Thank you for the opportunity to review the DEQ Heat Source Model documentation. I found the model to be quite informative and a very useful contribution to our available tools for water quality assessment and management. I believe it could, as mentioned below, be expanded in its scope – from reach to watershed – whereby it would find even more useful application, in particular for TMDL applications. Let me first get the typographical nit-picking of the model documentation out of the way.

1. There are several citations in the text to publications that do not appear among the literature references at the end of the document. Beschta 1984, for example, is repeatedly cited, and appears to be an important source, but is not included among the references listed. Other citations in the text that I did not find among the listed references include: Brown 1972; Chen 1996; Sullivan and Adams 1990; Boyd 1996 (likely also an important source); and Park 1993. Sinokrot and Stephan 1992 is cited in the text (p.2), but the reference listed is 1993; Sellers 1965 is cited (p.22, 23), but the Sellers reference listed is 1974; Weathered, in Beschta an Weathered 1984 (p23) is spelled Weathered in the reference list.

2. For equation 2-1 (p.8), $A_i$ is defined as a cross-sectional area when it should in this equation be surface area. In subsequent equations on p. 10, $A_i$ correctly is the cross-sectional area. For equation 2-5 (p.9), there are no definitions specified for $W$ or $U_s$. Some possible confusion would be eliminated from equations on p.10 if the term $A_i^2$ were instead $\Delta x^2$. In the last equation for $T$ on p.10, the superscript on $T$ in the second right-hand term should be $t+1$, rather than $t-1$. The same applies to the carryover of the term to the equations on p.11.

3. Perhaps some more could be written about the boundary conditions assume on p.11. I understand why the initial $\partial T / \partial x = 0$, but why temperature should remain constant ($\partial T / \partial t = 0$) at position $i_N$ escapes me. Also, am I correct in guessing that if travel time in the reach is more than about 8 hours, then temperature at $i_N$ will be independent of temperature change at $i_o$?

4. I have not examined closely the solar geometry described in pp.14-22, but assume all this is straightforward. The expression(s) for transmissivity, $\mathrm{TRANS}_{\text{stream}}$ on p.23 and $T_{\text{stream}}$ on p.29, could include an accounting for stream turbidity, or suspended solids, of the form:

$$T_{\text{stream}} = (T_{\text{h2o}}) (T_{\text{ss}})$$

$$T_{\text{ss}} = \exp(-k[SS])$$

Where $k$ is an empirical correlation coefficient (there are numerous sources for this correlation, from algal growth models for example).

5. The numerical dispersion described by equation 2-6 could be reduced or eliminated by reframing the present model as follows:

Redefine $\xi_i$ and $\lambda_i$ as:

$$\xi_i = \frac{\Delta x}{4\Delta x} U_i$$
Define:

\[ a_i^{+1} = -c_i^{+1} - \lambda \]
\[ b_i^{+1} = 1 + 2\lambda \]
\[ c_i^{+1} = c_i^{+1} - \lambda \]
\[ a_i^{+1} = c_i^{+1} + \lambda \]
\[ b_i^{+1} = 1 - 2\lambda \]
\[ c_i^{+1} = -c_i^{+1} + \lambda \]

\[ d_i = \frac{\Delta x \Phi_i}{c_y \lambda D_i} \]

Then:

\[ a_i^{+1} T_{i-1} + b_i^{+1} T_i^{+1} + c_i^{+1} T_{i+1}^{+1} = a_i^{+1} T_{i-1} + b_i^{+1} T_i + c_i^{+1} T_{i+1} + d_i \]

The series of simultaneous equations is expressed in matrix notation:

\[ [M_{ij}^{+1}][T_{j}^{+1}] - [M_{ij}^+] [T_{j}] + [d_j] \]

And is solved:

\[ [T_i^{+1}] = [M_{ij}^{+1}]^{-1} [M_{ij}^+] [T_j] + [M_{ij}^{+1}]^{-1} [d_j] \]

Note that each of the individual elements \( m_{ij} \) of the inverse matrix \([M^{+1}]^{-1}\) are usefully the derivatives of \( T_i^{+1} \) with respect to solar input \( d_j \), which provide indicators of the input reductions (shading) necessary to achieve the temperature objective. Both of the matrices \([M^{+1}] \) and \([M^+] \), the inverse \([M^{+1}]^{-1} \), and the product of matrix and inverse will be of the tri-diagonal form displayed on p.12, and thereby responsive to the solution algorithm on the page. However, most any commercially available spreadsheet program (e.g., Excel) will perform these matrix operations and their solution automatically and dynamically, and without restriction to tri-diagonality.

6. This latter observation opens the possibility of a relatively painless way of extending the Heat Source application from reach to watershed, i.e., application to tributary
networks with point source inflows. I have developed this watershed-scale application to what I think might be worth some discussion, and would like to talk with you – or maybe more appropriately Matt Boyd – about it. I would more fully present this extended application here, but this goes a bit beyond the review you requested. (Also, without substantial encouragement, I'm frankly weary of typing equations in this word-processor.)

Again, thanks for the opportunity to review this interesting model. I hope my comments will be helpful, and further that we will talk more about the potential for its extended application.

Best regards.

Sincerely yours,

Jack Douglas Smith, Ph.D.

DEQ response to comment #1

The corrected citations are listed below.


DEQ response to comment #2

- The reviewer is correct. Equation 2-1 incorrectly labels surface area ($A_y$) as cross-sectional area ($A_i$).
- Equation 2-5 has been further described in DEQ response to Review #1, Comment #6.
• The finite difference form has been modified as described in DEQ response to Review #1, Comment #5.

**DEQ response to comment #3**

The reviewer has correctly noted that the downstream boundary condition is not \( \frac{dT}{dx} = 0 \). The downstream boundary condition is assumed to equal that of the second to last finite difference cell \( T_n^t \). The model compensates by calculating one extra finite difference cell \( T_{n+1}^t \). The downstream boundary condition is then \( T_n^t = T_{n+1}^t \).

**DEQ response to comment #4**

Heat Source methodology does not account for turbidity. The algorithm suggested by the reviewer could easily be incorporated into the model. DEQ will investigate the sensitivity of the model to turbidity effects (SS), based on correlation coefficient values and calculated transmissivity values.

**DEQ response to comment #5**

The finite difference solution derived by the reviewer appears to be an improvement over the existing solution. For simplicity, the solution method has been changed to an explicit finite difference, which is detailed in DEQ Response to Review #1, Comment #5. If the solution method reverts back to an implicit form, these suggested changes will be made.

**DEQ response to comment #6**

Heat Source has been expanded to a quasi-network scale. Point sources and tributary mixing is performed at longitudinally defined locations. Tributary simulations are performed separately and then mixed with the mainstem. Simultaneous simulations of multiple tributaries have not been completed. DEQ would looks forward to continuing our working with the reviewer in model development and applications.