Grade K – Mathematics Standards and Guidance

K.NCC - Numeric Reasoning: Counting & Cardinality

CLUSTER: K.NCC.A - Know number names and the count sequence.

STANDARD: K.NCC.A.1

Standards Statement (JUNE 2021):

Orally count to 100 by ones and by ten in sequential order.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should count for authentic purposes which connect to their everyday experiences.
- Students should understand that each successive number name refers to a quantity that is one larger.
- When students are rote counting forward, start the count sequence at 1.
- When students are counting backward, start the count sequence beginning at 10 and progress to counting backward beginning at 20.

Boundaries

- When students count by tens, they are only expected to master counting by the decade (10, 20, ...).
- This expectation does not require recognition of numerals.

Teaching Strategies

- Beginning at number zero students can orally count in sequential order without skipping or repeating numbers to 100
- When students count backward from 20, they can use visual resources such as a number line, a 99-chart, or a 100-chart.

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STANDARD: K.NCC.A.2

Standards Statement (JUNE 2021):

Count forward beginning from a given number within 100 of a known sequence.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- This learning objective builds on the skill of naming numbers up to 20 in sequence.
- Know sequence can start at a number of known sequence instead of having to begin at 1.
- Students should count forward and backward from a given number using the known number word sequence for authentic purposes.
- Students should be able to begin and end with any given number.

Boundaries

- The "known sequence" for this standard can be within 20. The intent is to build toward addition and subtraction in first grade.
- This expectation does not require recognition of numerals

Example

- Counting forward can be demonstrated using manipulatives, or oral response. In written form would be beyond the intent of this standard.
- Given the number 54, a student will count "54, 55, 56, 57, 58…"

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STANDARD: K.NCC.A.3

Standards Statement (JUNE 2021):

Identify number names, write numbers, and the count sequence from 0-20. Represent a number of objects with a written number 0-20.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Recognizes numerals and uses counting as part of play and as a means for determining quantity.
- Students should be able to identify written numerals for authentic purposes.
- Students should be able to demonstrate the relationship between written numerals and a number of objects.

Teaching Strategies

- Common errors include transposing or skipping a written number. Example of transposing 13 in writing to 31 Students will know that 0 represents a count of no objects.

Example

- Know number names and the count sequence.
- Write numbers from 0-20 and represent a number of objects with a written number 0-20.

DRAFT

CLUSTER: K.NCC.B - Count to tell the number of objects.

STANDARD: K.NCC.B.4

Standards Statement (JUNE 2021):

Understand the relationship between numbers and quantities; connect counting to cardinality.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Counts at least 10 objects using one-to-one correspondence.
- Students should count objects using one-to-one correspondence saying the number names in the standard order and communicate quantities for authentic purposes. “Authentic purposes” refers to experiences students have in their everyday lives.

Terminology

- When counting objects, explain that the last number counted represents the total quantity in a set (cardinality), regardless of the arrangement and order.

Teaching Strategies

- Understand that objects are counted using 1:1 correspondence in sequential order to determine quantity with last number representing the total objects counted.
- Students should instantly see how many objects are in a group without counting (subitizing)

Examples

- Any type of counter, such as cereal, beads, rocks, and bears, are sample tools that can be used for counting objects.
- Dot cards, five-frames, ten-frames, and rekenreks can be used for subitizing

DRAFT

STANDARD: K.NCC.B.5

Standards Statement (JUNE 2021):

Count to answer "how many?" questions using up to 20 objects arranged in a variety of configurations or as 10 objects in a scattered configuration. Given a number from 1-20, count out that many objects.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Count from 1 to 20 objects in sequential order in a variety of configurations. Configurations can include ten frames, arrays, circles or a line.
- Students should be able answer questions of "how many?" objects using one-to-one correspondence.

Teaching Strategies

- Dot cards, five-frames, ten-frames, rekenreks, dominoes, and playing cards are some tools that can be used for subitizing.

Connections

- Students should be able to count to answer "how many?" questions with up to 20 objects arranged in a variety of ways (a line, a rectangular array, or a circle), or up to 10 objects arranged in a scattered configuration.

DRAFT

CLUSTER: K.NCC.C - Compare numbers.

STANDARD: K.NCC.C.6

Standards Statement (JUNE 2021):

Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Matches two equal sets using one- to-one correspondence and understands they are the same.
- Students should compare the number of objects in two groups in real-life situations and identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.
- Students should be able to explain that equal to is “the same” quantity.

Boundaries

- Group sizes limited to at most 10 objects each..
- The words greater than, less than, or the same as (equal to) should be used instead of the symbols.

Teaching Strategies

- Understand the intent is to use matching and counting strategies to reinforce vocabulary of greater than, less than or equal to

DRAFT

STANDARD: K.NCC.C.7

Standards Statement (JUNE 2021):

Compare two numbers between 1 and 10 presented as written numerals.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students identify numbers that come before and after a given number up to 10.

Boundaries

- Students should have had the experience of comparing the number of physical objects prior to comparing and ordering written numerals.

Example

- Compare numbers in a written format without manipulatives or visuals.
- Given the numerals 4 and 2, students would say, “Two is less than four, and four is greater than two.” Students would also put them in sequential order: “2, 4.”

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K.OA- Algebraic Reasoning: Operations

CLUSTER: K.OA.A - Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

STANDARD: K.OA.A.1

Standards Statement (JUNE 2021):

Represent addition as putting together and adding to and subtraction as taking apart and taking from using objects, drawings, physical expressions, numbers or equations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Practices combining, separating, and naming quantities.
- Uses simple strategies to solve mathematical problems and communicates how he/she solved it.
- Students should be able to represent real-life problems involving the addition and subtraction of whole numbers within 10 with objects and drawings.

Terminology

- Physical expressions can include, but not limited to, sounds (e.g., claps), acting out situations, or other types of physical movement.
- Pictorial drawings need not show details, but should show the mathematics in the problem.

Boundaries

- Exposure to equations is expected but mastery of equations is not required.
- Drawings do not need to show details but should show the mathematics in the problem.
- Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required. However, please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation.

Teaching Strategies

- Representations may include objects, fingers, mental images, drawings, expressions, or equations.
- Student drawings should show the mathematics of the solution from the given context. Equations should be derived from contexts.

Example

- Representation can include objects, fingers, mental images, drawings, sounds, acting out, verbal explanations, expressions or equations. An example of representational sounds can be clapping.
- Note: The student work above shows four different representations of the student's thinking. One with pictures (3 ladybugs + 2 ladybugs) and two equations with numerals (3 + 2 and 2 + 3). The student also represented the problem with words and numbers.

DRAFT

STANDARD: K.AO.A.2

Standards Statement (JUNE 2021):

Add and subtract within 10. Model authentic contexts and solve problems that use addition and subtraction within 10.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Use addition and subtraction within 10 to solve and represent problems in authentic contexts involving situations of adding to, taking from, putting together, and taking apart.
- Practices combining, separating, and naming quantities.
- Uses simple strategies to solve mathematical problems and communicates how he/she solved it.

Terminology

- Students should be provided with a variety of problem types including Join: Result Unknown, Separate: Result Unknown, and Part-Part-Whole: Whole Unknown; however, students are not required to know or use this terminology.
    - Join: Result Unknown
        - Example: 3 birds were sitting in a tree and 2 more birds flew onto the tree. How many birds were in the tree then?
    - Separate: Result Unknown
        - Example: Toni had 8 guppies. She gave 3 guppies to Roger. How many guppies does Toni have now?
    - Part-Part-Whole: Whole Unknown
        - Example: 6 girls and 4 boys were playing soccer. How many children were playing soccer?

Boundaries

- Exposure to equations is expected but mastery of equations is not required in Kindergarten.

Teaching Strategies

- Use objects and drawings to represent the word problem. In order to solve word problems within 10, use numbers 0-9
- Students should be able to solve real-life problems involving the addition and subtraction of single-digit whole numbers, using a variety of strategies such as:
    - counting on
    - counting backward
    - making 10

Example

- Note: The student work above shows four different representations that helped the student solve the problem: One with pictures (3 ladybugs + 2 ladybugs), two with numerals (3 + 2 = 5 and 2 + 3 = 5), and written form. The student also used the commutative property of addition to solve the problem.

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STANDARD: K.AO.A.3

Standards Statement (JUNE 2021):

Using objects or drawings, and equations, decompose numbers less than or equal to 10 into pairs in more than one way.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students practice combining, separating, and naming quantities.

Terminology

- Decomposition is the process of breaking apart a number into a variety of parts that all equal the same whole. Example 9 = 6 +3; 9 = 5 + 4 both equations equal 9.
- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Compose – put together numbers
    - Decompose – break apart numbers

Teaching Strategies

- Use objects or drawings to decompose numbers in at least two different ways. Record each decomposition with a drawing, number bond, or equation.
- Teachers should use dot card images for students to explain how they see different number combinations.

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STANDARD: K.AO.A.4

Standards Statement (JUNE 2021):

By using objects, drawings, or equations, find the unknown number that makes 10 when added to a given number from 1 - 9.

DRAFT Standards Guidance (JUNE 2021):

Example

- By using objects, drawings, or equations, find the unknown number that makes 10 when added to a given number from 1 - 9.

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STANDARD: K.AO.A.5

Standards Statement (JUNE 2021):

Fluently add and subtract within 5 with accurate, efficient, and flexible strategies.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Uses simple strategies to solve mathematical problems and communicates how he/she solved it.
- Students should be able to solve real-life problems involving the addition and subtraction of numbers within five.

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Fluently/Fluency -- To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Boundaries

- Fluency does not lend itself to timed tests or speed.
- Exposure to equations is expected but mastery of equations is not required.

Example

- Record the sum or difference with a drawing oral response, visual cue or equation. Can use an oral response to a verbal or visual cue to demonstrate fluency.
- When making toothpick designs to represent the various combinations of the number “5”, the student writes the numerals for the various parts (such as “4” and “1”) or selects a number sentence that represents that particular situation (such as 5 = 4 + 1)*.

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K.NBT - Numeric Reasoning: Base Ten Arithmetic

CLUSTER: K.NBT.A - Work with numbers 11-19 to gain foundations for place value.

STANDARD: K.NBT.A.1

Standards Statement (JUNE 2021):

Compose and decompose from 11 to 19 into groups of ten ones and some further ones using objects, drawings, or equations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to put together (compose) and break apart (decompose) numbers into a group of ten ones and some further ones to understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
- Students should use strategic thinking in order to communicate quantities for authentic purposes.

Terminology

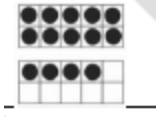
- Composition refers to putting numbers together, such as 10 + 2 = 12
- Decomposition refers to breaking multi-digit numbers apart, which would be into groups of tens and ones at this grade level, such as 14 = 10 + 4.

Teaching Strategies

- To compose and decompose numbers from 11 to 19 into ten, single ones (not "a ten;" this is in first grade) with some more ones by using objects, or drawings to record each composition or decomposition. An example would be a student recognizing that the number 13 is made up of ten single ones and three more. This is specific to teen numbers only.
- Use objects or drawings to explain and record each composition or decomposition with a drawing or equation.
- Students should be given the opportunity to use five frames, ten frames, and rekenreks with support to demonstrate each composition or decomposition.

Example

- Using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
- The teacher can provide students with a variety of tools to make sense of numbers during everyday instruction. One day, a teacher may ask during a Number Talk, "In what ways can you decompose the number 14?".
- Possible student response: "I decomposed 14 in my mind's eye into one full ten frame and four more on another ten frame." The teacher records the student's thoughts as follows:



K.GM - Geometric Reasoning & Measurement

CLUSTER: K.GM.A - Identify and describe shapes.

STANDARD: K.GM.A.1

Standards Statement (JUNE 2021):

Describe objects in the environment using names of shapes and describe the relative positions of these objects in their environment.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students *appropriate directional language to indicate where things are in their environment positions, distances, order.*
- Kindergarten students should be able to explain the location of an object in relation to another object using positional language, such as “above,” “below,” “beside,” “in front of,” “behind,” or “next to.”

Teaching Strategies

- Use positional terms such as above, below, beside, in front of, behind, and next to when describing position of an object. For example, the box is under the chair or the bear is next to the table.

Boundaries

- Students can identify the following two-dimensional shapes based on attributes:
    - *square*
    - *circle*
    - *triangle*
    - *rectangle*
    - *hexagon*
    - *oval*
    - *rhombus*
- Students can identify the following three-dimensional shapes based on attributes:
    - cubes
    - cones
    - cylinders
    - spheres

Examples

- “The cup is beside the pencil."
- “The boy is behind the girl in line.”
- In a sequence of pictures, the student would describe the position of a particular object.

DRAFT

STANDARD: K.GM.A.2

Standards Statement (JUNE 2021):

Correctly name basic two-dimensional and three-dimensional geometric shapes regardless of their orientations or overall size.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should distinguish between defining attributes of two-dimensional shapes and three-dimensional figures versus non-defining attributes (e.g., triangles are closed and three-sided, a defining attribute versus triangles are red, non-defining attribute).
- Students should be able to build and draw shapes based on defining attributes. Two dimensional shapes should be limited to triangles, squares, rectangles.
- Students should be able to identify a shape's attributes, regardless of its orientation (i.e., flipped) or position (i.e., turned).

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Attributes – characteristics of two-dimensional shapes and three-dimensional figures, including geometric properties.
    - Defining attributes – include number of sides, faces, vertices (corners), and angles.
    - Non-defining attributes – include size, orientation, texture, and color.

