Exploring the Evidence for Mathematics

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Alignment of standards, assessments, and materials



CCSS Shifts in Mathematics



Session Objective:



Part 1:

Taking a closer look at the CCSSM



Oregon Common Core State Standards for Mathematics

Grade 4 Overview

Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
- · Gain familiarity with factors and multiples.
- · Generate and analyze patterns.

Number and Operations in Base Ten

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying

Mathematical Practices

- Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

Oregon Common Core State Standards for Mathematics

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

- Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
- Multiply or divide to solve word problems involving multiplicative comparison, e.g., with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.¹
- 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Number and Operations in Base Ten²

4.NBT

4.0A

Generalize place value understanding for multi-digit whole numbers.

 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.



HB 2680 Work Group Report – Exhibit 5c

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Table 1. Progress to Algebra in Grades K-8

| к | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|--|---|---|--|---|--|--|---|
| Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value | Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units | Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length | Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition | Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multi- digit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of noerations Understand decimal notation for fractions, and compare decimal fractions | Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems* | Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables | Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations | Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities* |

*Indicates a cluster that is well thought of as part of a student's progress to algebra, but that is currently not designated as Major by one or both of the assessment consortia in their draft materials. Apart from the two asterisked exceptions, the clusters listed here are a subset of those designated as Major in both of the assessment consortia's draft documents.

Activity 1:

Taking a closer look at the CCSSM

- Take a closer look at the 4th grade standards, and find the following:
 - How many MATH PRACTICES are there in 4th grade?
 - How many DOMAINS are there in 4th grade?
 - What are they?
 - How many CLUSTERS are there in 4th grade?
 - Put a star by clusters identified in the PROGRESS TO ALGEBRA document.
 - What percentage of the 4th grade clusters did you put a star by?

Part 2: Taking a closer look at Smarter Balanced





Claim 1 – Concepts and Procedures

Claim 1 is the only claim that is directly linked to the Common Core

State <u>Content</u> Standards

They are linked by way of the targets (which are cluster headings)



Claims 2, 3 and 4 are directly linked to the Common Core State <u>Standards for Mathematical Practice</u>

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

Claims 2, 3 and 4 are directly linked to the Common Core State <u>Standards for Mathematical Practice</u>

Claim 2 - Problem Solving

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Claim 3 - Communicating Reasoning

Students clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Claim 4 – Modeling and Data Analysis

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.



Part 3: Taking a closer look at FOCUS



Focus: Narrow the scope of content in each grade

- Significantly reduce the range of content in each grade so that so that students can have more time to deeply experience the content that remains.
- Focus deeply on major content emphasized in the standards, so that students gain strong foundations.

Oregon Common Core State Standards for Mathematics

Operations and Algebraic Thinking

4.OA

Use the four operations with whole numbers to solve problems.

- Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
- Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.¹
- 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

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Number and Operations in Base Ten²

4.NBT

Generalize place value understanding for multi-digit whole numbers.

 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.

Activity 2: Label the Grade 4 cluster headings with Smarter **Balanced Claim 1 Assessment Targets**

Oregon Common Core State Standards for Mathematics

4.0A

Operations and Algebraic Thinking Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal

| | | Target Sampling Mathematics Grad | ad 1 th Grade |
|-------------------------------|---------------------|---|--------------------------|
| Claim | Content Category | Assessment Targets | Blueprint |
| | | A. Use the four operations with whole numbers to solve problems. | |
| 1. Concepts and Procedures | | E. Use place value understanding and properties of operations to perform multi-digit arithmetic. | |
| | Priority Cluster | F. Extend understanding of fraction equivalence and ordering. | |
| | | G. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. | |
| | | D. Generalize place value understanding for multi-digit whole numbers. | |
| | | H. Understand decimal notation for fractions, and compare decimal fractions. | |
| | | Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. | |
| | | K. Geometric measurement: understand concepts of angle and measure angles. | |
| | | P. Gain familiarity with factors and multiples | |

Activity 2: Taking a closer look at FOCUS

- Label the Grade 4 cluster headings (CCSSM) with Smarter Balanced Claim 1 Assessment Targets (Blueprint)
- Questions:
 - How much of the 4th grade content is identified?
 - What connection do you see between the blueprints & Progress to Algebra content?
 - What fraction or percent of the Claim 1 CAT test is represented by the Priority Clusters?

HB 2680 Work Group Report – Exhibit 5c

Table 1. Progress to Algebra in Grades K-8

| К | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|--|---|--|--|---|
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Table 1. Progress to Algebra in Grades K-8



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Attending to FOCUS

- Is there evidence that Smarter Balanced attends to the range of content in each grade level?
- Is there evidence that Smarter Balanced attends to the major content emphasized in the standards?

