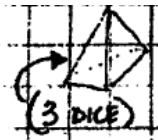
High School - Probability

Tetra Dice

A game requires each player to roll three specially shaped dice. Each die is a regular tetrahedron (four congruent, triangular faces). One face contains the number 1; one face contains the number 2; on another face appears the number 3; the remaining face shows the number 4. After a player rolls, the player records the numbers on the underneath sides of all three dice, and then calculates their sum. You win the game if the sum divides evenly by three. What is the probability of winning this game?



(3 DICE)

SIDES ARE NUMBERED 1, 2, 3, AND 4.
TO WIN, THE SUM HAS TO BE DIVISIBLE BY 3.

IF YOU LAND ON 4:

1st DIE: 4

2nd DIE: 1 2 3 4

3rd: 1234 1234 1234 1234

SUMS: 6789 78910 891011 9 10 11 12

÷3? 1 2 3 4 5 6

6/16 SUMS WIN

IF YOU LAND ON 3:

1st: 3

2nd: 1 2 3 4

3rd: 1234 1234 1234 1234

SUMS: 5678 6789 78910 89 1011

÷3? 1 2 3 4 5

5/16 WINS

IF YOU LAND ON 2:

1st: 2

2nd: 1 2 3 4

3rd: 1234 1234 1234 1234

SUMS: 4567 5678 6789 789 10

÷3? 1 2 3 4 5

5/16 WINS

IF YOU LAND ON 1:

1st: 1

2nd: 1 2 3 4

3rd: 1234 1234 1234 1234

SUMS: 3456 4567 5678 6789

÷3? 1 2 3 4 5 6

6/16 WINS

IF YOU ONLY ROLL 3 TETRA-DICE 1 TIME EACH THE CHANCES OF THE BOTTOM #'S EQUALING A SUM DIVISIBLE BY 3 IS 22/64

16 ← # CHANCES ON EACH TREE
x4 ← # TREES
64 ← SAMPLE SPACE

10 + 12 = 22 WINS

22/64

DICE #1 →	1	2	3	4
2nd 1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

→ DIVISIBLE BY 3

SUMS FROM DICE #3 ↓ 1,2 →	2	3	4	5	3	4	5	6	4	5	6	7	5	6	7	8	
1	1,2 3*	1,3 4	1,4 5	1,5 6*	1,3 4	1,4 5	1,5 6*	1,6 7	1,4 5	1,5 6*	1,6 7	1,7 8	1,5 6*	1,6 7	1,7 8	1,8 9*	(6*)
2	2,2 4	2,3 5	2,4 6*	2,5 7	2,3 5	2,4 6*	2,5 7	2,6 8	2,4 6*	2,5 7	2,6 8	2,7 9*	2,5 7	2,6 8	2,7 9*	2,8 10	(5*)
3	3,2 5	3,3 6*	3,4 7	3,5 8	3,3 6*	3,4 7	3,5 8	3,6 9*	3,4 7	3,5 8	3,6 9*	3,7 10	3,5 8	3,6 9*	3,7 10	3,8 11	(5*)
4	4,2 6*	4,3 7	4,4 8	4,5 9*	4,3 7	4,4 8	4,5 9*	4,6 10	4,4 8	4,5 9*	4,6 10	4,7 11	4,5 9*	4,6 10	4,7 11	4,8 12*	(6*)

AGAIN 6+5+5+6 = 22 OUT OF 64 OUTCOMES 22/64 ← 11/32

H-1-1

Scores and Commentary: Paper #1: Tetra Dice

Making Sense of the Task (MS)	Representing and Solving the Task (RS)	Communicating and Reasoning (CR)	Accuracy (Acc)	Reflecting and Evaluating (RS)
5	6	6	5	5

Making Sense of the Task: The translation of the key concepts (sample space related to the shape and number of dice, sums divisible by three, and theoretical probability) is thoroughly developed in both translations (tree and matrix). Then s/he shows the sample space in three ways (tree, calculations, matrix) and connects the pieces in the "tree" solution to those in the "matrix". This is not a 6 because it is not extended or connected to other mathematical ideas.

Representing and Solving the Task: The process of identifying each of the possible sums when starting with each possible roll of the dice, finding the sums divisible by three, comparing the number of successful sums to the total number of sums is elegant and insightful in the original approach. The reflection provides further evidence of the key concepts and the original representation is strengthened through its comparison to the second representation.