Boundaries

- Students can identify the following two-dimensional shapes based on attributes:
    - *square*
    - *circle*
    - *triangle*
    - *rectangle*
    - *hexagon*
    - *oval*
    - *rhombus*
- Students can identify the following three-dimensional shapes based on attributes:
    - cubes
    - cones
    - cylinders
    - spheres

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STANDARD: K.GM.A.3

Standards Statement (JUNE 2021):

Identify shapes as two-dimensional or three-dimensional.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students will name flat shapes as two-dimensional or solid shapes as three-dimensional. Example would be a square is flat but a cube is solid.

DRAFT

CLUSTER: K.GM.B - Analyze, compare, create, and compose shapes.

STANDARD: K.GM.B.4

Standards Statement (JUNE 2021):

Analyze and compare two and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts and attributes.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students recognize and name common two-dimensional and three-dimensional shapes, their parts and attributes.
- Students should be able to identify basic shapes, including squares, circles, triangles, rectangles, hexagons, octagons, cubes, cones, cylinders, and spheres.
- Students begin to understand how three-dimensional figures are composed of two-dimensional shapes.

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Attributes – characteristics (i.e., two- dimensional shapes (lying in a plane, "flat") and three-dimensional figures ("solid"), including geometric properties.). An example of an attribute is having sides of equal length.
    - Vertices – corners of a geometric figure

Example

- Example a square has 4 equal sides and 4 corners/vertices and a cube has 8 equal sides with 8 corners/vertices and 6 faces. The intent is not for students to yet have the formal language, but teachers can start to guide the transition from informal to formal mathematical language.
- The base and top of a cylinder is a circle.

DRAFT

STANDARD: K.GM.B.5

Standards Statement (JUNE 2021):

Represent shapes in the world by building shapes from components and drawing shapes.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- A variety of materials can be used to create models of shapes that exist in everyday life.

Examples

- Can build 2 dimensional shapes or 3 dimensional shapes using manipulatives and other components. Example building a house using marshmallows and toothpicks or Legos.
- *Models* – sticks with clay balls, toothpicks with marshmallows, popsicle sticks, technology, etc.

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STANDARD: K.GM.B.6

Standards Statement (JUNE 2021):

Compose simple shapes to form larger shapes.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students manipulate objects and describe the process for fitting objects together.
- Students combine simple shapes to form new shapes.

Teaching Strategies

- Students should be able to form (compose) larger shapes by putting together smaller shapes through exploration and play.

Examples

- What shapes can you create with these two triangles?"
- Use more than one shape to build a larger shape. Example two triangles make a rhombus or two trapezoids to make a hexagon.
- "Use the 7 tangram pieces to make a fox."

DRAFT

CLUSTER: K.GM.C - Describe and compare measurable attributes.

STANDARD: K.GM.C.7

Standards Statement (JUNE 2021):

Describe several measurable attributes of a single object using measurable terms, such as length or weight.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students use a variety of techniques and standard and non-standard tools to measure and compare length, volume (capacity) and weight
- Students independently orders objects using one characteristic and describes the criteria used.
- In Kindergarten, students should use language such as heavier, lighter, longer, taller, shorter, wider, larger, smaller.
- In Kindergarten, students may use a variety of techniques and tools to compare, describe, and order objects. Students may use a referent object being compared as a tool to describe the other object(s).

Terminology

- Measurable attributes can be vocabulary words that describe the length, weight or shape of an object.
- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Attributes – characteristics (i.e., length, height, width, or weight)
    - Referent object – an object used as the standard of comparison

Examples

- Directly compare the heights of two objects and describe one object as taller/shorter.
- A student may describe a shoe as, “The red shoe is heavier than the blue shoe (the blue shoe is the referent in this case)! The red shoe is also longer!”
- “The clock is round or the box is heavy.”

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STANDARD: K.GM.C.8

Standards Statement (JUNE 2021):

Directly compare two objects with a measurable attribute in common, and describe which object has "more" or "less" of the attribute.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students describe sets as having more, less, same as/equal.
- Students can tell numbers that come before and after a given number up to 10.
- Students should be able to understand that each successive number name refers to a quantity that is one larger and the previous number name is one less.

Teaching Strategies

- Use comparative vocabulary to directly compare two objects.

Example

- Example one child is shorter than the other child. Shorter being the identified attribute.

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K.DR – Data Reasoning

CLUSTER: K.DR.A – Pose investigative questions and collect/consider data.

STANDARD: K.DR.A.1

Standards Statement (JUNE 2021):

Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by sorting and counting.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.

Terminology

- Collecting data would refer to student generating data sets, such as counting and recording the frequency of an event.
- Considering data refers to existing data sets given to students by a teacher for consideration.
- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - A statistical investigative question is one that requires data that will vary.

Boundaries

- Limit category counts to be less than or equal to ten.
- At this grade level, more support is needed with formulating statistical questions. Students should be given guidance when developing statistical investigative questions.
- This standard should be taught throughout the year.

Teaching Strategies

- Students should be provided with support strategies for collecting and organizing their data.

Example

- "How did you get to school today?"
- "What is your favorite _____?"

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CLUSTER: K.DR.B - Analyze the data and interpret the results.

STANDARD: K.DR.B.2

Standards Statement (JUNE 2021):

Analyze data sets by counting the number of objects in each category and interpret results by classifying and sorting objects by count.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students sort and classify objects using one or more attributes or relationships.
- Kindergarten students should have experience sorting objects by characteristics such as heavier, lighter, longer, and shorter (compare to benchmark item).

Terminology

- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
- Attributes – characteristics (i.e., length, height, width, or weight)

Boundaries

- Categories should have no more than 10 objects. In later grades, students will represent data in pictographs and bar graphs.
- In Kindergarten, students should be able to use friendly language to explain their data and answer the overall question.
- The category can be determined by teacher or student prior to count.

Teaching strategies

- Students could display their data using objects and pictures.

Connections

- Represent the findings from generated questions using objects and pictures.
- Explain the findings based on the data collected and represented on graphs.
- Opportunity to connect with K.NCC.6 to reinforce comparison of group sizes *[Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.]*

Example

- When given a collection of buttons, the student separates the buttons into different piles based on color. Then, the student counts the number of buttons in each pile. Finally, the student organizes the groups by the quantity in each group: orange buttons (3), green buttons next (4), purple buttons with the green buttons because purple also had (4), blue buttons last

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Grade 1 – Mathematics Standards and Guidance

1.OA - Algebraic Reasoning: Operations

CLUSTER: 1.OA.A - Represent and solve problems involving addition and subtraction.

STANDARD: 1.OA.A.1

Standards Statement (JUNE 2021):

Use addition and subtraction within 20 to solve and represent problems in authentic contexts involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be given opportunities to use mental reasoning to solve problems involving number strings within 20. Click here for a listing of all problem types.
- Students should also solve problem situations with an unknown in all positions.
- Students should be given multiple opportunities to apply strategies developed through number strings to solve contextual problems.

Boundaries

- Students should not be encouraged to use key/clue words because they will not work with subsequent problem types.
- The unknown quantity should be represented in all positions.

Teaching Strategies

- Symbols can be used to represent unknown amounts in equations.
- Students should be provided with learning experiences to develop strategies such as:
  - Advanced Counting; Counting On
  - Making Ten
  - Decomposing a number leading to a ten
  - Using the relationship between addition and subtraction within 20 (knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (6 + 7 is the same as 6 + 6 + 1 = 12 + 1
  - = 13).
  - Counting All 5 + 2 = □. The student counts five counters. The student adds two more. The student counts 1, 2, 3, 4, 5, 6, 7 to get the answer.
  - Counting Back 12 – 3 = □. The student counts twelve counters. The student removes a counter and says 11, removes another counter and says 10, and removes a third counter and says 9. The student knows the answer is 9 since they counted back 3.

Examples

- Represent addition and subtraction word problems using objects, drawings, and equations. Write an addition or subtraction equation with a symbol for the unknown number in different position, such as:
  - 13 + 5 = n, 13 - 5 = n, 13 + n= 18, 18 - n= 13.
- Recognize and represent adding to and putting together situations as addition.

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- Recognize and represent taking from, taking apart, and comparing situations as either subtraction or addition with a missing addend.

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STANDARD: 1.OA.A.2

Standards Statement (JUNE 2021):

Solve problems that call for addition of three whole numbers whose sum is less than or equal to 20 using objects, drawings or equations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should understand subtraction as an unknown-addend problem.
- Students are not expected to know nor use the term inverse.

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Addend – a number that is added to another number in an addition expression or equation. For example, in the expression 5 + 8, 5 and 8 are both addends.
    - An inverse relationship shows the relationship between addition and subtraction where addition can be used to find the quantity of a set after some in the set are removed. For example, 3+2 = 5 is related to 5 - 3 = 2 because of the inverse relationship.

Boundaries

- Problems should be within 20.

Examples

- Solve word problems by using objects, drawings or equations to represent the quantities in the problem.
- Solve word problems with an equation where a symbol stands for the unknown. For example, 5 + 4 + 6 = ___.
- Understand that objects, drawings, and equations are interchangeable representations of a story problem.
- There are 14 birds in the tree. 8 of them flew away. How many birds are left in the tree?
    - The student thinks of 14 – 8 = □ as 8 + □ = 14
- Jenny had 10 pencils and gave some to Eric. Jenny now has 8 pencils. How many pencils did she give to Eric?
    - The student thinks of 10 - □ = 8 as □ + 8 = 10

DRAFT

CLUSTER: 1.OA.B - Understand and apply properties of operations and the relationship between addition and subtraction.

STANDARD: 1.OA.B.3

Standards Statement (JUNE 2021):

Apply properties of operations as strategies to add and subtract.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should solve problem situations with an unknown in all positions.
- Understand that numbers can be added flexibly.

Terminology

- Properties of operations used as strategies include:
    - Commutative property of addition: For example, if 8 + 3 = 11 is known, then 3 + 8 = 11 is also known.
    - Associative property of addition: For example, add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12.
- Addend – any number that is added to another number in an addition expression or equation. For example, in the expression 7 + 3, 7 and 3 are addends.

Boundaries

- Students should not be encouraged to use key/clue words because they will not work with subsequent problem types.
- The unknown quantity should be represented in all positions.
- The terminology above is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.

DRAFT

STANDARD: 1.OA.B.4

Standards Statement (JUNE 2021):

Understand subtraction as an unknown-addend problem.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Restate a subtraction problem as a missing addend problem using the relationship between addition and subtraction.
- Recognize the inverse relationship between subtraction and addition within 20 and use this inverse relationship to solve real-life problems.

Examples

- Subtract 10 – 8 by finding the number that makes 10 when added to 8.
- Understand that subtraction is equivalent to an unknown-addend problem because both ask for the unknown part in a situation where the total and another part are known.

DRAFT

CLUSTER: 1.OA.C - Add and subtract within 20.

STANDARD: 1.OA.C.5

Standards Statement (JUNE 2021):

Relate counting to addition and subtraction.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to relate counting to addition and subtraction by counting all, counting on, and counting back when making sense of contextual addition and subtraction problems within 20.

Teaching Strategies

- Students should understand how addition and subtraction relate by solving situations in context. Students should use strategies to count up, count back, etc., to model this relationship on tools such as ten frames, rekenreks, number lines (predetermined and open), etc.

Connections

- Relate counting on to addition. For example, recognize counting on two after 15 as solving 15+2.
- Relate counting back to subtraction. For example, recognize counting back two from 15 as solving 15-2
- Relate counting between two numbers finds their difference. For example, recognize counting two number between 15 and 17 as solving 17-15.
-

Example

- When students count on 3 from 4, they should write this as 4+3=7.
- When students count on for subtraction, 3 from 7, they should connect this to 7−3=4. Students write "7−3= ?" and think "I count on 3+ ?=7."

DRAFT

STANDARD: 1.OA.C.6

Standards Statement (JUNE 2021):

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10 with accurate, efficient, and flexible strategies.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Fluently/Fluency – To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.
- Accuracy includes attending to precision.
- Efficiency includes using well-understood strategy with ease.
- Flexibility involves using strategies such as making 5 or making 10.

Boundaries

- Fluency does not lend itself to timed tests or speed.

Example

- Use strategies such as counting on; making ten, for example 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14; decomposing a number leading to a ten for example, 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9;
- Use the relationship between addition and subtraction, for example, knowing that 8 + 4 = 12, one knows 12 – 8 = 4;
- Create equivalent but easier or known sums, for example, adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13.

DRAFT

CLUSTER: 1.OA.D - Work with addition and subtraction equations.

STANDARD: 1.OA.D.7

Standards Statement (JUNE 2021):

Use the meaning of the equal sign to determine whether equations involving addition and subtraction are true or false.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should explore and explain the relationship of the equal sign to quantities and orally justify if equations involving addition and subtraction are "true" (equal) or "false" (not equal).

Teaching Strategies

- Use the meaning of the equal sign ("is the same as") to determine if two expressions involving a whole number and/or addition or subtraction expressions are equivalent. In other words,

Example

- Determine if the equation is true or false, for example determining that 3-1 = 2+3 is false because the expressions do not have equal values.
- Which of the following equations are true and which are false? How do you know?
  - 6 = 6 (True/Correct Statement)
  - 7 = 8 – 1 (True/Correct Statement)
  - 5 + 2 = 2 + 5 (True/Correct Statement)
  - 4 + 1 = 5 + 2 (False/Incorrect Statement)

DRAFT

STANDARD: 1.OA.D.8

Standards Statement (JUNE 2021):

Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Determine the unknown whole number relating three whole numbers, with the unknown in any position.

Teaching Strategies

- Symbols can be used to represent unknown amounts in equations.

