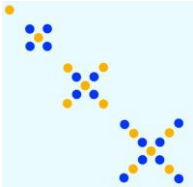
K-3 FORMATIVE ASSESSMENT CONSORTIUM



Assessment Means Form: Mathematical Patterns

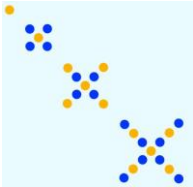
Assessment Overview: Teachers should aim to assess students in the most naturalistic environment first (i.e., observation) before moving on to more intentionally structured activities (i.e., the Situation).

What Teachers Need to Know and Observe: The purpose of this construct progression is to determine how children understand and use sequential and growing patterns. The focus is not on a child's ability to accurately count objects, perform calculations, or use correct mathematical terminology (e.g., children should not be penalized for saying pentagon instead of hexagon). There are a variety of terms related to patterning (see Key Terms below). Although it is important for teachers to understand these terms (e.g., recursive rule, step, unit, element), children do not need to understand or use them. Children also do not need to use abstract labels for patterns (e.g., ABAB).

Patterns can be made with physical objects, sounds, movements, or visual representations. Children do not need to display a skill with multiple types of patterns (e.g., if a child demonstrates a skill with an auditory pattern, the child does not also need to demonstrate the skill with a visual pattern). Children can be stronger in one type of pattern than another. If a child is struggling with one type of pattern (e.g., visual, spatial), look for opportunities to observe the child interacting with other types of patterns (e.g., kinesthetic, numerical). All patterns used to document children's place on the construct progression should be grade appropriate. There are a variety of sequential and growing patterns that can be used with children. A list of examples can be found at the end of this document (Patterns Cheat Sheet).

General Teacher Instructions: Many of the steps in this progression require that a model pattern be used. For sequential patterns, models used must include at least two units (e.g., red-blue, red-blue and not just red-blue). For growing patterns, models used (including those represented in a t-chart) must include at least three completed steps. Teachers should take care to allow children to demonstrate their skills independently in order to accurately place children on the progression. However, teachers can provide some information to child and/or ask some questions as needed and as appropriate to elicit evidence to determine the child's level on the progression, such as the following:

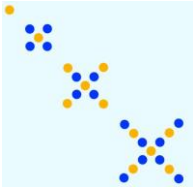
- <u>All skills</u>
  - Teachers can ask children to explain their actions or thoughts in order to verify the requirements of a skill are met (e.g., "How did you come up with that answer? ", "Can you tell me what you're doing?").
  - Teachers can describe that growing patterns are changing or that sequential patterns are repeating.
  - Teachers can point out when there is a pattern (e.g., "This is a pattern") and define a pattern (e.g., For sequential patterns: "Some patterns have parts that repeat over and over again; For growing patterns: "Some patterns have parts that change in a predictable way").
- <u>Skills A, B, and E</u>
  - Teachers can point to each element in a pattern as a child is duplicating it or use another method to help child track pattern elements.
- <u>Skills A-E</u>
  - When duplicating or extending a pattern, teachers may prompt the child to continue (e.g., "Can you keep going?"; "What comes next? ").
- <u>Skill E</u>
  - To encourage pattern abstraction, teachers can say something like, "Look at this pattern. Can you make the same pattern using different materials?"



- Skill F
  - When identifying the repeating unit of a pattern, if a child describes the pattern unit multiple times (e.g., mama-baby- baby, mama-baby- baby), teachers can ask a clarifying question to encourage child to identify the pattern unit (e.g., "What's the part that repeats over and over again?").
  - Teachers can model identifying a pattern unit using a pattern that has a different repeating unit than the child's pattern (e.g., if the child is working with an ABC pattern, the teacher can model with an AB pattern).
- Skills G and H
  - When asking children to extend or describe a growing pattern (SKILLS G and H), teachers may ask clarifying questions (e.g., "How does it change from one step to the next? "; "How is this step different from this step?").
- Skill H
  - Teachers can indicate the place in a growing pattern where a step is missing. When asking a child to communicate a recursive rule for a growing pattern, teachers can use child-friendly language: "How does this pattern change from one step to the next? "
  - Teachers can model communicating a recursive rule using a growing pattern with a different rule than in the child's pattern (e.g., if the rule in the child's pattern is "increase by 2 every time", the teacher can model with a pattern that has a different rule).
- Skill I
  - Teachers can provide children with an empty t-chart that they can fill in.
- Skills I and J
  - Teachers can remind children what a t-chart is and what it looks like: "This is a t-chart. It has two columns where we enter information."
- Skills K and L
  - When asking a child to communicate a functional rule for a growing pattern, teachers can use child-friendly language: "When working with growing patterns, we think about how each number in one set of values relates to a number in another set of values. For example, if my first set of values is 1, 2, 3 and my second set of values is 10, 20, 30, then the rule is that I multiply each number in the first set by 10 to get the second set."
  - Teachers can provide children with a calculator.
  - Teachers can provide children with the correct calculation if they make a calculation error.
- Skills K-M
  - When asking a child to communicate or symbolize a functional rule for a growing pattern (SKILLS K, L, M), the teacher may provide a completed t-chart or encourage children to create a t-chart.
- Skill M
  - Teachers can remind children of the order of operations and how to symbolize which operation comes first (NOTE: Child must first express the order correctly).

