.

Note: The flow measuring devices at POTWs are typically, but not always, accurate to 2 significant figures. Inaccurate flow measurements can serve to mask compliance problems. They should not be allowed to do so.

Example 4: Calculate the geometric mean of the following data set (data in cts./100ml).

Result type	Data Reported	Data for Calculation
No colony growth	<4	4
# of colonies <20	15 est.	15
Colonies between 20-60	40	40
Colonies > 60	150 est.	150
Colonies TNTC	>6000	6000

TNTC = too numerous to count

The geometric mean may be calculated as the nth root of the results:

$$\text{Geometric Mean} = \overline{X_g} = \sqrt[n]{X_1 \times X_2 \times X_3 \times \ldots \times X_n}$$

Where: n = the number of values observed / analyzed
$X_1, X_2, \ldots X_n$ = the sample results or values

For example data set:

$$\overline{X_g} = \sqrt[5]{4 \times 15 \times 40 \times 150 \times 6000} = \sqrt[5]{2{,}160{,}000{,}000} = 74 \text{ cts./100 ml}$$

Comments

The results of bacteria calculations are always whole numbers, and the number of significant figures is equal to the number of digits of a particular result.

5. Sample Permit Language

The following is suggested permit language:

Permit Language: (Schedule B, Monitoring and Reporting Requirements)

2. Reporting Requirements

(f) The permittee shall report the same number of significant digits as the permit limit for a given parameter. Regardless of the rounding conventions used by the permittee (i.e., rounding 5 up for calculated results or, in the case of laboratory results, rounding 5 to the nearest even number), the permittee shall use the conventions consistently, and shall ensure that consulting laboratories employed by the permittee use the same conventions.

Appendix A Revision History

Revision	Date	Changes	Editor
1.0	10/31/2011	New Document	SBH
1.1	1/24/2012	1) Section 4.2 on rounding has been made more concise. 2) Section 4.3 on precision has been modified as follows: a. "Significant figures" has been replaced with "decimal places" where relevant. b. Information on the correct order of arithmetic operations has been added. 3) Appendix B has been modified so that the number of decimal places in the columns entitled "Typical Permit Limit Range" and "Significant Figures" are consistent with each other.	SBH, SCH
1.2	5/31/2013	Section 4.2 has been modified to state that if a permit holder chooses to use the same convention for calculated values as for measured values, the permit holder is allowed to do so, provided the permit holder is willing to commit to doing so on a consistent basis.	SBH
1.3	12/05/2013	Corrected typos - Editorial changes only	SCH

Appendix B Table of Significant Figures for Conventional and Toxic Parameters

Conventional Parameters	Typical Permit Limit Range	Standard Laboratory Technique	Significant Figures	DMR reporting
BOD	5.0 to 50 mg/L	DO Probe	2	< 10 (0.1 mg/L) > 10 (whole numbers)
CBOD	2.0 to 45 mg/L	DO Probe	2	< 10 (0.1 mg/L) ≥ 10 (whole numbers)
TSS	5.0 to 80.0 mg/L	Filtration/Gravimetric	<10 = 2 sig figures ≥ 10 = 3 sig figures	Report to 0.1 mg/L
Temperature	77 F as a maximum	Various	Various	± 0.1 degrees F or C
Bacteria (Fecal, e. Coli, etc.)	126/406 for e coli or 14 for fecal for sewage effluent, and 2 total bacteria for Class A recycled water	Various	<10 = 1 ≥10 <100 = 2 ≥100 = 3	Whole numbers only
DO	8.0 to 10.0 mg/L	DO Probe	<10 = 2, ≥10 = 3	Report to 0.1 mg/L
Total Chlorine Residual (method dependent)	0.02 mg/L to 1.0 mg/L 0.1 mg/L to 1.0 mg/L	Amperometric Titr. DPD – colorimetric	< 0.1=1 <10 = 2 ≥10 = 3	Report to 0.1 mg/L
Minimum UV dose	35 millijoules			
pH	6.0 to 9.0	pH Probe	<10 = 2 ≥10 = 3	0.1 pH unit
Nutrients				
TKN	5.0 to 20.0 mg/L	Digest w/ ISE or Colorimetric	<10 = 2 ≥10 = 3	Report to 0.1 mg/L
Total Ammonia as N	1.0 to 30.0 mg/L	Distill w/ ISE or Colorimetric IC	<10 = 2 ≥10 = 3	Report to 0.1 mg/L
Nitrate and Nitrite	1.0 to 20.0 mg/L	Colorimetric or IC	<10 = 2 ≥10 = 3	Report to 0.1 mg/L
Total Phosphorus	0.01 to 3.0 mg/L	Colorimetric	< 0.1=1 <10 = 2 ≥10 = 3	<0.1 (0.01 mg/L) ≥ 0.1 (0.1 mg/L)
Dissolved Orthophosphate as P	0.01 to 3.0 mg/L	Colorimetric	< 0.1=1 <10 = 2 ≥10 = 3	<1 (0.01 mg/L) ≥ 1 (0.1 mg/L)
Toxics				
Various	Various	Various	<10 = 2 ≥10 = 3	Various. Refer to QLs in Appendix C of RPA IMD