Example

- Students should be given the opportunity to find missing part given a known part and total, such as:
    - A missing addend in an addition equation, for example 3+_=5.
    - A missing subtrahend in a subtraction equation, for example 5-_=2.
    - A missing difference in a subtraction equation, for example 5-3=_
    - Students should be given the opportunity to find missing totals given known parts, such as:
    - A missing sum in an addition equation, for example 3+2=_.
    - A missing minuend in a subtraction equation, for example _-2=3.
- Determine the unknown number that makes the equation true in each of the equations: 8 + ? = 10, 5 = □ – 3, 3 + 4 = Δ. These are some possible ways to record equations that indicate an unknown number.

DRAFT

1.NBT - Numeric Reasoning: Base Ten Arithmetic

CLUSTER: 1.NBT.A - Extend the counting sequence.

STANDARD: 1.NBT.A.1

Standards Statement (JUNE 2021):

Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should understand that as the counting sequence increases, the value of each number increases by one or ten. As the counting sequence decreases, the value of each number decreases by one or ten.
- Count forward and backward, starting at any number less than 120.
- In this range, read and write numerals and represent a number of objects with a written numeral.

Teaching Strategies

- Students should count forwards and backwards by 1s and 10s from any number within 120.
- Skip count by twos to 20, by fives to 100, and by tens to 120.
- Students should have opportunities to explore the counting sequences using a variety of tools. These tools can include, but are not limited to 99 charts, hundred charts, number paths, number lines (predetermined and open), etc.

Example

- Understand that two digit numbers are composed of tens and ones.
- Understand that 3 digit numbers are composed of hundreds, tens, and ones.
- Understand that numbers increase in consistent patterns because of the place value system.

DRAFT

CLUSTER: 1.NBT.B - Understand place value.

STANDARD: 1.NBT.B.2

Standards Statement (JUNE 2021):

Understand 10 as a bundle of ten ones and that the two digits of a two-digit number represent amounts of tens and ones.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to recognize the relationship of a digit to its place indicates the number of groups represented in that place. For example: In the number 33, the digit "3" in the tens place has a value that is equivalent to three groups of ten. Students interpret the value of each digit. The number 33 has three tens and three remaining ones. They should also see this as equivalent to 33 ones.
- Students should understand the following as special cases:
    - 10 can be thought of as a bundle of ten ones — called a "ten." Bundles could include groups of pennies, bundles of straws, or other hands-on manipulatives.
    - The numbers from 11 to 19 are composed or decomposed as a ten and one, two, three, four, five, six, seven, eight, or nine ones.

Boundaries

- Students should be able to explain that the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

Teaching Strategies

- The numbers 11 to 19 can be represented on ten frames, double ten frames, rekenreks, and with pennies and dimes, etc.
- The numbers 10, 20, 30, 40, 50, 60, 70, 80, and 90, can be represented using a variety of tools (popsicle sticks, linking cubes, straws, etc.)

Example

- Understand the following as special cases:
    - 10 can be thought of as a bundle of ten ones — called a "ten."
    - The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
    - The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

DRAFT

STANDARD: 1.NBT.B.3

Standards Statement (JUNE 2021):

Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should understand whole numbers to 100 based on meanings of the tens and ones and record the results of comparisons with the symbols >, =, and <.

Boundaries

- Students should have ample experiences communicating their comparisons using words, models AND context before using only symbols in the learning objective.
- Students need practice justifying comparisons with words and models, prior to exposure and use of the comparison symbols.

Teaching Strategies

- Representations should include the use of physical materials such as number paths, base-ten materials, number lines (predetermined and open), dimes and pennies, etc.

Examples

- Students should be given the opportunity to provide explanations of their results based on their understanding of place value.
    - -2 tens + 9 ones < 3 tens + 2 ones
    - -2 tens and 9 ones < 92
- Understand that a greater value in a given place supersedes any amount in a place with a smaller value.

DRAFT

CLUSTER: 1.NBT.C - Use place value understanding and properties of operations to add and subtract.

STANDARD: 1.NBT.C.4

Standards Statement (JUNE 2021):

Add within 100 using concrete using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain why sometimes it is necessary to compose a ten.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Problems provided should be contextual, mathematical problems.
- Students should be able to interpret and manipulate concrete mathematical models.
- Students should be given opportunities to justify their solutions to meet this learning objective.
- Students should use estimation as a strategy to find numbers that are close to the numbers they are using to add and subtract.
- Students should be able to use numerical reasoning to add and subtract within 100.
- The numerical reasoning developed should include an understanding of the base-ten structure and properties of operations.
- Students should reason that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to put together (compose) or break apart (decompose) a ten.

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Compose – put together numbers
    - Decompose – break apart numbers
    - Estimate – find a value that is close

Boundaries

- The properties of operation that should be explored in this objective are the commutative and associative properties. Students are not expected to identify properties.

Teaching Strategies

- Students should use concrete models, drawings, estimation, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction to explain their reasoning.
- Strategies may include reasoning involving making a ten, doubles and near-doubles, think addition, and using benchmark numbers.
- Examples of different strategies and representations can be found within the Computational Strategies for Whole Numbers document found in the appendices.

Examples

- Adding a two-digit number and a one-digit number adding a two-digit number and a multiple of 10 adding two two-digit numbers.

DRAFT

STANDARD: 1.NBT.C.5

Standards Statement (JUNE 2021):

Without having to count, mentally find 10 more or 10 less than a given two-digit number and explain the reasoning used.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- This expectation requires students to apply this mental strategy and become fluent through purposeful practice. The goal is automaticity built on a deep understanding of the patterns of tens within our base-ten system.

Example

- Find 10 more than a given two digit number, for example 34 + 10.
- Find 10 less than a given two digit number, for example 34-10.
- Understand that adding or subtracting multiples of 10 from a number changes only the tens digit because the addition or subtraction changes only the quantity of tens.
- There were 74 birds in the park. 10 of the birds flew away. How many birds are in the park, now?
    - I pictured 7 ten-frames and 4 left over in my head. Since 10 birds flew away, I took one of the ten-frames away. That left 6 ten-frames and 4 left over. So, there are 64 birds left in the park.

DRAFT

STANDARD: 1.NBT.C.6

Standards Statement (JUNE 2021):

Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy and model used to a written method and explain the reasoning used.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should use concrete models; drawings, and strategies based on place value, properties of operations, and or/the relationship between addition and subtraction to explain their reasoning.
- Students should describe sums and differences, using concrete models (tools and manipulatives), drawings, and strategies based on place value, properties of operations and/or the relationship between addition and subtraction to explain (verbally and/or written) the reasoning used.

Boundaries

- By the end of first grade, students should be able to state and write their justifications showing the relationship between their solution path and their reasoning. The focus of this standard is on thought processes, not merely on computational accuracy.
- (positive or zero differences)

Example

- Represent subtraction of multiples of 10 with concrete and/or visual models based on place value. For example, represent 30 as 3 groups of ten and no ones.
- Understand that the inverse relationship between subtraction and addition exists because both are different representations of the same part-part-whole relationship. For example, understand that both 20+30=50 and 50-20=30 represent the same parts and whole.

DRAFT

1.GM - Geometric Reasoning & Measurement

CLUSTER: 1.GM.A - Reason with shapes and their attributes.

STANDARD: 1.GM.A.1

Standards Statement (JUNE 2021):

Distinguish between defining attributes versus non-defining attributes for a wide variety of shapes. Build and draw shapes to possess defining attributes.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Attributes – characteristics of two-dimensional shapes and three-dimensional figures, including geometric properties.
    - Defining attributes – include number of sides, faces, vertices (corners), and angles.
    - Non-defining attributes – include size, orientation, texture, and color.

Clarifications

- Students should identify these two-dimensional shapes based on attributes:
    - half circles
    - quarter circles
    - circles
    - triangles
    - squares
    - rectangles (Students should know that a square is a type of rectangle, based on its attributes.)
    - hexagons
- Students should identify these three-dimensional shapes based on attributes:
    - cubes
    - cones
    - cylinders
    - spheres
    - rectangular prisms
- Students should distinguish between defining attributes of two-dimensional shapes and three-dimensional figures versus non-defining attributes (e.g., triangles are closed and three-sided, a defining attribute versus triangles are red, non-defining attribute).
- Students should be able to build and draw shapes based on defining attributes. Two dimensional shapes should be limited to triangles, squares, rectangles.
- Students should be able to identify a shape's attributes, regardless of its orientation (i.e., flipped) or position (i.e., turned).

Example

- For example, defining attributes are sides, angles, and faces (triangles are closed and three-sided). For example, non-defining attributes color, orientation, and overall size.

DRAFT

STANDARD: 1.GM.A.2

Standards Statement (JUNE 2021):

Compose common two-dimensional shapes or three-dimensional shapes to create a composite shape, and create additional new shapes from composite shapes.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- It is important to note that the size of the shape does not necessary distinguish between common and composite.

Terminology

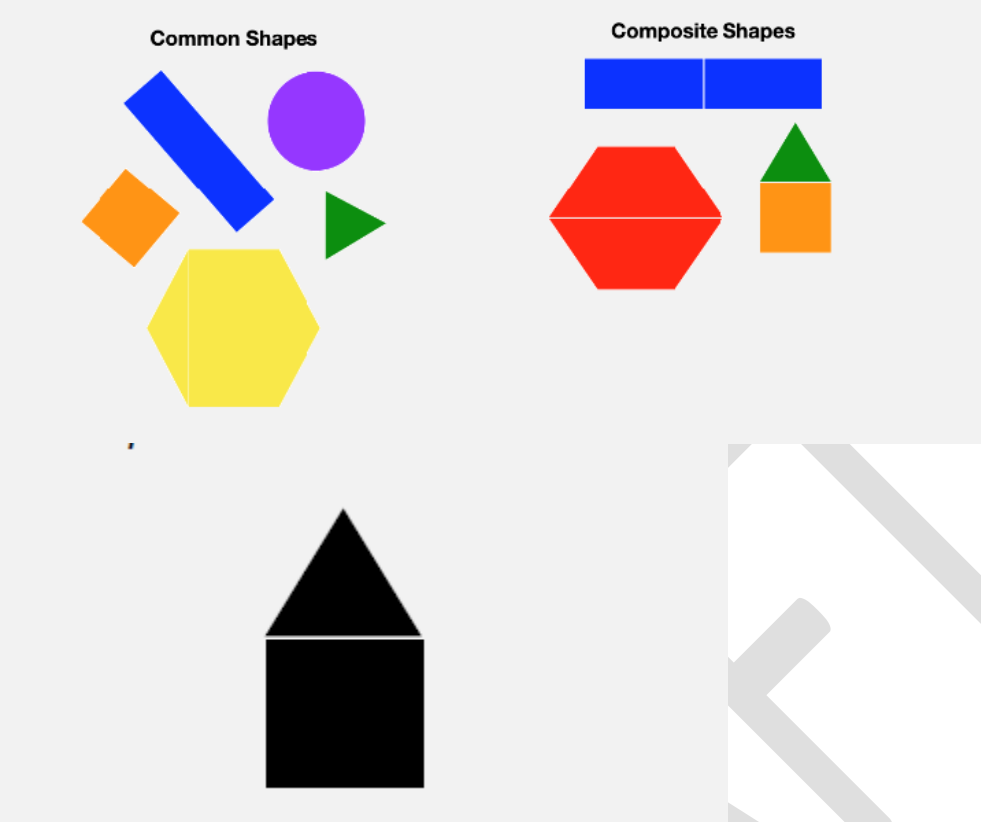
- Shapes that are made up of two or more common shapes are called composite shapes.
- Students will be working with shapes to compose and decompose shapes to form new shapes.
    - Compose – put together
    - Decompose – break apart

Boundaries

- Students should use these common two-dimensional shapes to create composite shapes:
    - circles
    - half-circles
    - quarter-circles
    - triangles
    - squares
    - rectangles (Students should know that a square is a type of rectangle, based on its attributes.)
    - hexagons
    - trapezoids
- Students should use these common three-dimensional shapes to create composite shapes:
    - cubes
    - cones
    - cylinders
    - spheres
    - rectangular prisms
    - right circular cones
    - right circular cylinders
- Students do not need to learn formal names, such as, “right rectangular prism”.

DRAFT

Examples



(Students may compose a pentagon using a triangle and square as above.)

DRAFT

STANDARD: 1.GM.A.3

Standards Statement (JUNE 2021):

Partition circles and rectangles into two and four equal shares. Describe the equal shares and understand that partitioning into more equal shares creates smaller shares.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should explore and justify reasoning about the relationship of parts to the whole.
- Students should describe the shares using the words "halves," "fourths or quarters."
- Students should describe the whole as "two of" or "four of" the shares.
- Students should reason that partitioning a shape into more equal shares creates smaller shares.

Boundaries

- No shading of the shares is needed for this learning objective because the student is only required to partition the whole shape into equal shares.
- Students are not expected to write the fraction using fraction notation in first grade.

Examples

- Describe the equal shares created using the words halves, fourths, and quarters.
- Relate the equal shares to the whole using the phrases half of, fourth of, and quarter of.
- Describe the whole as two of, or four of the shares.
- Understand that halves and fourths are equal parts of a partitioned whole.

DRAFT

CLUSTER: 1.GM.B - Measure lengths indirectly and by iterating length units.

STANDARD: 1.GM.B.4

Standards Statement (JUNE 2021):

Order three objects by length; compare the lengths of two objects indirectly by using a third object.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end, by using non- standard units.
- Students should explore this concept with objects found in the real world to develop solid measurement reasoning.
- Students should explore this concept with objects.
-

Terminology

- Length measurement of an object is the number of same- sized length units that span an object with no gaps or overlaps (iteration).
- Iteration –the process of repeating a unit length end to end along an object to obtain a measurement.