Teacher should NOT:

- Provide any modeling or demonstrations of the skills to be observed, except when helping children understand what is meant by the terms "pattern units" or "recursive rule."

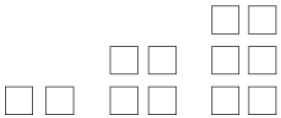


- Read patterns to children (e.g., "This pattern is orange-white, orange-white," "This pattern is AB, AB").
- Describe how sequential patterns are repeating or how growing patterns are changing.
- Prompt children to say the next element if they only name one when asked to identify the pattern unit (e.g., when asked to identify the repeating unit in a green-white pattern, the child says "green").
- Identify the ordinal position of the steps in the pattern when children are entering data into a t-chart (e.g., "This is the first step in the pattern").

Key Terms:

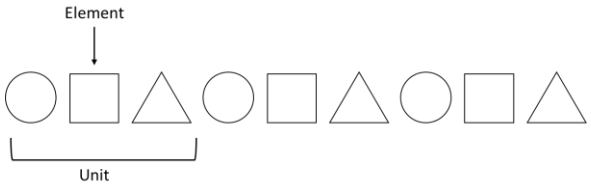
General Terms:

- **Model pattern:** The initial pattern that children refer to when completing a pattern task. For example, if a child is shown an AB pattern made out of red and blue cubes and asked to create the same type of pattern using round and square wooden shapes, then the initial pattern made out of red and blue cubes is called the model pattern.
- **Element:** In sequential patterns, the smallest component of a unit (such as A or B in an AB pattern). In growing patterns, the smallest component of a step in the pattern. For example, in the growing pattern below, there are two elements (squares) in step 1, four elements in step 2, and 6 elements in step 3.

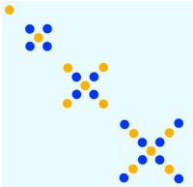


Terms Used in Reference to SEQUENTIAL Patterns:

- **Sequential pattern:** A linear sequence with a group of elements (i.e., a unit) that repeats. Also called a repeating pattern.
- **Unit:** The repeating part of a pattern (e.g., AB, ABB). A unit includes two or more elements.



- **Pattern abstraction:** Making the same kind of pattern (e.g., ABA) using a different set of materials. Pattern abstraction requires that children attend to the structure of a pattern, not just its attributes (such as color or shape).



- **Different materials:** Materials that are non-identical to those used in the model pattern. Different materials may be the same type of objects, as long as the attribute that is used to make the pattern changes. More commonly, different materials will be different types of objects altogether. Consider this ABA pattern of red-blue-red blocks:

  - This pattern could be abstracted with any of the following:
    - ABA pattern of square-cylinder-square blocks:

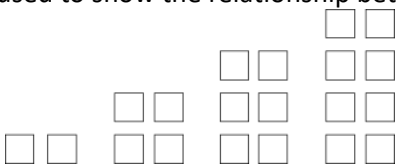
    - ABA pattern of green-orange-green blocks:

    - ABA movement pattern:

      Stomp – Clap – Stomp
  - This pattern could **NOT** be abstracted with an ABA pattern of red-blue-red *tiles*:

<u>Terms Used in Reference to GROWING Patterns:</u>

- **Growing pattern:** A sequence with elements that increase (or decrease) in a systematic or predictable way. The elements can be depicted spatially (e.g., ☐, ☐☐, ☐☐☐) or numerically (e.g., 1, 2, 3).
- **Step:** An individual instance in a growing pattern. In each successive step, elements increase (or decrease) in a predictable way.
- **Recursive rule:** A rule for determining the next step in a growing pattern based on the previous step (e.g., "you add two every time"). A recursive rule specifies how the number of elements changes from one step to the next. It relies on a change within one variable.
- **T-chart:** A chart used to document the relationship between the position of a step in a growing pattern and the number of elements at that position (e.g., step 1 has 2 elements, step 2 has 4 elements, step 3 has 6 elements). The chart represents the functional rule governing a growing pattern. The chart has two columns. When used to represent growing patterns, the first column (sometimes called X) documents the position of a step in a growing pattern and the second column (sometimes called Y) documents the number of elements at that position. T-charts can vary in appearance, but they are always used to show the relationship between two variables that are related by a functional rule. Consider the following pattern:



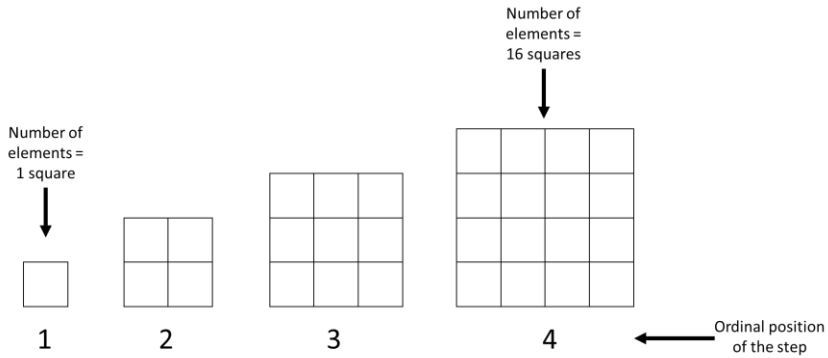
- Sample t-charts for this pattern include:

Position of step	Number of squares
1	2
2	4
3	6
4	8

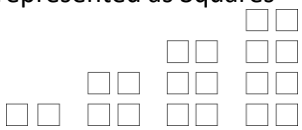
OR

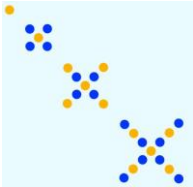
Position of step	Number of squares
1	2
2	4
3	6
4	8

- **Ordinal position:** The location of a step in a growing pattern, such as first, second, third. In growing patterns, there is a relationship between the ordinal position of the step and the number of elements in that step (e.g., the first step has four dots, the second step has eight dots, and the tenth step has forty dots). Understanding the ordinal position of the step, which is sometimes referred to as the "hidden variable," is essential to understanding and working with growing patterns.
- **Functional rule:** A rule for determining a step in a growing pattern based on its ordinal position in the pattern (e.g., "you multiply its position by two"). A functional rule describes the relationship between two variables: (1) the ordinal position of the step (hidden variable) and (2) the number of elements in that step. For the pattern below, the functional rule can be described as, "To get the number of squares, you multiply the position of the step times itself." An equation for this rule is: Number of Squares = Ordinal Position of Step X Ordinal Position of Step.

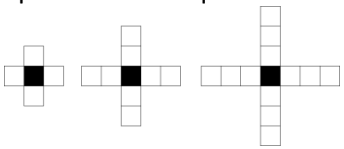


- **One-operation functional rule:** A rule that requires the use of one operation, such as addition OR multiplication. The functional rule for the following pattern can be represented as Squares = 2 X Step.





- **Two-operation functional rule:** A rule that requires the use of two operations, such as addition AND multiplication. The functional rule for the following pattern can be represented as Squares = 4 X Step + 1.

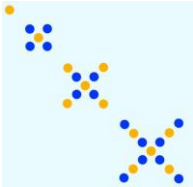


- **Equation:** A representation of an equal relationship between two quantities. Equations contain one or more variables that are unknown and represented symbolically (e.g., with a letter or picture). Equations always contain an equal sign.

Observation Instructions: Observe throughout the day while child is engaged in activities involving patterns, such as: (1) working in a block area (e.g., children making patterns with blocks of different sizes, shapes, or colors); (2) interacting with math manipulatives (e.g., making patterns with play coins); (3) engaging in art, music, and gym/outdoor play (e.g., making a rainbow bracelet, snapping fingers or clapping, or describing patterns in the bricks of the school); and (4) during transitions (e.g., identifying how other children are lined up).

Potential Opportunities for Observation	Potential Materials
Any setting where children are engaged in activities involving patterns (e.g., classroom, art/music room, playground, library, cafeteria, PE class)	Any materials children use to demonstrate their understanding of patterns: • math manipulatives (e.g., blocks, tiles) • concrete objects (e.g., colored beads, crayons, markers, chairs, books, leaves) - food items (e.g., crackers, fruit) • drawing/painting materials • musical instruments Materials used in pattern activities should have the following attributes: • easily distinguishable • interesting and engaging to children • stable enough to stay in place (e.g., buttons rather than marbles) • large enough for children to see and easily manipulate • of sufficient contrast to the background (e.g., floor or table) • familiar to most children • (for growing patterns) be placed with enough space between each step so that the steps are clearly distinguishable

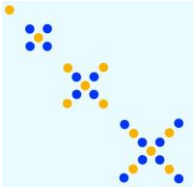
Placing a Child on this Progression: With all progressions, the goal is to identify the level at which the child is solidly performing. If the child is inconsistent at a given level, as children often are when they are learning a new skill, the correct placement is at a lower level. The teacher needs to collect enough evidence to be confident that the child is correctly placed on the progression. This will include multiple pieces of evidence where the child demonstrates the skill level at which he/she is placed and at least one documented instance of allowing the child the opportunity to demonstrate his/her skills/behavior at the next highest



level. It will be difficult to place some children on a progression. Children who are not yet at Skill A should be marked as "Emerging" for that progression. Children who have reached the highest level of a progression should be marked at that highest level.