Communicating and Reasoning: The use of mathematical language (sample space "tree", outcomes, sum, divisible) is enhanced by the use of clearly executed graphics which allow the reader to move easily from one thought to another. The insightful way the student chose to break the tree diagram apart further enhances the flow.

Accuracy: 22/64 is a mathematically justifiable solution to the task and is supported by the work. The solution is enhanced by the two slightly different approaches and connection between the sample space and the outcomes to produce a probability.

Reflecting and Evaluating: The student completely justifies the solution by reviewing the interpretation, concepts, strategies, calculations, and reasonableness (starting with the words after the tree diagrams that mention sample space). The reflection is strengthened by reworking the problem using a different recording method.

I need to find the probability of winning a game where you roll 3 dice and you win if the sum of the 3 divides evenly by 3

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

+	1	2	3	4
2	3	4	5	6
3	4	5	6	7
3	4	5	6	7
4	5	6	7	8
4	5	6	7	8
4	5	6	7	8
5	6	7	8	9
5	6	7	8	9
5	6	7	8	9
5	6	7	8	9
6	7	8	9	10
6	7	8	9	10
6	7	8	9	10
7	8	9	10	11
7	8	9	10	11
8	9	10	10	12

/ = not divisible by 3

$\frac{22}{64}$ are divisible by 3

conclusion - you have a $\frac{22}{64}$ chance to win

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

16 #'s from

+	1	2	3	4
2	3	4	5	6
3	4	5	6	7
3	4	5	6	7
4	5	6	7	8
4	5	6	7	8
4	5	6	7	8
5	6	7	8	9
5	6	7	8	9
5	6	7	8	9
5	6	7	8	9
6	7	8	9	10
6	7	8	9	10
6	7	8	9	10
7	8	9	10	11
7	8	9	10	11
8	9	10	11	12

/ = not divisible by 3

$\frac{22}{64}$ are divisible by 3

you have a $\frac{22}{64}$ chance to win this game

H-1-2

Scores and Commentary: Paper #2: Tetra Dice

Making Sense of the Task (MS)	Representing and Solving the Task (RS)	Communicating and Reasoning (CR)	Accuracy (Acc)	Reflecting and Evaluating (RS)
5	4	4	4	4