Part 4: Taking a closer look at RIGOR



Rigor: Expect Fluency, Deep Understanding and Application

- Oregon Common Core Standards for Mathematics require a balance of:
 - Solid conceptual understanding
 - Procedural skill and fluency
 - Application of skills in problem solving situations
- Pursuit of all three requires equal intensity in time activities, and resources.

Check for Understanding: Questions about any of the components of RIGOR in the CCSSM?



Closer look at Item Specification Documents

Grade 5 Mathematics Item Specification C1 TI



Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency. Content Domain: Measurement and Data Target I [m]: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. (DOK 1, 2) Tasks for this target will ask students to find the volume of right rectangular prisms with whole-number edge lengths using unit cubes and formulas. Some tasks should ask students to consider the effect of changing the size of the unit cube (e.g., doubling the edge length of a unit cube) using values that do not cause gaps or overlaps when packed into the solid. Other tasks will ask students to find the volume of two non-overlapping right rectangular prisms, often together with targets from Claim 2 or Claim 4. 5.MD.C Geometric measurement: understand concepts of Standards: volume and relate volume to multiplication and to addition. 5.MD.C, 5.MD.C.3, 5.MD.C.3a, 5.MD.C.3b, 5.MD.C.4, 5.MD.C.5, 5.MD.C.3 Recognize volume as an attribute of solid figures and 5.MD.C.5a, 5.MD.C.5b, understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to 5.MD.C.5c have "one cubic unit" of volume, and can be used to measure volume. **b**. A solid figure which can be packed without gaps or overlaps

Closer look at Range Achievement Level Descriptors (ALD) (page 2)

| Achievement LEVEL Descriptors: | | | | |
|--------------------------------|---|--|--|--|
| | | | | |
| RANGE Achievement | Level 1 Students should be able to use unit cubes to find the | | | |
| Level Descriptors | volume of rectangular prisms with whole-number edge lengths. | | | |
| (Range ALD) | Level 2 Students should be able to understand the concept that | | | |
| Target I: | the volume of a rectangular prism packed with unit cubes is | | | |
| Geometric | related to the edge lengths. | | | |
| measurement: | Level 3 Students should be able to use the formulas $V = I \times w \times v$ | | | |
| understand concepts of | h and $V = b \times h$ to find the volume of rectangular prisms. They | | | |
| volume and relate | should be able to find the volume of two non-overlapping right | | | |
| volume to multiplication | rectangular prisms. | | | |
| and addition. | Level 4 Students should be able to find the volume of a right | | | |
| | rectangular prism after doubling the edge length of a side and | | | |
| | compare it to the original. | | | |

Activity 3: Looking for RIGOR



Find copies of Range ALDs for: <u>"Grade 5, Target I"</u>

- Look for evidence of Rigor in
 - wording of the Range ALDs for Claim 1, 2, 3, & 4
 - Within example items given
- Highlight evidence of attending to one of the elements or rigor
 - Pink Conceptual Understanding
 - Yellow Procedural Fluency
 - Blue Application

Attending to RIGOR

- Is there evidence of attending to:
 - Conceptual understanding?
 - Procedural skill and fluency?
 - Application of skills in problem solving situations?

Part 5:

Taking a closer look at COHERNECE



Coherence: Think Across Grades, and Link to Major Topics Within Grades

- Carefully connect content across grades so that students can build new understanding on foundations built in previous years.
- Carefully connect the learning within a given grade so that students can develop a solid foundation of core content that is reinforced by connections across content expectations.

Different ways to visualize coherence

Student achievement coherence map

http://achievethecore.org/coherence-map/



Visualizing Coherence https://www.turnonccmath.net/ Kindergarten Grade 8

Activity 4: Looking for Coherence



Grade 5 Mathematics Item Specification C1 TI





Multiplication and Division Progression

Step 1:

Circle -- Targeted Standards for Grade 3 Claim 1 Target A

Step 2: **Highlight in Blue -- Below Grade Standards** Step 3: Highlight in Yellow -- Above Grade Standards

Step 4:

Are 3.OA.3 and 3.OA.4 assessed via Claims 2, 3, and 4? If so, write the claim number in the hexagon(s).













Attending to Coherence

• Is there evidence in the Smarter Balanced documentation of attending to Coherence?

Attending to Focus, Coherence, Rigor



Alignment to CCSSM



End of Session Activity

- What questions do you have relating to the content of this part of today's presentation?
- What additional evidence would you like to see in order to answer those questions?