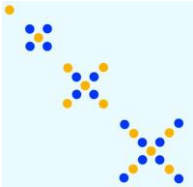
Observation Examples

Skill Progression	Observation Examples
A. Duplicates sequential AB patterns with the same materials.	*Twins Faye and Lily are playing with the stringing beads. Faye makes a necklace in a pattern of orange-yellow, orange-yellow, orange-yellow. Lily says, "We are twins who like to dress alike so I'm going to make mine like yours," and makes an identical necklace with the same colors, same pattern, and same number of beads.*
B. Duplicates sequential three-element patterns (e.g., ABB, ABC) with the same materials.	*During a study of bears, the class is reviewing the "Learned" column of their K-W-L chart, including that most black bear mothers have two cubs each season. Mrs. Hoke uses large and small counting bears to make a pattern of large-small-small, large-small-small, large-small-small. Sonya duplicates the pattern with her own counting bears.*
C. Extends sequential AB patterns at least one unit.	*Some friends have been playing hopscotch on the playground. When the teacher walks by, Cora says, "To get all the way to the end, my feet go one, two, one, two, one, two. If we want to make it longer, we can add another one and another two!"* *After studying the American flag, Gervase and Axl decide to make their own flag. Gervase paints a sequence of six alternating purple and yellow stripes. Axl then adds three more stripes (purple-yellow-purple).*
D. Extends sequential three-element patterns (e.g., ABB, ABC) at least one unit.	*Phillip and Beth are playing a "snake-making game" with interlocking plastic cubes. Beth makes a snake with six cubes using green-green- red, green-green- red. She passes the snake to Phillip, who adds three more cubes (green-green-red) to the end.*
E. Duplicates sequential patterns (e.g., AB, ABC, AABB) using materials different from those used in the model pattern (pattern abstraction).	*Jorge and Melinda are playing with some of the math manipulatives. After Jorge makes a pattern of red-blue-green, red-blue-green, red-blue- green cubes, Melinda says, "I'm going to make the same pattern but with different colors." She then takes some stringing beads and makes a necklace with the same pattern but using different colors (orange-white-black).* *During art class, Jacob says to the teacher, "I made the same kind of pattern as you. I have circle-square, circle-square on my paper." The teacher asks, "What do you mean the same kind of pattern as me?" Jacob replies, "Your shirt has a pattern of red-green, red-green stripes."*
F. Identifies repeating unit in sequential patterns.	*During a study of bears, the class is reviewing the "Learned" column of their K-W-L chart. After the teacher notes that most black bear mothers have two cubs each season, Ms. Bryan asks one child to come up front and stand, then two more children to kneel next to the first child. She then repeats the pattern with three more children. Maria says, "That's an ABB pattern with mama-baby-baby."*



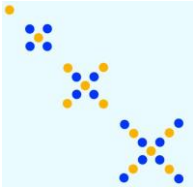
Assessment Means Form: Mathematical Patterns

Skill Progression	Observation Examples
	During a music activity in which the children clap their hands and stomp their feet along with the music, Diego says, "The pattern is clap-stomp-stomp."
G. Extends by at least one step or determines the missing step in spatial or numerical growing patterns.	*Remy is building a set of stairs with one-inch cubes [a spatial growing pattern]. He has reached four cubes high when Morrison sits down with him. Morrison asks if he can build, too, and uses cubes to build the next step five cubes high.* *At the beginning of the day, the teacher writes a numerical growing pattern with a missing step on the board (10 20 _ 40). After lunch, Sophia says to the teacher, "I figured out your puzzle. A 30 goes in the missing space."*
H. Communicates a recursive rule governing the next step in spatial or numerical growing patterns.	*Alik and Denise are working with blocks. Alik is making a series of block towers where the 1st tower has three blocks, the 2nd tower has five blocks, and the 3rd tower has seven blocks. Denise says, "Oh, look - you're adding two every time."*
I. Creates or enters data into a t-chart to document the relationship between the ordinal position of a step in a growing pattern (i.e., first, second, third) and an important feature of the step.	*During a study of bears, the class is reviewing the "Learned" column of their K-W-L chart and using it to talk about growing patterns. The teacher says, "Most black bear mothers have two new cubs each season" and draws two circles on the board. She then says, "Two mama bears would have four cubs" and draws four circles underneath the two circles. She continues with, "And three mama bears would have six cubs" and draws six circles underneath the four circles. Sonya says, "We can make a t-chart that shows that one mama bear has two cubs, two mama bears have four cubs, and three mama bears have six cubs." The teacher says, "Can you draw the t-chart on the board?" Sonya then creates the following t-chart:* Mama bears | Cubs 1 | 2 2 | 4 3 | 6
J. Applies the relationship between the two variables in a t-chart to extend a numerical growing pattern by at least one step.	*During a study of bears, the class has made a t-chart to show that if one mama bear has two cubs, two mama bears will have four cubs, and three mama bears will have six cubs. Then the teacher writes a four in the 1st column*:



Assessment Means Form: Mathematical Patterns

Skill Progression	Observation Examples
	<table><tr><th>Mama bears</th><th>Cubs</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>6</td></tr><tr><td>4</td><td></td></tr></table> *The teacher says, "How many cubs will there be when there are four mama bears?" Charisse says, "You multiply the number of Mama bears times two. So, there will be eight cubs. That's a lot of cubs!"*
K. Communicates a <u>one-operation functional rule</u> governing spatial or numerical growing patterns and uses it to determine a <u>far step</u>. NOTE: a "far step" is more than 5 steps from the last represented step in the pattern.	*The class is planning for field day. The teacher says, "There will be 16 stations and each station can have 10 children." The teacher writes a row of numbers (10, 20, 30) on the board and says, "One station can have 10 children, two stations can have 20 children, and three stations can have 30 children. How could we find out how many children can participate in all 16 stations."? Jake says, "We can multiply the number of stations by ten to find out how many children can play. 16 times 10 is 160, so 160 children can participate."*
L. Communicates a <u>two-operation functional rule</u> governing spatial or numerical growing patterns and uses it to determine a <u>far step</u>. NOTE: a "far step" is more than 5 steps from the last represented step in the pattern.	*Benny is making a string necklace out of beads. He puts on two blue beads and one red bead, then four blue beads and one red bead, and then six blue beads and one red bead. He says, "For each step, the number of blue beads increases by 2. I multiply 2 times the step to figure out the number of blue beads. And I add 1 for the red bead. So, the rule is 2 times the number of steps plus 1. I'm going to try to get to 10 steps, so I'll need 20 blue beads and 1 red bead for that last step."*
M. <u>Creates an equation</u> that symbolizes a <u>functional rule</u> governing a spatial or numerical growing pattern.	*Emily overhears Benny talking about his necklace and says, "I can make an equation to show many beads go in each step of your necklace." Emily writes the following equation on a piece of paper and shows it to Benny: Number of beads = 2 x step +1 [the relationship may also be represented by other equations including y=2x+1]*



Mathematical Patterns Situation 1

Pattern Calisthenics

Suggested Instructions: Teacher explains how the game is played by saying, "We are going to play Pattern Calisthenics. I'll make some patterns by clapping my hands, slapping my knees, and stomping my feet and you'll make patterns too. Let's start the game!" Teacher engages the children in a variety of activities based on the skills in the progression (i.e., duplicating, extending, identifying). The teacher creates a variety of action patterns with different repeating units (e.g., clap-slap, clap-clap-slap, stomp-clap-clap) and documents children's performance. The teacher should call on children individually and ask them to perform activities based on the skills in the progression.

Notes:

- Teachers may repeat the model pattern for the child.
- The teacher should not ask two different children to perform the same skill with the same pattern (e.g., if Child A is asked to duplicate the AB pattern clap-slap, clap-slap, Child B should not be asked to duplicate the same pattern but could be asked to extend it).
- To keep children who struggle with patterns engaged, the teacher should consider varying the order of the skills rather than presenting them in order from easiest to hardest.
- Teachers do not need to include all of the steps in the progression if some steps have already been observed.

Suggested Group Size: 2-4 children who, based on prior observation and professional judgement, likely have skills within the same Understanding.

Suggested Materials: Actions used in kinesthetic pattern activities should have the following attributes:

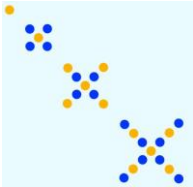
- simple enough for children to easily do
- easily distinguishable
- interesting and engaging to children
- familiar to most children

Skill	Example Teacher Instructions*	Examples
A. Duplicates sequential AB patterns with the same materials.	Teacher provides child with a model AB pattern and then says, "Can you copy this pattern?"	*Mr. Garcia says to Luis, "Listen to this pattern" and makes the following model pattern: clap-stomp, clap-stomp. He then says to*

Assessment Means Form: Mathematical Patterns

Skill	Example Teacher Instructions*	Examples
		Luis, "Can you copy this pattern?" Luis makes the same pattern (clap-stomp, clap-stomp).
B. Duplicates sequential three-element patterns (e.g., ABB, ABC) with the same materials.	Teacher provides child with a model three-element pattern and then says, "Now you make the same pattern."	*Mr. Garcia says to Emily, "I am going to create a pattern. Listen to it. After, I am going to ask you to copy it." Mr. Garcia makes the following three-element action pattern: stomp-clap- clap, stomp-clap-clap and says to Emily, "Now you make the same pattern." Emily repeats the pattern exactly: stomp-clap-clap, stomp-clap-clap.*
C. Extends sequential AB patterns at least one unit.	Teacher provides child with a model AB pattern and then says, "What comes next in the pattern?" [Can repeat question until child completes a pattern unit or makes a mistake]	*Mr. Garcia says to Tommy, "Listen to this pattern" and makes the following model pattern: table knock-nose "beep", table knock-nose "beep." He then says to Tommy, "What comes next in the pattern?" Tommy knocks on the table and stops. Mr. Garcia says, "Can you keep going?" Tommy "beeps" his nose.*
D. Extends sequential three-element patterns (e.g., ABB, ABC) at least one unit.	Teacher provides child with a model three-element pattern and then says, "Can you keep the pattern going?"	*Mr. Garcia says to Tabitha, "I am going to make a pattern and ask you to extend the pattern by keeping it going." He says, "Listen to this pattern" and makes the following model three-element pattern: clap-knock-knock, clap-knock-knock, clap-knock-knock. Mr. Garcia asks Tabitha, "Can you extend the pattern? Can you keep it going?" Tabitha extends the pattern with clap-knock-knock, clap-knock-knock.*
E. Duplicates sequential patterns (e.g., AB, ABC, AABB) using materials different from those used in the model pattern (pattern abstraction).	Teacher provides child with a model pattern and then says, "Can you make the same kind of pattern using different actions?"	*Mr. Garcia says to Beckett, "I am going to make a pattern with these blocks and ask you to copy the pattern by making sounds with your body like we have done with the other patterns." He says, "Look at this pattern" while making the following pattern with the blocks: blue-green, blue-green. Mr. Garcia then says to Beckett, "Now, you copy this pattern by making sounds with your body." Beckett copies the pattern with the action pattern: clap-snap, clap-snap.*
F. Identifies repeating unit in sequential patterns.	Teacher provides child with a model pattern and then says, "What's the part of the pattern that repeats over and over again?"	*Mr. Garcia says to Lucia, "Listen to this pattern" and makes the following pattern: clap-stomp, clap-stomp. After Mr. Garcia asks, "What's the part of the pattern that repeats over and over again?", Lucia makes the repeating unit: clap-stomp.*