Boundaries

- Students should learn through exploration that the length measurement of an object is the number of same-sized length units that span it with no gaps or overlaps (iteration). For example, when students are measuring the height of a vegetable plant in their classroom garden, they may use snap cubes put together to determine how tall the plant is.

Teaching Strategies

- Students should use terminology such as, but not limited to, “longer than”, “shorter than”, “same length as”, “taller than”, and “equal to”.
- Appropriate tools to measure non-standard units can be items such as one-inch paper clips, one-inch tiles, centimeter cubes, etc. The units need to correspond to standard units of measurement.

Example

- Determine when an object is longer or shorter than another object.
- Compare two objects to a third and use those comparisons against the third object to compare the two objects.
- Students at an elementary school are maintaining an aquaponics garden. To measure the heights of the plants growing in their garden, they use snap cubes to determine how many cubes high the plant have grown.

DRAFT

STANDARD: 1.GM.B.5

Standards Statement (JUNE 2021):

Express the length of an object as a whole number of non-standard length units, by laying multiple copies of a shorter object (the length unit) end to end.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Estimate, measure, and record lengths of objects using non-standard units, and compare and order up to three objects using the recorded measurements.
- Use a shorter object to measure the length of a longer object.
- Record the length of an object as the total number of shorter objects it takes to span the longer object without gaps or overlaps.

Boundaries

- Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.
- Include use of standard units such as inch-tiles or centimeter tiles.

DRAFT

CLUSTER: 1.GM.C - Tell and write time.

STANDARD: 1.GM.C.6

Standards Statement (JUNE 2021):

Tell and write time in hours and half-hours using analog and digital clocks.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- The familiarity of the number line provides students with an opportunity to make sense of the concept of elapsed time. The connection to the traditional clock can be made by bending the clock number line into a circle.

Boundaries

- Students should tell and write time to the hour and half hour in everyday settings, paying attention to a.m. and p.m.
- Problems presented to students should avoid crossing over a.m. and p.m.
- Students are not required to know the term elapsed time at this grade level.

Teaching Strategies

- Begin with a one-handed clock (just the hour hand) and use a lot of approximate language such as:
    - "It's close to 10:00."
    - "It's half-way between 11:00 and 12:00."
    - "It's just a little after 1:00."
    - Video showing how to use a number line to tell time and how the number line can be curved to look like a circular clock – Click Here.

Examples

- Tell time in hours and half hours using an analog clock.
- Tell time in hours and half hours using a digital clock.
- Write time in hours and half-hours.
- At 3:00 PM we are going to the trampoline park. We will be there for 4 hours. What time will we be leaving the trampoline park? Represent this on a number line.
- It will be 7:00 when we leave the trampoline park.

DRAFT

1.DR – Data Reasoning

CLUSTER: 1.DR.A - Pose investigative questions and collect/consider data.

STANDARD: 1.DR.A.1

Standards Statement (JUNE 2021):

Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by organizing data with visual representations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.

Terminology

- Collecting data would refer to student generating data sets, such as counting and recording the frequency of an event.
- Considering data refers to existing data sets given to students by a teacher for consideration.
- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - A statistical investigative question is one that requires data that will vary.

Boundaries

- Determine strategies for collecting and organizing data within 20 to answer a statistical investigative question.
- This standard should be taught throughout the year.

Teaching Strategies

- Students should use tally marks and numerical values within 20 to organize and represent the data.
- Developing strategies for collecting data include students collaborating to determine ways to collect data.
- Data can be gathered from a variety of sources to answer the statistical investigative question posed.

Connections

- Students should formulate a statistical investigative question to explore a real-life situation in their classroom.

Example

- “How many pets do you have?” is a statistical investigative question because it anticipates variability in students’ responses.

DRAFT

CLUSTER: 1.DR.B – Analyze, represent, and interpret data.

STANDARD: 1.DR.B.2

Standards Statement (JUNE 2021):

Analyze data sets with up to three categories by organizing data with visual representations, and interpret information presented to answer investigative questions.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to organize the data collected, represent the data on a table, and ask questions about the data generated."
- Understand that the sum of the data points in all categories is the total number of data points.
- Understand that data representations make data points easier to read, count, and compare.
- Understand that the number of data points in different categories can be compared using subtraction, counting on, or counting back between the quantities.

Boundaries

- This expectation is limited to data with up to three categories presented in tables and charts.
- Students should be using tally marks and numerical values to organize and represent data.
- Students should be able to summarize the number of tally marks in each category.

Teaching Strategies

- Create a picture graph and a bar graph (with single-unit scale) to represent a data set with up to three categories. Analyze the information by asking and answering questions about the data.
- Interpret categorical data to answer the statistical investigative question created, including total number of data points, how many in each category, and how many more or less are in one category than another.

Connections

- Students should be able to organize the data collected, represent the data on a table, and ask questions about the data generated.
- Students should be able to analyze and interpret categorical data on a provided pictograph or bar graph to answer the formulated statistical investigative question.

Example:

- On a picture graph, one symbol stands for a value of 1 at this grade level.

DRAFT

Grade 2 – Mathematics Standards and Guidance

2.OA - Algebraic Reasoning: Operations

CLUSTER: 2.OA.A - Represent and solve problems involving addition and subtraction.

STANDARD: 2.OA.A.1

Standards Statement (JUNE 2021):

Use addition and subtraction within 100 to solve one- and two-step word problems in authentic contexts by using drawings and equations with a symbol for the unknown.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Students should work with contextual, mathematical problems involving standard units of linear measurement (inches). Note: This is an ongoing process that will take much of the year.
- The sum of the numbers should be no greater than 1000.

Teaching Strategies

- Opportunities to engage with problem types should include adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.
- Equations should use a symbol for the unknown number to represent the problem. (See Problem Type Table in Glossary)
- Students should be given the opportunity to explore and develop a variety of flexible strategies and algorithms.
- Students should be able to solve one and two step mathematical problems within 100 and represent the problem by using concrete materials, drawings, and equations with a symbol for the unknown number.
- Students should be able to use strategies that are based on a deep understanding of place-value in order to meet this expectation.
- When solving problems, students should be given the opportunity to use concrete materials, drawings, tools, and part-whole reasoning strategies.
- Students should be able to solve contextual, mathematical problems involving the addition of up to four two-digit numbers using strategies based on place value, properties of operations and the relationship between addition and subtraction.

Connections

- Problems should be presented through contexts to provide students with the opportunity to make sense of the mathematics.
- Problems presented may include money as a context.
- Students should be able to solve problems presented in contextual, mathematical situations using addition and subtraction within 1,000.
- Students should be able to use numerical reasoning to solve contextual, mathematical problems involving all problem types.

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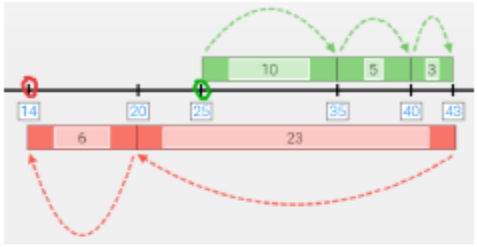
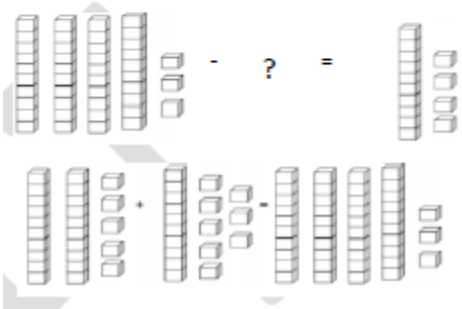
Example

- In the morning, there are 25 students in the cafeteria. 18 more students come in. After a few minutes, some students leave. If there are 14 students still in the cafeteria, how many students left the cafeteria? Write an equation for your problem.





DRAFT

CLUSTER: 2.OA.B - Add and subtract within 20.

STANDARD: 2.OA.B.2

Standards Statement (JUNE 2021):

Fluently add and subtract within 20 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Fluently/Fluency – To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.
- Accuracy includes attending to precision.
- Efficiency includes using well-understood strategy with ease.
- Flexibility involves using strategies such as making 5 or making 10.

Boundaries

- Students bring mental strategies and fluency within 10 from first grade to build towards fluency to 20.
- This standard does not require timed assessments.
- Ample opportunity to develop efficient, accurate, and flexible understanding is essential for operating with larger numbers.

Teaching Strategies

- Students add and subtract within 20 using a variety of mental, part-whole strategies.
- Students should explain their approaches and produce accurate answers efficiently and appropriately using mental strategies that include counting on, making ten, decomposing a number leading to a ten, using the relationship between addition and subtraction, creating equivalent but easier or known sums.

DRAFT

CLUSTER: 2.OA.C - Work with equal groups of objects to gain foundations for multiplication.

STANDARD: 2.OA.C.3

Standards Statement (JUNE 2021):

Determine whether a group up to 20 objects has an odd or even number by pairing objects or counting them by 2s; record using drawings and equations including expressing an even number as a sum of two equal addends.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Addend – any number that is added to another number in an addition expression or equation. For example, in the expression 16 + 4, 16 and 4 are addends.

Teaching Strategies

- Students should explore strategies such as pairing objects, counting by 2s, and drawing arrays to express doubles.
- Students should write an equation to express an even number as a sum of equal addends and as a sum of repeated pairings
- Students can group by pairing objects or counting them by 2s.
- Students may also use doubles to determine if a quantity is even. For example, 18 is even because adding two nines equals 18 or 9 + 9 =18.

Examples

- 12 is even because 6+6= 12 and also because 2+2+2+2+2+2=12 so that 12 is being represented as two groups of six or six groups of two.

DRAFT

STANDARD: 2.OA.C.4

Standards Statement (JUNE 2021):

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- The terms below are used to clarify Expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Rectangular array – an arrangement of objects into rows and columns that form a rectangle
    - Addend – any number that is added to another number in an addition expression or equation. For example, in the expression 2 + 7 + 5, 2, 7 and 3 are addends.

Boundaries

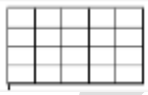
- The intent of the standard is to provide students the opportunities to work with arrays and connect them to repeated addition and equal groupings as a foundation to multiplication.

Teaching Strategies

- Students should model using rectangular arrays to determine the number of objects and discuss their reasoning.

Example

- Students should have the opportunity to recognize that the total in the array is the same whether adding by row or adding by column. For example, an array with 5 rows and 4 columns could be represented as 5+5+5+5 and 4+4+4+4+4 and results in the same total of 20.
- Beth put 5 purses on each shelf. She has 4 shelves. Draw an array to model this. Write an equation to match the array.



5+5+5+5 =20

DRAFT

2.NBT - Numeric Reasoning: Base Ten Arithmetic

CLUSTER: 2.NBT.A - Understand place value.

STANDARD: 2.NBT.A.1

Standards Statement (JUNE 2021):

Understand 100 as a bundle of ten tens and that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to put together (compose) and break apart (decompose) three-digit numbers.
- Students should have multiple opportunities use concrete materials to develop an understanding of the place value structures, the relationship between numbers, and the value of quantities.

Teaching Strategies

- Students should be given the opportunity to discover base-ten units can be broken down and built back up in different ways. For example, understand the number 706 can be represented as:
  - 7 hundreds, 0 tens, and 6 ones where a 0 is used as a placeholder.
  - 70 tens and 6 ones.
  - 706 ones.
- Students should be able to explain that a bundle of ten 10s is equal to 100.

Example

- The number 241 can be expressed as 2 $hundreds$ + 4 $tens$ + 1 or as 24 $tens$ + 1 one or as 241 $ones$.

DRAFT

STANDARD: 2.NBT.A.2

Standards Statement (JUNE 2021):

Count within 1000; skip-count by 5's, 10's, and 100's.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students need to be provided the opportunity to count and skip count both forward and backward starting from any number within 1000 to notice patterns within the number system.
- Students should explore patterns on a hundred-chart, starting from a given number 10-90.
- Students should be able to use coins to count, including nickels, dimes, quarters, and dollars. Half-dollars may also be used, if available.

DRAFT

STANDARD: 2.NBT.A.3

Standards Statement (JUNE 2021):

Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Students should be able to represent a quantity from word form.

Teaching Strategies

- Representations should include concrete materials (i.e., base ten blocks, counters, etc.), base ten numerals, words, expanded form, and pictures.

Example

- The number 706 in base-ten numerals is represented as 7 hundreds, 0 tens, and 6 ones, in number names is represented as "seven hundred six" and in expanded form is represented as 700 + 6.
- The number two-hundred forty-one written in standard form is 241 and in expanded form is 200+40+1.

DRAFT

STANDARD: 2.NBT.A.4

Standards Statement (JUNE 2021):

Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Tools such as a hundred chart and visual number lines may be used to help students compare three digit numbers.

Example

- Students should be given the opportunity to provide explanations of their results based on their understanding of place value, for example:
    - 2 hundreds + 3 ones > 5 tens + 9 ones
    - 9 tens + 2 hundreds + 4 ones < 924
    - 456 < 5 hundreds

DRAFT

CLUSTER: 2.NBT.B - Use place value understanding and properties of operations to add and subtract.

STANDARD: 2.NBT.B.5

Standards Statement (JUNE 2021):

Fluently add & subtract within 100 using accurate, efficient, & flexible strategies base on place value, properties of operations, and/or the relationship between addition and subtraction.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Fluently/Fluency – To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Boundaries

- Students should be given multiple opportunities to solve contextual, mathematical problems as they work to build fluency.
- The sum of the number should be no greater than 100.