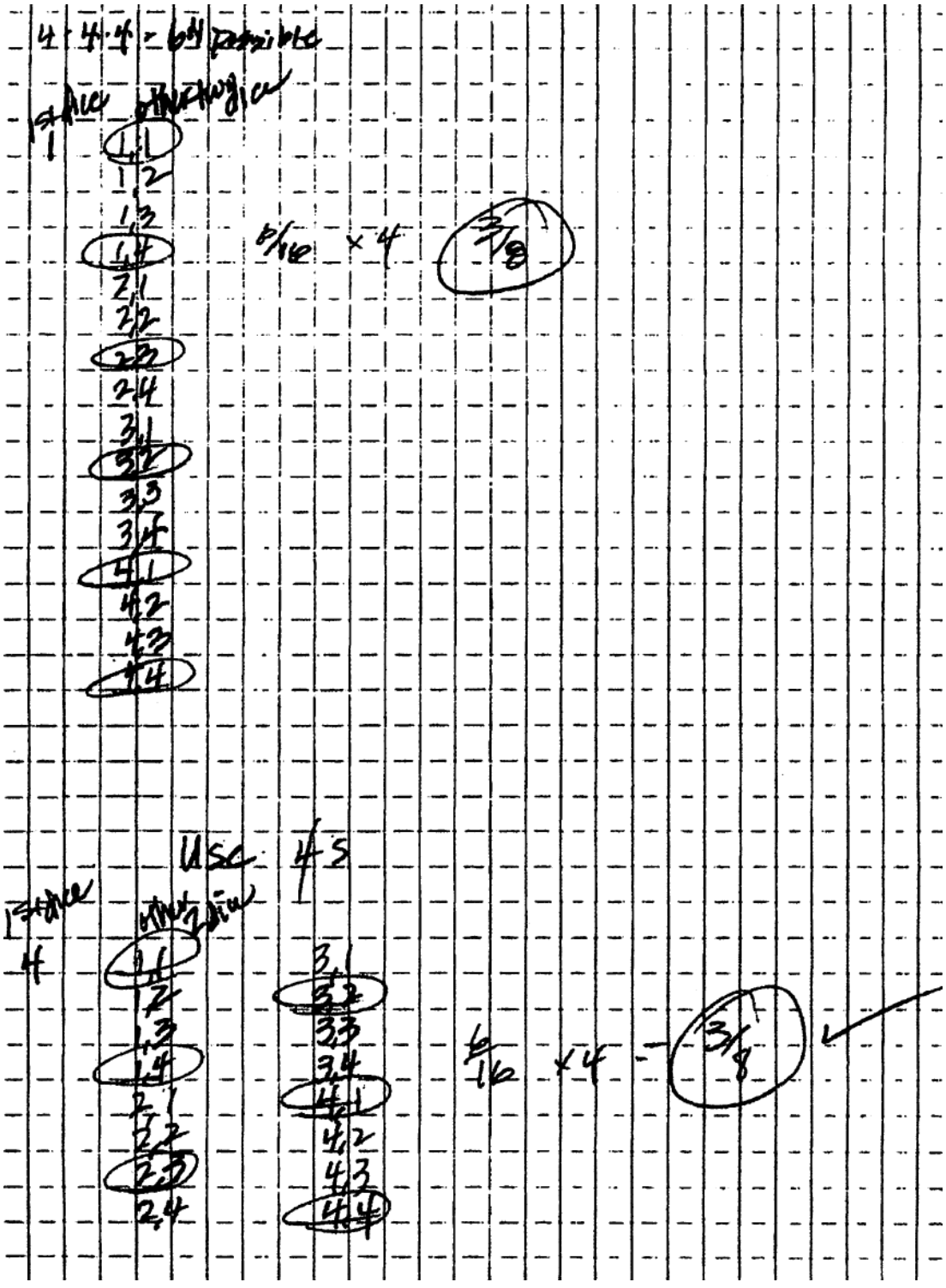
Making Sense of the Task: The interpretation and translation of sample space related to the shape of the dice, sums divisible by three, and theoretical probability is thoroughly developed. The table is systematic and complete and the student connects the outcomes to the sample space to create the probability. The student was also able to take what made sense to them for two dice and extend it to make sense of a problem with three dice.

Representing and Solving the Task: The strategy of creating an organized list of all possible sums making up the sample space is complete. Crossing out those with a sum not divisible by three and finding the number not crossed out and comparing it to the size of the sample space, is effective.

Communicating and Reasoning: The communication follows a clear and coherent path throughout and leads to a clearly identified solution. It is not a 5 because very little mathematical language is used and the reasoning is not enhanced or elegant.

Accuracy: The correct solution is given and is mathematically justified and supported by the work.

Reflecting and Evaluating: The solution is stated within the context of the problem and the reflection (under the line) reviews the interpretation of the problem, concepts, strategies, and calculations. The student defends the process by completely reworking the problem, therefore the review is complete. Enough elements from the "4" are evident to make this a complete review.



H-1-3

Scores and Commentary: Paper #3: Tetra Dice

Making Sense of the Task (MS)	Representing and Solving the Task (RS)	Communicating and Reasoning (CR)	Accuracy (Acc)	Reflecting and Evaluating (RS)
3	3	3	2	2

Making Sense of the Task: The interpretation of the key concepts for the task (sample space related to the shape and number of the dice, sums divisible by three, and theoretical probability) is present It appears that the student correctly interprets the results for a first roll of "1" or "4" (6/16 or 3/8) and incorrectly assumes that the results from a first roll of "2" or "3" will be the same. Therefore, the translation is partially developed.

Representing and Solving the Task: The strategy of assuming a first roll and going through the different permutations of rolls 2 and 3 is partially complete.

Communicating and Reasoning: The reasoning is partially displayed with significant gaps. The sample space of 64 is not connected to the solution and the reader has to infer that they assumed all 4 situations would have the same result (because of the "x4").

Accuracy: The solution is incomplete. If the student had continued with the same strategy for all 4 "first rolls" they may have solved it correctly. As a result, the error is more than minor.

Reflecting and Evaluating: The justification is ineffective because the choice of a first roll of 4 as the defense supported their answer, but using a first roll of 2 or 3 would have exposed the error in the student's thinking.