*Note: Variations in language in the example teacher instructions are used to provide teachers with ideas of how activities can be presented.



Mathematical Patterns Situation 2

Pattern Game

Suggested Instructions: Teacher explains how the game is played by saying, "We are going to play the Pattern Game. We'll be making patterns with different materials. Let's start the game!" Teacher engages the children in a variety of activities based on the skills in the progression (i.e., duplicating, extending, identifying). The teacher creates a unique model pattern for each child (e.g., red-yellow blocks for one child and green-white blocks for another child). The model patterns selected for this activity will vary based on the skills the teacher wants to observe. The teacher can have all children perform the same skill OR give each child individual instructions based on the skills to be observed.

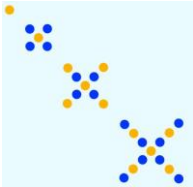
Notes:

- A child can use the same model pattern for multiple skills in the progression.
- To keep children who struggle with patterns engaged, teacher should consider varying the order of the skills rather than presenting them in order from easiest to hardest.
- Teachers do not need to include all of the steps in the progression if some steps have already been observed.

Suggested Group Size: 2-4 children who, based on prior observation and professional judgement, likely have skills within the same Understanding.

Suggested Materials: See Potential Materials for observation above. Additionally, each child should have his/her own unique set of materials. The teacher should have his/her own matching set for each child.

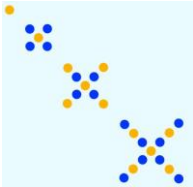
Skill	Example Teacher Instructions*	Examples
A. Duplicates sequential AB patterns with the same materials.	Teacher provides child with a model AB pattern and then says, "Can you copy this pattern?"	*Mrs. Young provides Jenna with an AB pattern made of red and green blocks and says, "Can you copy this pattern?" Jenna selects a red block from a pile of multicolored blocks and places it in front of her. Then, she places a green block next to it. Jenna continues on until she has duplicated the entire pattern.*
B. Duplicates sequential three-element patterns (e.g., ABB, ABC) with the same materials.	Teacher provides child with a model three-element pattern and then says, "Now you make the same pattern."	*Mrs. Young provides Diego with an ABB pattern made of orange and blue blocks and says, "Can you make this same pattern?" Diego selects an orange block from a pile of multicolored blocks and places it in front of him. Then, he places two blue blocks next to it. Diego continues on until he has duplicated the entire pattern.*



Assessment Means Form: Mathematical Patterns

Skill	Example Teacher Instructions*	Examples
C. Extends sequential AB patterns at least one unit.	Teacher provides child with a model AB pattern and says, "What comes next in this pattern?" [Can repeat question until child completes a pattern unit or makes a mistake]	*Mrs. Young uses interlocking blocks to make an AB pattern that varies based on color (red block - blue block). Mrs. Young tells Omar to make the pattern longer. Omar adds six more blocks in the correct order, successfully extending the pattern three units.*
D. Extends sequential three-element patterns (e.g., ABB, ABC) at least one unit.	Teacher provides child with a model three-element pattern and then says, "Can you keep the pattern going?" [Can repeat question until child completes a pattern unit or makes a mistake]	*Mrs. Young gives Ayora a string of beads in an ABB pattern (yellow-black-black) and says, "Make this pattern longer." Ayora adds a yellow bead. Mrs. Young says, "Can you keep going?" Ayora then adds two black beads.*
E. Duplicates sequential patterns (e.g., AB, ABC, AABB) using materials different from those used in the model pattern (pattern abstraction).	Teacher provides child with a model pattern and then says, "Can you make the same kind of pattern using different materials?" [Make sure child uses materials different than those in the model pattern]	*Mrs. Young gives Inez a model ABC pattern (black-white-purple cubes) and tells her to make the pattern using the basket of shells, stones, and leaves. Inez uses the shells for the black cubes, the stones for the white cubes, and the leaves for the purple cubes and creates an ABC pattern underneath the original sequence of cubes.*
F. Identifies repeating unit in sequential patterns.	Teacher provides child with a model pattern and says, "What's the part that repeats over and over again in this pattern?"	*Mrs. Young shows Stella a model ABC pattern using colored chips and says, "What is the group of chips that you see repeating one after the other?" Stella says, "I see yellow-green-blue repeating."*
G. Extends by at least one step or determines the missing step in spatial or numerical growing patterns.	EXTENDING - Teacher provides child with a model growing pattern and says, "What comes next?" DETERMINING A MISSING STEP - Teacher provides child with a model growing pattern with a missing step and says, "There's a missing step here. Can you complete the missing step?"	*EXTENDING - Mrs. Young makes a spatial growing pattern out of squares of construction paper (2, 4, 6, 8) and tells Cecilia, "I made a pattern that grows. Can you make the next step in my pattern?" Cecilia makes the next step by placing ten squares of paper at the end of the pattern.* *DETERMINING A MISSING STEP - Mrs. Young makes a numerical growing pattern and deliberately leaves out the third step (2, 4, space, 8, 10). She asks Masha, "Does this look right to you? Please help me fix my growing pattern." Masha fills in the third step of the pattern with the number six.*
H. Communicates a recursive rule governing the next step in spatial or numerical growing patterns.	Teacher provides child with a model growing pattern and says, "How does this pattern change from one step to the next?"	*Mrs. Young shows Shane a picture of a spatial growing pattern (2 circles, 4 circles, 6 circles) and says, "Tell me how the pattern grows." Shane says, "You add two each time."*
I. Creates or enters data into a t-chart to document the relationship between the ordinal	CHILD-CREATED T-CHART - Teacher provides child with a model growing pattern and says,	*CHILD-CREATED T-CHART - Mrs. Young shows Alice a spatial growing pattern made out of shapes (2 circles, 4 circles, 6 circles). Mrs. Young*