Connections

- Students should be able to use numerical reasoning to solve contextual, mathematical problems involving all problem types.

Example

- Students should move from count all toward strategies that are efficient, accurate, and flexible based on the math situation presented. For example:
    - 56+48=50+40+6+8=90+14=104
    - 56+48=54+2+48=54+50=104
    - 56-48 can be thought of as 48+x=56

DRAFT

STANDARD: 2.NBT.B.6

Standards Statement (JUNE 2021):

Add up to four two-digit numbers using strategies based on place value and properties of operations and describe how two different strategies result in the same sum.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should investigate repeating patterns to make predictions and build algebraic reasoning.
- Patterns may include exposure to even and odd.
- Students should be using any tools available such as a number line, hundred-chart, 99-chart, etc., to create and analyze the patterns.
- Patterns should be extended from 1st grade, where they explore intervals of 1s, 2s, 5s, and 10s, to also include intervals of 25s and 100s.

Boundaries

- Patterns involving addition and subtraction should include sums within 1,000 through models and representations.

Teaching Strategies

- Students should be given the opportunity to use a variety of strategies to identify, describe, and create numerical patterns.
- Students describe how two different strategies result in the same sum

Connections

- Problems should be presented through contexts to provide students with the opportunity to make sense of the mathematics.
- Problems presented may include money as a context.

Example

- Students should be given the opportunity to connect representations. For example:
    - -42 + 31 + 12 + 83 may be decomposed into tens and ones to add 40 + 30 + 10 + 80 and then 2 + 1 + 2 + 3.
    - -42+31= 73 and 12+83= 95 so 73+95= 168.
- Start with 3 and jump by 5s to create a pattern. Change the start number and create another pattern. What do you notice about the two patterns? How did they change?

DRAFT

STANDARD: 2.NBT.B.7

Standards Statement (JUNE 2021):

Add and subtract within 1000 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain why sometimes it is necessary to compose or decompose tens or hundreds.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should be encouraged to use place value language such as hundreds, tens and ones, when connecting their representation to their explanation.
- Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Example:

- Students may use equations to represent their strategies based on place value such as: 324+515=(300+500)+(20+10)+(4+5)=839.

DRAFT

STANDARD: 2.NBT.B.8

Standards Statement (JUNE 2021):

Without having to count, mentally find 10 more or 10 less and 100 more or 100 less than a given three-digit number.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Mental addition and subtraction is limited to adding or subtracting by 10 or 100 for numbers between 100-900.

Teaching Strategies

- Add and subtract within 1000 using properties of operations and/or the relationship between addition and subtraction, including mentally adding or subtracting 10 or 100 to a given number;
- Relate the strategies used to a written method.
- Tools such as a hundred chart and visual number lines may be used to help students discover the patterns of ten more and ten less.

DRAFT

STANDARD: 2.NBT.B.9

Standards Statement (JUNE 2021):

Explain why strategies to add and subtract work using properties of operations and the relationship between addition and subtraction.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Explanations may be supported by drawings or objects.

DRAFT

2.GM - Geometric Reasoning & Measurement

CLUSTER: 2.GM.A - Relate addition and subtraction to length.

STANDARD: 2.GM.A.1

Standards Statement (JUNE 2021):

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
- Sizes are compared directly or visually, not compared by measuring.

Terminology

- Attributes – characteristics of a two-dimensional or three-dimensional shape
- Vertices – corners of a geometric figure

Connections

- Students should be able to use spatial reasoning to analyze shapes in the environment.

Example

- Describe a shape based on its attributes and compare and sort a collection of shapes based on the number of angles, vertices, sides, and equal faces.

DRAFT

STANDARD: 2.GM.A.2

Standards Statement (JUNE 2021):

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- As a foundation for multiplication and meaning of area, students should draw and build these arrays.

Boundaries

- Rectangles should be no larger than 5 rows and 5 columns.

DRAFT

STANDARD: 2.GM.A.3

Standards Statement (JUNE 2021):

Partition circles and rectangles into two, three, or four equal parts. Recognize that equal parts of identical wholes need not have the same shape.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students have explored quarters and halves in first grade and are extending their understanding of fractions to thirds.
- As a foundation of the meaning of fractions, students should describe the shares using the words halves, thirds, fourths, half of, a third of, a fourth of and describe the whole as two halves, three thirds, four fourths.

Boundaries

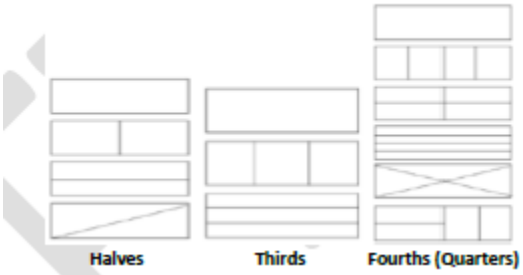
- Partitioning shapes prepares students to reason about fractions in upper grades.
- No shading should occur within images for this grade because the student is only required to partition the whole shape into equal shares.

Teaching Strategies

- Students are not expected to precisely partition circles into thirds, but rather partition circles and rectangles into thirds close enough to be described as three equal parts.

Examples

Examples



Below is a student work sample showing a second grade student's two attempts at partitioning a circle into thirds during a mini lesson. As she is making sense of what happens when you partition a circle into thirds, she realizes that each part represents the same quantity and is one third of the whole circle (approximate partitions are sufficient for beginning phases of understanding development related to quantity):

CLUSTER: 2.GM.B- Measure and estimate lengths in standard units.

STANDARD: 2.GM.B.4

Standards Statement (JUNE 2021):

Measure the length of an object by selecting and using appropriate measurement tools.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Appropriate standardized measurement tools include rulers, yardsticks, meter sticks, and measuring tapes.
- Students should determine which measuring tool is appropriate for a given object.
- Units of measure include inches, feet, and yards

DRAFT

STANDARD: 2.GM.B.5

Standards Statement (JUNE 2021):

Measure the length of an object using two different length units and describe how the measurements relate to the size of the unit chosen.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- In Grade 1, students used one-inch items as non-standard units of measure for length. In Grade 2, students compare constructed ruler with standard rulers and compare the use of the devices.

Teaching Strategies

- Students may use objects but tools from different systems of measurement, such as inches and centimeters, will allow them to begin to compare these systems.
- Students should discuss how measurement with iterating individual one-inch units, such as one-inch tiles, compares with measurement using an instrument such as a standard ruler.

DRAFT

STANDARD: 2.GM.B.6

Standards Statement (JUNE 2021):

Estimate lengths using units of inches, feet, yards, centimeters, and meters.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should be encouraged to use real world objects and body benchmarks for estimations.

DRAFT

STANDARD: 2.GM.B.7

Standards Statement (JUNE 2021):

Measure two objects and determine the difference in their lengths in terms of a standard length unit.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- This is the first time students are introduced to a standard-length unit such as an inch.

Teaching Strategies

- Students should use tools such as rulers, measuring tapes, and yardsticks to obtain measurements.
- Comparisons in length are recorded in standard length units such as inches, feet or yards, as well as metric length units such as meters, centimeters, or millimeters.

Example

- I measured my two pet parakeets. One was 7 inches long and one was 15 inches long. The larger one is 8 inches longer than the smaller one.

DRAFT

CLUSTER: 2.GM.C - Relate addition and subtraction to length.

STANDARD: 2.GM.C.8

Standards Statement (JUNE 2021):

Use addition and subtraction within 100 to solve word problems in authentic contexts involving lengths that are given in the same units.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should represent the problem using drawings and equations with a symbol for the unknown number.

DRAFT

STANDARD: 2.GM.C.9

Standards Statement (JUNE 2021):

Represent whole number lengths on a number line diagram; use number lines to find sums and differences within 100.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to represent sums and differences presented in contextual, mathematical problems on a number line diagram.

Boundaries

- This prepares students to use number lines for fractions in higher grades

Teaching Strategies

- Students should understand length as the distance on a number line where equally spaced points correspond to the numbers 0, 1, 2 and so on.
- The length of an object is the amount of space on this diagram.
- Students will use a number line to show how to move up and down the number system while representing sums and difference (100-28 means you would jump down 20 and 8 to land on 72).

DRAFT

CLUSTER: 2.GM.D - Work with time and money.

STANDARD: 2.GM.D.10

Standards Statement (JUNE 2021):

Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to categorize daily activities by a.m. and p.m.

Boundaries

- Problems involving elapsed time in second grade should be written so as to avoid crossing over a.m. and p.m.

Teaching Strategies

- Video showing how to use a number line to tell time and how the number line can be curved to look like a circular clock – Click Here.

Example

- Denise had soccer practice after school today. Practice began at 3:30 and ended at 6:00. How much time did she spend at soccer practice?

DRAFT

STANDARD: 2.GM.D.11

Standards Statement (JUNE 2021):

Solve word problems in authentic contexts involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and c (cents) symbols appropriately.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to identify the values of pennies, nickels, dimes, and quarters. Half-dollars may also be investigated, if available.

Boundaries

- This is the first time students are required to find the value of a group of coins.
- The total quantity should be based on cents and the value of a group of coins should be less than 100 cents.
- Use of written decimal numbers is not an expectation for this grade level.
- The $ symbol should only be used when referring to whole dollar amounts at this grade level.
- Students should be able to solve contextual, mathematical problems that either have only dollars or only cents.
- Dollar bills may include $1, $5, $10, $20, and $100.

Teaching Strategies

- Students should be given opportunities to explore this concept using hands-on manipulatives. Virtual manipulatives may also be used.

Example

- If you have 2 dimes and 3 pennies, how many cents do you have?
- If you have $3 and 4 quarters, how many dollars or cents do you have?

DRAFT

2.DR – Data Reasoning

CLUSTER: 2.DR.A - Pose investigative questions and collect/consider data.

STANDARD: 2.DR.A.1

Standards Statement (JUNE 2021):

Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by using measurements with whole-number units.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Students should formulate a statistical investigative question to explore a real-life situation in their classroom.
- Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object.
- Data could be organized and recorded on a line plot (dot plot) where the horizontal scale is marked off in whole-number units.

Terminology

- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - A statistical investigative question is one that requires data that will vary.

Boundaries

- The data collection can occur through the use of surveys and scientific observations.
- Tables and tally marks can be used to organize data.
- Developing strategies for collecting data include students collaborating to determine ways to collect data.
- Data can be gathered from a variety of sources to answer the statistical investigative question posed.

Teaching Strategies

- Students should display data set with up to four categories and solve problems that put-together, take-apart, and compare the information presented in the graph.

Connections

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.

DRAFT

CLUSTER: 2.DR.B – Analyze, represent, and interpret data.

STANDARD: 2.DR.B.2

Standards Statement (JUNE 2021):

Analyze data with a single-unit scale and interpret information presented to answer investigative questions.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Investigative question identified can include those created by students or posed by the teacher.

Terminology

- Single unit scale -each unit represents one data point
- Pictograph – uses a symbol to represent the items being graphed. A pictograph has one picture represented in each category and has a key to give a numerical value to each picture.
- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Categorical data take on values that are names or labels, such as colors, foods, and types of pets
    - Pictograph – uses a symbol to represent the items being graphed. A pictograph has one picture represented in each category and has a key to give a numerical value to each picture.

Boundaries

- Pictographs and bar graphs used at this grade level should represent a data set with no more than four categories.

Teaching Strategies

- Students should solve simple join, separate, and compare problems using information presented.
- Students should use addition and subtraction to create and obtain information from tables, pictographs, bar graphs, and tally charts.

Example

- Create a picture graph and a bar graph (with single- unit scale) to represent a data set with up to four categories.

DRAFT

Grade 3 – Mathematics Standards and Guidance

3.OA - Algebraic Reasoning: Operations

CLUSTER: 3.OA.A - Represent and solve problems involving multiplication and division.

STANDARD: 3.OA.A.1

Standards Statement (JUNE 2021):

Represent and interpret multiplication of two factors as repeated addition of equal groups.

DRAFT Standards Guidance (JUNE 2021):

Boundaries:

- Interpret the factors as representing the number of equal groups and the number of objects in each group. Describe a context in which a total number of objects can be expressed as __ x __.
- This standard does not include calculating products. It is about understanding the meaning of each of the factors in 5 x 7, not the product of 5 x 7.

DRAFT

STANDARD: 3.OA.A.2

Standards Statement (JUNE 2021):

Represent and interpret whole-number quotients as dividing an amount into equal sized groups.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should solve multiplication problems including single-digit factors and division problems including single- digit divisors and quotients.

Terminology

- This standard focuses on two models of division: partition models and measurement (repeated subtraction) models.
    - Partition models focus on "How many in each equal-sized group?"
    - Measurement (repeated subtraction) models focus on "How many groups can you make?".
- This standard does not include calculating. It is about understanding the meaning of what does 56 ÷ 8 mean, not the quotient of what does 56 ÷ 8 equal.

Boundaries

- Students should be able to use numerical reasoning to learn multiplication and division facts through playing games and solving contextual, mathematical problems.
- Fluency does not lend itself to timed tests or speed. Students should be given opportunities to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.
- Fluency can be assessed in different ways.

Teaching Strategies

- Multiplication strategies may include repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line and skip counting. Multiplication tables may be used to help students discover patterns and relationships.
- Division strategies may include repeated subtraction, equal sharing, and forming equal groups.
- Examples of different strategies and representations can be found within the Computational Strategies for Whole Numbers document found in the appendices.