<table>
<tr><th>Skill</th><th>Example Teacher Instructions*</th><th>Examples</th></tr>
<tr><td>position of a step in a growing pattern (i.e., first, second, third) and an important feature of the step.</td><td>"Can you make a t-chart to show how this pattern is changing?"
CHILD GIVEN A BLANK T-CHART - Teacher provides child with a model growing pattern and an empty t-chart, and says, "Here is a t-chart. Can you fill in the t-chart to show how this pattern is changing?"</td><td>says, "Alice, make a t-chart for this pattern." Alice creates the following t-chart:
<table><tr><th>Step</th><th>Circles</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>6</td></tr></table>CHILD GIVEN A BLANK T-CHART - Mrs. Young shows Alice a numerical growing pattern: 10, 20, 30. Mrs. Young gives Alice an empty t-chart and says, "Can you fill in this t-chart to show how this pattern is changing?" Alice enters the following information in the t-chart:
<table><tr><th>Step</th><th>Number</th></tr><tr><td>1</td><td>10</td></tr><tr><td>2</td><td>20</td></tr><tr><td>3</td><td>30</td></tr></table></td></tr>
<tr><td>J. <u>Applies the relationship</u> between the two variables in a <u>t-chart</u> to extend a numerical growing pattern by at least one step.</td><td>Teacher provides child with a partially completed t-chart and says, "What would be in the next row of this t-chart?" A follow-up question may be needed to determine if a child used a functional rule (e.g., "How did you get that number?").</td><td>CHILD-CREATED T-CHART - After Alice creates the t-chart, Mrs. Young says, "What would be in the next row?" Alice writes a 4 at the end of the 1st column (NOTE: writing the "4" is not necessary) and an 8 at the end of the 2nd column (Alice could also verbalize the answer). Mrs. Young then says, "How did you get the number 8?" Alice says, "I multiplied the step by 2."
(NOTE: The follow-up question is asked to determine if the child used a functional rule or a recursive rule.)</td></tr>
<tr><td>K. Communicates a <u>one-operation functional rule</u> governing spatial or numerical growing patterns and uses it to determine a <u>far step</u>. NOTE: a "far step" is more than 5 steps from the last represented step in the pattern.</td><td>Teacher provides child with a model growing pattern governed by a one-operation functional rule and says, "Tell me the rule for finding the value of any step in the pattern." After child describes a correct functional rule for the pattern, teacher says, "How many squares would be in step 12 in the pattern?"</td><td>Mrs. Young presents Ezra with a numerical growing pattern (2, 4, 6) and says, "Tell me the rule for finding the value of any step in the pattern." Ezra says, "Well, the rule is that you multiply the step by 2." Mrs. Young then says, "What number would be at the 10th step in the pattern?" Ezra says, "The number at the tenth step would be 20."</td></tr>
<tr><td>L. Communicates a <u>two-operation functional rule</u> governing spatial</td><td>Teacher provides child with a model growing pattern governed by a two-operation</td><td>Mrs. Young presents Ricky with a spatial growing pattern (3 pentagons, 5 pentagons, 7 pentagons, 9 pentagons) and says, "Tell</td></tr>
</table>