Connections

- Students will have an opportunity to use modeling and repeated reasoning to show conceptual understanding of partition and measurement models. (MP 4, 8)

Examples

- Interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each

DRAFT

STANDARD: 3.OA.A.3

Standards Statement (JUNE 2021):

Use multiplication and division within 100 to solve problems in authentic contexts involving equal groups, arrays, and/or measurement quantities.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to solve practical, real-life division problems including "how many in each group" and "how many groups" using efficient and flexible strategies.

Boundaries

- Solve multiplication word problems with factors up to and including 10.
- Solve division word problems with a divisor and quotient up to and including 10.
- These properties should be used organically within the daily number sense routines.
- Students at this grade level are not expected to identify the specific properties.
- Third grade students do not need to know the formal names for these properties.
- Multiplication and division within 100 means multiplication and division of two whole numbers with whole number answers, and with product or dividend in the range 0-100 (e.g., 39 ÷ 3 = 13).

Teaching Strategies

- Students should use a variety of representations for creating and solving one-step word problems, including using drawings and equations with a symbol for the unknown number.
- Some problems should include reading bar graphs, pictographs, and dot plots, as well as measurements in grams, kilograms, liters. Dot plots and line plots can be used interchangeably.

Examples

- 7 x 3 is known, then 3 x 7 is also known (Commutative Property)
- 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or 5 x 2 = 10, then 3 x 10 = 30 (Associative Property)
- Knowing 8 x 5 = 40 and 8 x 2 = 16, 8 x 7 can be found as the sum of these partial products: 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive Property)
- The store had video games on sale for $15 each. If you bought 4 games, how much would you spend?

DRAFT

STANDARD: 3.OA.A.4

Standards Statement (JUNE 2021):

Determine the unknown number in a multiplication or division equation by applying the understanding of the inverse relationship of multiplication and division.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- The focus of 3.OA.4 goes beyond the traditional notion of fact families by having students explore the inverse relationship of multiplication and division.

Examples

- Determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \_\_ \div 3$, $6 \times 6 = ?$.

DRAFT

CLUSTER: 3.OA.B - Understand properties of multiplication and the relationship between multiplication and division.

STANDARD: 3.OA.B.5

Standards Statement (JUNE 2021):

Apply properties of operations as strategies to multiply and divide.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Students need not use formal terms for these properties.

Examples

- If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.)
- If 3 × 5 × 2 can be found by 3 × 5 = 15 then 15 × 2 = 30, or by 5 × 2 = 10 then 3 × 10 = 30. (Associative property of multiplication.)
- Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

DRAFT

STANDARD: 3.OA.B.6

Standards Statement (JUNE 2021):

Understand division as an unknown-factor in a multiplication problem.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Solve an unknown factor problem, by using division strategies or changing the division problem to an equivalent multiplication problem.
- Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems.

Examples

- Divide 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. (8 x ? = 32)

DRAFT

CLUSTER: 3.OA.C - Multiply and divide within 100.

STANDARD: 3.OA.C.7

Standards Statement (JUNE 2021):

Fluently multiply and divide within 100 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- This standard uses the word fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property).
- Fluently/Fluency – To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Connections

- Students should be able to use numerical reasoning to solve contextual, mathematical problems involving all problem types. Click here for a listing of all problem types.

Teaching Strategies

- Ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9 x 9).
- Some problems should include reading bar graphs, pictographs, and dot plots. Some problems should involve grams, kilograms, and liters. Dot plots and line plots can be used interchangeably.
- Strategies may be based on place value, properties of operations, and/or the relationship between addition and subtraction.

Boundaries

- By the end of Grade 3, know from memory all products of one-digit numbers. “Know from memory” should not focus only on timed tests and repetitive practice.
- This standard does not require timed assessments. Ample opportunity to develop efficient, accurate, and flexible strategies is essential.
- Students should be allowed to choose an appropriate strategy to demonstrate fluency.
- Finding and using key words is not an appropriate strategy.

DRAFT

CLUSTER: 3.OA.D - Solve problems involving the four operations, and identify and explain patterns in arithmetic.

STANDARD: 3.OA.D.8

Standards Statement (JUNE 2021):

Solve two-step problems in authentic contexts that use addition, subtraction, multiplication, and division in equations with a letter standing for the unknown quantity.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should use the four operations to solve contextual, mathematical problems.
- Students should represent problems using equations with a variable standing for the unknown quantity and justify their answers.

Boundaries

- This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
- This is limited to problems posed with whole numbers and having whole-number answers. Situations involving money should not include decimal numbers.

Teaching Strategies

- Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- Some problems should include reading bar graphs, pictographs, and dot plots, as well as measurements in grams, kilograms, liters. Dot plots and line plots can be used interchangeably.
- Students should solve multi-step problems.
- Represent these problems using equations with a letter standing for the unknown quantity.
- Students should use numerical reasoning to assess the reasonableness of answers.

Example

- At the movies, tickets cost $11 each, popcorn costs $7 each, and drinks costs $4 each. If I have $25, do I have enough to purchase 1 ticket, 1 popcorn, and 2 drinks?

DRAFT

STANDARD: 3.OA.D.9

Standards Statement (JUNE 2021):

Identify and explain arithmetic patterns using properties of operations, including patterns in the addition table or multiplication table.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Identifying patterns can help students derive and automatize multiplication facts.
- Multiplication tables may be used to help students discover patterns and relationships.
- A student looking at a multiplication table may discover that multiples of even numbers (2, 4, 6, and 8) are always even; the products in each row and column increase by the same amount (skip counting); the multiples of 6 are double the multiples of 3; the multiples of any number fall on a horizontal and a vertical line due to the commutative property, etc.
- Patterns may include exposure to even and odd extending from previous work in 2nd grade.

Teaching Strategies

- Opportunities for students to examine numerical patterns.
- The ability to recognize and explain patterns in mathematics leads students to developing the ability to make generalizations, a foundational concept in algebraic thinking.

Example

- Students investigate multiplication tables in search of patterns and explain why these patterns make sense mathematically.
    - The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
    - The doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
    - On a multiplication chart, the products in each row and column increase by the same amount (skip counting).
    - The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
    - All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.
- Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.
- A student highlighting the multiples of 9 on a hundreds chart might notice 2 x 9 is 2 away from 20, 3 x 9 is 3 away from 30, and so forth.
- Arithmetic patterns are patterns that change by the same rate, such as adding the same number the series 2,4,6,8,10 is an arithmetic pattern that increases by 2 between each term.

DRAFT

3.NBT - Numeric Reasoning: Base Ten Arithmetic

CLUSTER: 3.NBT.A - Use place value understanding and properties of operations to perform multi-digit arithmetic.

STANDARD: 3.NBT.A.1

Standards Statement (JUNE 2021):

Use place value understanding to round whole numbers within 1000 to the nearest 10 or 100.

DRAFT Standards Guidance (JUNE 2021):

Connections

- Students should be able to use place value understanding to round whole numbers for an authentic purpose within contextual situations.

Teaching Strategies

- Students should locate numbers on a number line to determine the nearest multiple of 10 or 100.

Clarifications

- Students should be given opportunities to build understanding by exploring the concept within 100 first and then progressing to applying the same mathematical thinking within 1000.

Example

- On a road trip, there is a gas station at the 700-mile mark and the 800-mile mark. You have about 50 miles left in the tank when you hit the 765-mile mark, which gas station is the closest for you to go to?

DRAFT

STANDARD: 3.NBT.A.2

Standards Statement (JUNE 2021):

Fluently add and subtract within 1000 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should add and subtract multi-digit whole numbers within 10,000 to solve contextual, mathematical problems using efficient and generalizable procedures, based on knowledge of place value and properties of operations.

Teaching Strategies

- Students will have opportunities to use strategies based on place value and properties of operations.
- This standard uses the word fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies).
- This standard does not require timed assessments. Ample opportunity to develop efficient, accurate, and flexible strategies is essential.
- Students should be given opportunities to use variety of models and representations when extending their understanding of part-whole reasoning strategies.
- Students should be given the choice of which strategy they can use.

Examples

- Students will use estimation strategies to assess reasonableness of answers.
    - Use the relationship between addition and subtraction can be applied to solve addition and subtraction problems.
    - Use expanded form to decompose numbers and then find sums and differences

DRAFT

STANDARD: 3.NBT.A.3

Standards Statement (JUNE 2021):

Find the product of one-digit whole numbers by multiples of 10 in the range 10-90, such as 9 x 80. Students use a range of strategies and algorithms based on place value and properties of operations.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Students should be given an opportunity to explore that when a number is 10 times larger than another number, this does not come from adding zero.
- Students should understand that adding zero does not change the overall quantity.
- Students should explore the patterns of multiplying by ten and notice how the magnitude of the number changes. Exploring the pattern, students should uncover as numbers are multiplied by a multiple of 10, the digit shifts left, making the value ten times more with each shift.

Example

- 6 times 20 is 120 because 6 groups of 20 is 120; or 6 x 20 = 6 x (10 x 2) = (6 x 10) x 2 = 60 x 2 = 120.

Teaching Strategies

- Students extend their work in multiplication by applying understanding of place value. The special role of 10 in the base-ten system is important in understanding multiplication of one-digit numbers with multiples of 10.
- Using the properties of operations (commutative, associative, and distributive) and place value, students are able to explain their reasoning.
- Use concrete and pictorial models, based on place value and the properties of operations, to find the product of a one-digit whole number by a multiple of 10 in the range 10–90.

DRAFT

3.NF - Numeric Reasoning: Fractions

CLUSTER: 3.NF.A - Develop understanding of fractions as numbers.

STANDARD: 3.NF.A.1

Standards Statement (JUNE 2021):

Understand the concept of a unit fraction and explain how multiple copies of a unit fraction form a non-unit fraction.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Understand a unit fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts;
- Understand a fraction a/b as the quantity formed by a parts of size 1/b.

Boundaries

- Grade 3 expectations are limited to denominators of 2, 3, 4, 6, and 8 as quantities formed when a whole is partitioned into equal parts.

Teaching Strategies

- Students should investigate unit fractions using area models, parts of a set, linear models, and points on a number line.
- Students should be given the opportunity to explore this concept using a variety of visual tools such as Cuisenaire rods, fraction tiles, fraction strips, fraction bars, fraction towers, number lines, etc.
- Students should determine that two fractions are equal when they are the same size or on the same location on a number line.
- Students should express whole numbers as fractions recognize fractions that are equivalent to whole numbers.

Connections

- Students should explore the relationship between halves, fourths, and eighths, as well as thirds and sixths to generate simple equivalent fractions.
- Represent and identify unit fractions using visual models.

Example

- If there are six equal parts, one of those parts is ⅙. The unit fraction is 1/6.
- Understand that 3/4 is composed of three pieces, each with a size of 1/4

DRAFT

STANDARD: 3.NF.A.2

Standards Statement (JUNE 2021):

Understand a fraction as a number on the number line; Represent fractions on a number line diagram.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Grade 3 expectations are limited to fractions with denominators 2, 3, 4, 6, and 8.
- Set sizes should not exceed 24.

Teaching Strategies

- Students should investigate unit fractions using area models, set models (parts of a set), linear models, and points representing distances on a number line.
- Students should be given the opportunity to explore this concept using a variety of visual tools such as Cuisenaire rods, fraction tiles, fraction strips, fraction bars, fraction towers, number lines, etc.

Connections

- Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts.
- Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
- Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0.
- Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

Example

- There are 6 keys in Stephanie's collection. She gives two of them to her friend. What fraction of her collection did she give?
    - Possible Solution:
    - She gave 1/3 of her collection.

DRAFT

STANDARD: 3.NF.A.3

Standards Statement (JUNE 2021):

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Equivalence of fractions in special cases include:
    - Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
    - Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3),
    - Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
    - Compare two fractions with the same numerator or the same denominator, by reasoning about their size

Teaching Strategies

- Student should have opportunity to :
- Represent equivalent fractions with visual models.
    - Compose and decompose fractions into equivalent fractions using related fractions: halves, fourths and eighths; thirds and sixths.
    - Explain that a fraction with the same numerator and denominator equals one whole.
    - Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
    - Recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.
- Students should be able to use numerical reasoning strategies when comparing unit fractions.
- Tools and strategies could include visual fraction models.
- Students should record the results of comparisons with symbols >, =, or <, and justify the conclusions.

Clarifications

- Students should be able to recognize that comparisons are valid only when the two fractions refer to the same whole.

Examples

- Express 3 in the form 3 = 3/1; recognize that 6/1 = 6;
- locate 4/4 and 1 at the same point of a number line diagram.
- 1/2=2/4, 4/6=2/3

DRAFT

3.GM - Geometric Reasoning & Measurement

CLUSTER: 3.GM.A - Reason with shapes and their attributes.

STANDARD: 3.GM.A.1

Standards Statement (JUNE 2021):

Understand that shapes in different categories may share attributes and that shared attributes can define a larger category.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).
- Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- There should be a focus on the investigation of quadrilaterals, specifically, but other polygons should also be explored.

Examples

- Compare and classify shapes by their sides and angles.
- Recognize rhombi, rectangles, squares, and trapezoids as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

DRAFT

STANDARD: 3.GM.A.2

Standards Statement (JUNE 2021):

Partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.

DRAFT Standards Guidance (JUNE 2021):

Example

- This could include partitioning a shape into 4 parts with equal area and describe each part as 1/4 of the area of the total shape.
- Draw lines to separate a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

DRAFT

CLUSTER: 3.GM.B- Solve problems involving measurement and estimation.

STANDARD: 3.GM.B.3

Standards Statement (JUNE 2021):

Tell, write, and measure time to the nearest minute. Solve problems in authentic contexts that involve addition and subtraction of time intervals in minutes.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be given opportunities to determine relative time and predict time to the nearest fifteen minutes using only the hour hand of an analog clock.