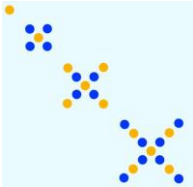


Assessment Means Form: Mathematical Patterns

Skill	Example Teacher Instructions*	Examples
or numerical growing patterns and uses it to determine a far step. NOTE: a "far step" is more than 5 steps from the last represented step in the pattern.	functional rule and says, "Can you tell me how the number of stars relates to their position in the pattern?" After child describes a correct functional rule for the pattern, teacher says, "If I was at step 10 in the pattern, how many stars would there be?"	*me the rule for finding the value of any step in the pattern." Ricky says, "For each step, the pentagons increase by two. First, I should multiply 2 times the step. Then, I have to add the pentagon on the top. So, the rule is 2 times the step plus 1." Mrs. Young then says, "How many pentagons would be in the 14th step in the pattern?" Ricky says, "The 14th step would be 14 times 2 plus 1. That is 29."*
M. Creates an equation that symbolizes a functional rule governing a spatial or numerical growing pattern.	Teacher presents child with a model growing pattern and says, "Write an equation that shows the rule for finding the value of any step in the pattern."	*Mrs. Young presents Rejane with a spatial growing pattern (3 pentagons, 5 pentagons, 7 pentagons, 9 pentagons) along with a completed t-chart and says, "Write an equation that shows the rule for finding the value of any step in the pattern." Rejane writes the following equation: Pentagons = 2 X Step + 1.*

*Note: Variations in language in the example teacher instructions are used to provide teachers with ideas of how activities can be presented.

NOTE: There is no Task for this Construct.



Assessment Means Form: Mathematical Patterns

Patterns Cheat Sheet

Sequential Patterns

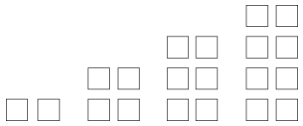
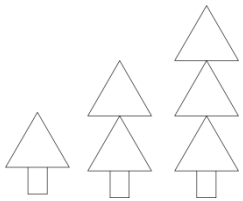
Common examples:

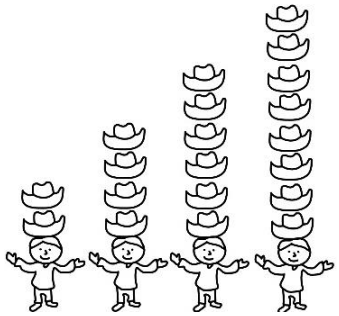
- AB
- ABA
- ABB
- AAB
- ABC
- AABB

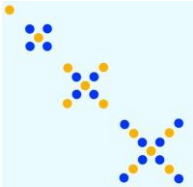
Pattern categories:

- Visual (e.g., color, shape, size, orientation)
- Kinesthetic (i.e., body movements)
- Auditory (i.e., sounds)

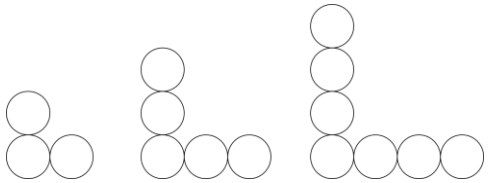
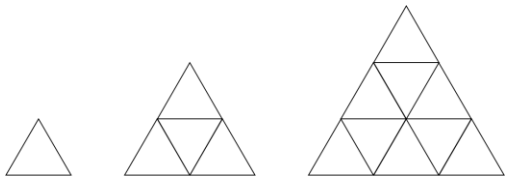
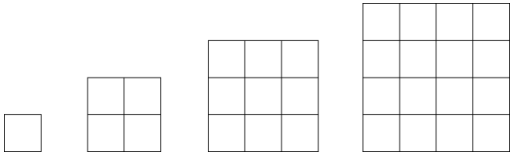
Spatial Growing Patterns

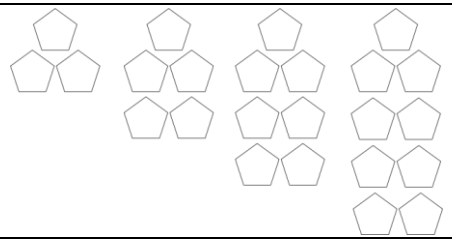
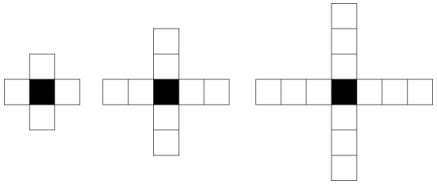
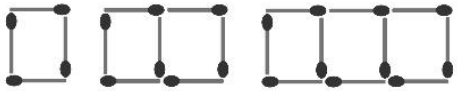
Pattern example	Equation example(s) (not an exhaustive list)
	Circles = Step OR y = x
	Squares = 2 X Step OR Squares = Step + Step OR y = 2x
	Triangles = Step OR Height = Trunk + Step OR Height = 1 + Step

Pattern example	Equation example(s) (not an exhaustive list)
	Hats = 2 X Step OR Hats = Step + Step OR Height = Boy + Step + Step OR Height = 1 + (2 X Step)



Assessment Means Form: Mathematical Patterns

Pattern example	Equation example(s) (not an exhaustive list)
	Circles = Step + Step + 1 OR Circles = 1 + (2 X Step) OR y = 1 + 2x
	Triangles = Step X Step OR Triangles = Step^2 OR $y = x^2$
	Squares = Step X Step OR Squares = Step^2

Pattern example	Equation example(s) (not an exhaustive list)
	Pentagons = 2 X Step + 1 OR y = 2x + 1
	Squares = 4 X Step + 1 OR y = 4x + 1
	Matchsticks = 4 + 3 X (Step – 1) OR y = 4 + 3(x – 1)