Boundaries

- Students may use tools such as clocks, number line diagrams, and tables to solve problems involving time intervals.

Teaching Strategies

- Students will have opportunities to representing the problems in different ways, including using a number line diagram.
- Problems should include am/pm, start unknown, end unknown, and change unknown and addition/subtraction of time intervals.
- Students should be given opportunities to use number lines to find unknowns.

Examples

- The bus comes at 7:00 a.m. It takes me 15 minutes to eat breakfast and 30 minutes to get ready. What time do I need to wake up? (e.g., start unknown)
- I went to the movies at 3:15 p.m. The movie lasted 1 hour 45 minutes. What time did the movie end? (e.g., end unknown)
- After school I went to the park at 2:30 p.m. and left to go home at 3:45 p.m. How long was I at the park? (e.g., change unknown)

DRAFT

STANDARD: 3.GM.B.4

Standards Statement (JUNE 2021):

Measure, estimate and solve problems in authentic contexts that involve liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should have an opportunity to compare capacity by filling one container with something and then pouring this amount into the comparison container.
- Students may use drawings (such as a beaker with a measurement scale) to represent the problem. This standard does not include conversions between units.
- The focus is on measuring and reasonable estimates, use benchmarks to measure weight, and capacity.

Terminology

- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Customary measurement units include weight (oz., lbs., tons) capacity (fl. oz, cups, pints, quarts, gallons), length (in., ft., yds., miles).

Boundaries

- Excludes compound units such as cm^3 and finding the geometric volume of a container.
- Excludes multiplicative comparison problems (problems involving notions of “times as much”).
- Students are not required to memorize the conversion factors.
- Students extend understanding of measuring length in inches to measuring in feet and yards.

Connections

- Identify and use the appropriate tools and units of measurement, both customary and metric, to solve one-step word problems using the four operations involving weight, mass, liquid volume, and capacity (within the same system and unit).

Teaching Strategies

- Add, subtract, multiply or divide to solve one-step word problems involving masses or volumes that are given in the same units.
- Students should have opportunities to physically measure objects.
- Record measurement equivalents in a two-column table and/or double number line.

DRAFT

CLUSTER: 3.GM.C - Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

STANDARD: 3.GM.C.5

Standards Statement (JUNE 2021):

Recognize area as an attribute of plane figures and understand concepts of area measurement presented in authentic contexts by tiling and counting unit squares.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Concept of area is measured with unit squares tiling a plane without gaps or overlaps.
    - A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
    - A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

Teaching Strategies

- Students should use numerical and spatial reasoning to determine the area of rectangles in contextual, mathematical problems by counting or tiling.

Example

- A laptop cover is being made with square vinyl stickers. There are four rows of stickers. There are 9 stickers in each row. How many square stickers were used to create the laptop cover?

DRAFT

STANDARD: 3.GM.C.6

Standards Statement (JUNE 2021):

Measure areas by counting standard and non-standard unit squares.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Standard unit squares use standard measurement units, such as square feet or square meters.
- Non-standard unit squares could be improvised unit measurements such as sticky notes or floor/countertop tiles.

Teaching Strategies

- Area can be counted in square cm, square m, square in, square ft, and improvised units.
- Students should use numerical and spatial reasoning to determine the area of rectangles in contextual, mathematical problems.

Example

- Students can determine the area of the top of their desk or other rectangle outlined by tape on the desk by covering it using non-standard units, such as index cards, sticky notes, tiles, etc.

DRAFT

STANDARD: 3.GM.C.7

Standards Statement (JUNE 2021):

Relate area to multiplication and addition. Use relevant representations to solve problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- The dimensions of a rectangle can be referred to as length and width OR base and height.
- A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area (e.g., square cm, square m, square in, square ft).

Teaching Strategies

- Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
- Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving problems, and represent whole-number products as rectangular areas in mathematical reasoning.
- Use tiles and/or arrays to illustrate and explain that the area of a rectangle can be found by partitioning it into two smaller rectangles and that the area of the larger rectangle is the sum of the two smaller rectangles.
- Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding.

Example

- The area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c; 4 x 7 is the same as 4 x (2 + 5) and is the sum of 4 x 2 and 4 x 5.
- In a rectangular garden, you have four rows of peanut plants. There are 9 peanut plants in each row. How many peanut plants are there in the garden?

DRAFT

CLUSTER: 3.GM.D - Geometric measurement: Recognize perimeter.

STANDARD: 3.GM.D.8

Standards Statement (JUNE 2021):

Solve problems involving authentic contexts for perimeters of polygons.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be given opportunities to develop a conceptual understanding of perimeter of all types of polygons including regular and irregular.
- Students should investigate perimeters of polygons with a focus on quadrilaterals.
- Students should be able to find the perimeter given the side lengths.
- Students should be able to find the unknown side length given the perimeter.

Terminology

- The focus of this learning objective should be on developing the conceptual understanding of perimeter, rather than on terminology.
- A polygon is a closed figure with at least three straight sides and angles; a polygon is regular only when all sides are equal and all angles are equal; and a polygon is irregular when all sides are not equal or all angles are not equal.

Teaching Strategies

- Finding the perimeter given the side lengths;
- Finding an unknown side length;
- Showing rectangles with the same perimeter and different area;
- Showing rectangles with the same area and different perimeters.

Connections

- Students should solve contextual, mathematical problems involving perimeter and area of rectangles.
- Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Example

- I have eighteen 1-foot panels to build a raised garden bed. How many different ways can I put these eighteen panels together to build a rectangular raised garden bed? Which rectangle will have the greatest area?

DRAFT

3.DR – Data Reasoning

CLUSTER: 3.DR.A - Pose investigative questions and collect/consider data.

STANDARD: 3.DR.A.1

Standards Statement (JUNE 2021):

Generate questions to investigate situations within the classroom, school or community. Collect or consider measurement data that can naturally answer questions by using information presented in a scaled picture and/or bar graph.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.
- Students should formulate a statistical investigative question to explore a real-life situation in their classroom.

Terminology

- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - A statistical investigative question is one that requires data that will vary.
- Numerical data: How many siblings does each student in the class have?
- Categorical data: Out of football, basketball, baseball, soccer, none of these, what is your favorite sport to watch?

Boundaries

- Rulers measurement can be limited to those marked with halves and fourths of an inch.
- Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Teaching Strategies

- Students should be provided with learning experiences to collect and analyze both numerical data and categorical data.
- Some problems should include reading bar graphs, pictographs, and dot plots, as well as measurements in grams, kilograms, liters. Dot plots and line plots can be used interchangeably.
- Developing strategies for collecting data include students collaborating to determine ways to collect data.
- Data can be gathered from a variety of sources to answer the statistical investigative question posed.
- Data could be generated from measurement data by measuring lengths using rulers marked with halves and fourths of an inch.

Examples

- Questions can be created to help students get to know each other better in the first weeks of school.
- How many siblings does each student in the class have?

DRAFT

CLUSTER: 3.DR.B – Analyze, represent, and interpret data.

STANDARD: 3.DR.B.2

Standards Statement (JUNE 2021):

Analyze measurement data with a scaled picture graph or a scaled bar graph to represent a data set with several categories. Interpret information presented to answer investigative questions.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Dot plots and line plots can be used interchangeably.
- Dot plots should be used for numerical data representation on a number line.

Terminology

- Numerical data - data that can be expressed in numbers rather than natural language. An example of numerical data that could be collected is the number of people who attended the movie theater over the course of a month.
- Categorical data - a type of data used to group information with similar characteristics. Examples of categorical data that could be collected might be marital status, favorite sport, or favorite type of movie.

Boundaries

- Students should use a ruler that is marked at halves and fourths only to create an evenly spaced number line for the dot plot.
- Data sets for categorical data created by students may include several categories.
- The scales of the pictographs, bar graphs, and dot plots should depend on the data collected.

Teaching Strategies

- Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve problems using information presented in these graphs.
- Collect data by asking a question that yields data in up to four categories.
    - Represent and interpret data in a scaled picture graph, and/or scaled bar graph with axes provided
    - Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.

Connections

- For dot plots and bar graphs, students may analyze the data presented making connections with single-digit multiplication or division in their explanations.
- On a pictograph, one symbol may stand for a value greater than 1 to allow students to apply their understanding of single digit multiplication and division facts.
- On a pictograph, one symbol may stand for a value greater than 1.

Example

- How many more kids in the school chose vanilla and chocolate combined than strawberry ice cream?”
- Draw a bar graph in which each square in the bar graph might represent 5 pets.
- The scales of the pictographs, bar graphs, and dot plots should depend on the data collected.

DRAFT

Grade 4 – Mathematics Standards and Guidance

4.OA - Algebraic Reasoning: Operations

CLUSTER: 4.OA.A - Use the four operations with whole numbers to solve problems.

STANDARD: 4.OA.A.1

Standards Statement (JUNE 2021):

Interpret a multiplication equation as comparing quantities. Represent verbal statements of multiplicative comparisons as equations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to solve contextual, mathematical problems involving multiplicative comparison.
- Students should be able to distinguish multiplicative comparison from additive comparison.

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Multiplicative comparison – a comparison situation based on one set of a quantity being a particular multiple of the other set within the comparison.
    - Additive comparison – involves two distinct quantities and the difference between them.

Teaching Strategies

- Students should be able to demonstrate an understanding of simple multiplicative relationships by using concrete materials, drawings, and equations with a variable for the unknown number to represent the problem.

Example

- Interpret 35 = 5 x 7 as 35 is the same value as 5 times as many as 7 and 7 times as many as 5.
- MP2 - Represent verbal statements of multiplicative comparisons as multiplication equations.
- Mara has four pencils. Josh has three times as many pencils as Mara. How many pencils does Josh have?

DRAFT

STANDARD: 4.OA.A.2

Standards Statement (JUNE 2021):

Multiply or divide to solve problems in authentic contexts involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Students may use drawings and equations with a symbol for the unknown number to represent the problem.
- Distinguish between multiplicative comparison from additive comparison.

Terminology

- Students should recognize that additive comparison refers to the difference between two numbers and multiplicative comparison refers to how much or how many times larger the bigger number is from the smaller number as a comparison.

Examples

- If the cost of a red hat is three times more than a blue hat that costs $5 then a red hat cost $15.
- MP6 - Use drawings and equations with a symbol for the unknown number to represent the problem.

DRAFT

STANDARD: 4.OA.A.3

Standards Statement (JUNE 2021):

Solve multistep problems in authentic contexts using whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to use the four operations with whole numbers to solve contextual, mathematical problems.
- Students should assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Boundaries

- Problems should include solutions in which remainders must be interpreted.

Teaching Strategies

- Students should represent and model problems using equations and diagrams with a variable for the unknown quantity.

Examples

- "How many busses are needed to transport 250 students, if each bus holds 36 students?" In which the remainder of 34 would be interpreted to include an additional bus.
- MP2 - Interpret and use remainders, and assess the reasonableness of answers using mental computation and estimation strategies.
- MP4 - Represent these problems using equations with a letter standing for the unknown quantity.
- MP6 - Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

DRAFT

CLUSTER: 4.OA.B- Gain familiarity with factors and multiples.

STANDARD: 4.OA.B.4

Standards Statement (JUNE 2021):

Find all factor pairs for a whole number in the range 1-100. Determine whether a given whole number in the range of 1-100 is a multiple of a given one-digit number, and whether it is prime or composite.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should recognize that a whole number is a multiple of each of its factors.
- Students should be able to determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number.
- Students should be able to determine whether a given whole number in the range 1–100 is prime or composite.

Terminology

- Prime number – A whole number greater than 1 that with two unique factors, 1 and itself.
- Composite number – A whole number greater than 1 that has at least one whole-number factor other than 1 and itself.

Examples

- The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.
- MP8 - Recognize that a whole number is a multiple of each of its factors (e.g., 24 is a multiple of 1, 2, 3, 4, 6, 8, 12, and 24).
- If there are 24 students in a class, how many unique ways can they be arranged into equal-sized groups?
- Every 8th person of the first hundred people in line for a concert will get a free T-shirt. Which places in the line will get a T-shirt?

DRAFT

CLUSTER: 4.OA.C- Generate and analyze patterns.

STANDARD: 4.OA.C.5

Standards Statement (JUNE 2021):

Analyze a number, visual, or contextual pattern that follows a given rule.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Within numeric patterns, students should be able to connect each term in a growing or shrinking pattern with its term number (e.g., in the sequence 1, 4, 7, 10, …, the first term is 1, the second term is 4, the third term is 7, and so on), and record the patterns in a table of values that shows the term number.
- Students should be provided with opportunities to explore and extend growing patterns using shapes.
- Students should be provided with opportunities to explore and extend numerical patterns using a given rule.
- Students should be able to identify features of the pattern that were not explicit in the rule itself.
- Students should be able to explain, informally, why a pattern will continue to develop as it does.

Boundaries

- Students are not expected to determine the rule but instead are expected to extend the pattern or complete a pattern.
- Patterns are limited to 8 elements.

Examples

- Given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
- Connection to MPs:
- MP3 - Identify and describe features of the rule not explicit in the rule itself.
- Given the rule “Add 3” and a starting number of 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers.
- Use square tiles to generate a growing pattern that shows multiples of four.
- Use the rule, multiply by 3 and add 1 to find the next two stages in the following growing pattern:
- Where does the pattern multiply by 3? Where is the “1” that is being added as this pattern grows? Create a different growing pattern using this rule. Identify where it multiplies by three and where one is added.

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4.NBT - Numeric Reasoning: Base Ten Arithmetic

CLUSTER: 4.NBT.A - Generalize place value understanding for multi-digit whole numbers.

STANDARD: 4.NBT.A.1

Standards Statement (JUNE 2021):

Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to use numerical reasoning to represent and explain using concrete materials, the relationship among the numbers 1, 10, 100, and 1,000. Students should be able to extend the pattern to the hundred-thousands place.
- Students should be able to recognize the relationship of same digits located in different places in a whole number.

Boundaries

- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Example

- Recognize that 700 ÷ 70 = 10 by applying concepts of place value and division
- The population of Atlanta is about 500,000 people and the population of Valdosta is about 50,000 people. How many times greater is the population of Atlanta than Valdosta?

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STANDARD: 4.NBT.A.2

Standards Statement (JUNE 2021):

Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Use understandings of place value within these forms to compare two multi-digit numbers using >, =, and < symbols.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
- Students are not expected to write numbers in word form.

Connections

- Make connections across representations of multi-digit whole numbers using base ten numerals, number names, and expanded form.
- Develop rules for comparing the multi-digit numbers.

Examples

- The number two hundred seventy-five thousand eight hundred two written in standard form is 275,802 and in expanded form is 200,000+70,000+5,000+800+2 or (2×100,000)+(7×10,000)+(5×1,000)+(8×100)+(2×1).

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STANDARD: 4.NBT.A.3

Standards Statement (JUNE 2021):

Use place value understanding to round multi-digit whole numbers to any place.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
- Grade 4 students should explore rounding within contextual situations.

Teaching Strategies

- Students rounding to 348 to the nearest hundred may mistakenly round initially to 350 and then 400 by applying rules such as if the digit is 0-4 then round down and 5-9 and round up. Models can help them see that 348 is closer to 300 than 400.
- Students should locate numbers on a number line to determine the nearest multiple of 1,000s, 10,000s or 100,000s.

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CLUSTER: 4.NBT.B - Use place value understanding and properties of operations to perform multi-digit arithmetic.

STANDARD: 4.NBT.B.4

Standards Statement (JUNE 2021):

Fluently add and subtract multi-digit whole numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should fluently (flexibly, accurately, and efficiently) add and subtract multi-digit whole numbers, to solve contextual, mathematical problems using efficient and flexible procedures, based on knowledge of place value and properties of operations.
- Students should use efficient algorithms that make sense for the given numbers and draw upon their understanding of multi-digit whole numbers, the properties of operations, and place value.

Terminology

- Efficiency in mathematics is the ability to produce answers relatively easily with a minimal number of steps.
- Flexibility is the ability to think about a problem in more than one way and to adapt or adjust thinking, if necessary.
- Accuracy is the ability to produce mathematically precise answers.
- Appropriateness is the ability to select and apply a strategy that is appropriate for solving a given problem efficiently.

Content Boundaries

- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1, 000,000. A range of algorithms may be used.
- Efficiency means the student is able to flexibly use strategies appropriate for the given problem with ease.
- Efficiency does not mean students should be timed.
- An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and making use of intermediate results to solve the problem. Efficiency means the student is able to flexibly use strategies appropriate for the given problem with ease.
- Students should be given the choice of which procedure they can use.
- Students should add and subtract multi-digit whole numbers within 100,000, to solve contextual, mathematical problems using efficient and generalizable procedures, based on knowledge of place value and properties of operations.

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STANDARD: 4.NBT.B.5

Standards Statement (JUNE 2021):

Use representations and strategies to multiply a whole number of up to four digits by a one-digit number, and a two-digit number by a two-digit number using strategies based on place value and the properties of operations.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Students should be familiar with multiple strategies but should be able to select and use the strategy with which they most closely connect and understand, with the ultimate goal of supporting students to use more efficient strategies.
- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
- A range of efficient algorithms may be used.

Teaching Strategies

- Illustrate and explain calculations using rectangular arrays, area models, and/or equations, along with strategies based on place value and properties of operations.
- Students should be able to solve contextual, mathematical problems involving the multiplication of a number with up to four digits by a 1-digit whole number.
- Students should be able to illustrate and explain their calculations using equations, rectangular arrays, and/or area models for all numbers included in the learning objective.

Examples

- Connect numeric and visual models such as those created by representing 285 with base 10 pieces and repeating three times. Use this area model with dimensions of 285 and 3 to find partial products.
- Connecting to MPs:
- MP3 - Illustrate and explain the calculation using equations, rectangular arrays and /or area models.
- MP7 - Use expanded form of the whole number and the distributive property of multiplication to simplify calculations.
- There are 7 boxes of chocolates. Each box contains 16 chocolates. How many chocolates are there all together?
- The school bought thirty-nine cases of popcorn for the school carnival. Each case contained 15 bags of popcorn. How many bags of popcorn is that all together?

DRAFT

STANDARD: 4.NBT.B.6

Standards Statement (JUNE 2021):

Use representations and strategies to find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to solve contextual, mathematical problems involving division of whole numbers.
- Students should be familiar with multiple strategies but should be able to select and use the strategy with which they most closely connect and understand, with the ultimate goal of supporting students to use more efficient strategies.

Content Boundaries

- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.
- Long division is not an expectation at this grade level.

Teaching Strategies

- Students should be able to illustrate and explain their calculations using equations, rectangular arrays, and/or area models.

Example

- Apply knowledge of decomposing whole numbers into divisible parts. Such as, connect numeric and visual models such as those created by representing 136 with base 10 pieces and dividing into groups of 4 or 4 groups to determine either the size of the group or the number of groups; or building/sketching a rectangle with an area of 136 and one dimension of 4 and finding partial quotients.
- MP3 - Illustrate and explain the calculation using equations, rectangular arrays and /or area models.
- Antonio won a jar of 373 jellybeans in a school contest. He wants to share them. He and his 7 friends will share them. How many jellybeans will each of the friends get?
- Possible solution: $373 \div 8 = (368 \div 8) + (5 \div 8) = 46$ with 5 jellybeans left over.
- Examples of different strategies and representations can be found within the Computational Strategies for Whole Numbers document found in the appendices.

DRAFT

4.NF - Numeric Reasoning: Fractions

CLUSTER: 4.NF.A - Extend understanding of fraction equivalence and ordering.

STANDARD: 4.NF.A.1

Standards Statement (JUNE 2021):

Use visual fraction representations to recognize, generate, and explain relationships between equivalent fractions.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to describe how the number and size of the parts differ even though the fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- Students should be able to explain fraction equivalence as a multiplicative relationship, not additive.
- Students should be able to explain why $ab = (n \times a)(n \times b)$ is a true mathematical statement, whereas $ab = (n+a)(n+b)$ is NOT a true mathematical statement.

Boundaries

- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.
- This expectation includes fractions greater than 1.
- Fractions should be limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Teaching Strategies

- Students should be provided with opportunities to demonstrate mastery of this expectation through contextual, mathematical problems.
- Concrete materials may include fraction circles, fraction strips, pattern blocks.
- Students may represent their problems and explain their reasoning with drawing and number lines.
- Students should be able to discover, explain, and generalize the relationship between the identity property of multiplication and equivalent fractions (i.e., paper folding activities, number lines, etc.).
- drawings, and number lines,

Example

- MP3 - Use visual models to justify why a fraction a/b is equivalent to a fraction (n × a)/(n × b) with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.
- Peter is giving half of his candy bar to four friends. Provide a mathematical representation to show this scenario.
- Possible student response: 12 = (4 × 1)(4 × 2) = 48

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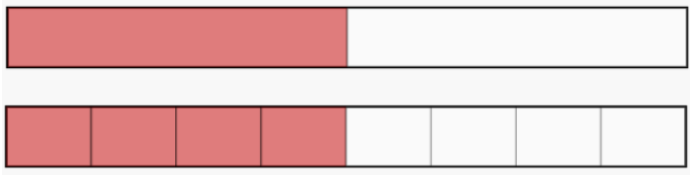


STANDARD: 4.NF.A.2

Standards Statement (JUNE 2021):

Compare two fractions with different numerators and/or different denominators, record the results with the symbols >, =, or <, and justify the conclusions.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to recognize that comparisons are valid only when the two fractions refer to the same whole.
- Students should record the results of comparisons with symbols >, =, or <, and justify the conclusions.

Boundaries

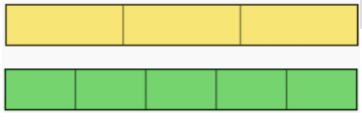
- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.
- Students should be given fractions with common numerators to compare.

Teaching Strategies

- MP2 - Recognize that comparisons are valid only when the two fractions refer to the same whole.
- MP3 - Justify using conceptual and procedural strategies. Conceptual strategies should include using visual models; comparing benchmark fractions such as 0, ½, 1; and attending to the size of the piece for the like numerators or number of pieces for like denominators. Procedural strategies should include finding a common denominator to directly compare the number of pieces.
- MP6 - Record with comparison symbols >, =, or <.

Examples

- Jamie and Kendra each had the same grid to color using any pattern they wished. Jamie colored 23 of her grid pattern and Kendra colored 25 of her grid pattern. Who colored more?
    - Jamie colored more because thirds are bigger than fifths and since they both colored two parts, 23 has to be bigger than 25 .



Each third (yellow) is larger than each fifth (green)

- Kennedy ran 58 of a mile during practice and Alice ran 78 of a mile. Who ran farther? o Alice ran farther because the distances they ran were both the same unit (eighths), so whoever had more eighths ran the greatest distance.
-

Each section above represents one-eighth of a mile. All 8 pieces represent the whole mile. Kennedy ran the length of 5 pieces and Alice ran the length of 7 pieces, so Alice ran the greater distance.

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CLUSTER: 4.NF.B - Build fractions from unit fractions.

STANDARD: 4.NF.B.3

Standards Statement (JUNE 2021):

Understand a fraction (a/b) as the sum (a) of fractions of the same denominator (1/b). Solve problems in authentic contexts involving addition and subtraction of fractions referring to the same whole and having like denominators.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to break apart (decompose) whole numbers and fractions as the sum of unit fractions.
- Break apart (decompose) a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.

Content Boundaries

- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.
- Extend understanding addition and subtraction to include fractions.
- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.
- Add and subtract mixed numbers with like denominators.

Teaching Strategies

- Students should be able to add and subtract fractions and mixed numbers with the same (like) denominators by joining and separating parts referring to the same whole while solving contextual, mathematical problems.
- Tools include fraction concrete materials, such as Cuisenaire rods, drawings, and number lines.
- Students should be flexible in their choice of strategy when subtracting fractions. Reasoning about the sizes of the fractions and their relationships is the expectation here rather than memorizing regrouping procedures.
- Students can justify their work using a visual fraction representation.

Example

- MP2 - Decompose and recompose 3/8 as 1/8 + 1/8 + 1/8 or 1/8 + 2/8 and 2 1/8 as 1 + 1 + 1/8 or 8/8 + 8/8 + 1/8.
- MP4 - Use visual fraction models and equations to represent problems.
- MP7 - Replace mixed numbers with equivalent fractions and/or use properties of operations and the relationship between addition and subtraction to solve problems.
- Alex has a whole pizza. How can it be cut so that it can be shared with (4, 6, 8, 12) people? What fraction of the whole pizza will each person get?
- Express 1 in the form 1 = 44 (1 whole is equal to four fourths 14+ 14+ 14+ 14= 44 = 1) recognize that additional wholes cut into fourths can also be written as the sum of

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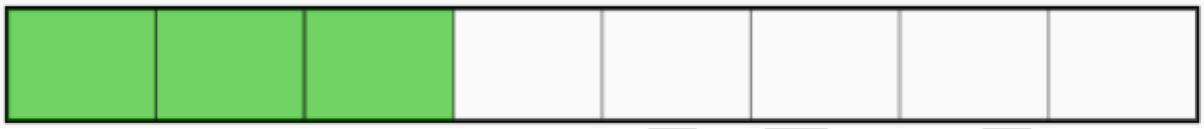
- Locate 44 and 1 at the same point of a number line diagram.

- Express 54 as the sum of unit fractions. o 54= 14+ 14+ 14+ 14+ 14

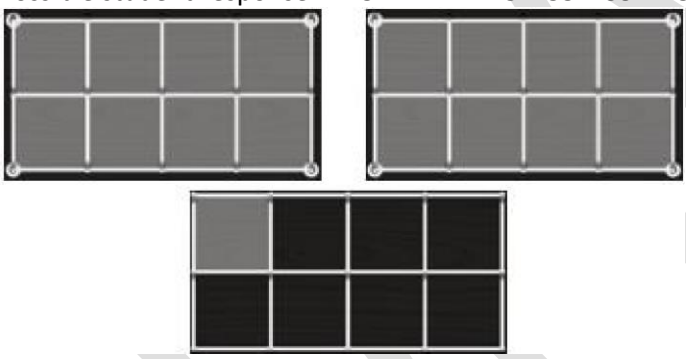
- A piece of ribbon was cut into eighths for a classroom art project. Three pieces were left at the end of the day. Show a mathematical representation of the ribbon that is left.

Possible student response: 38 = 18 + 18 + 18 ; 38 = 18 + 28



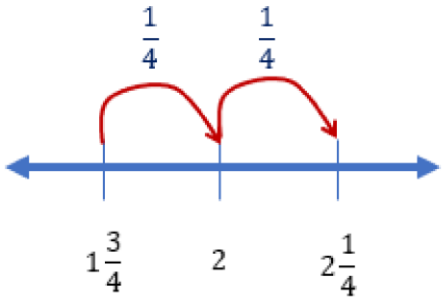
- Three pans of brownies were cut into eighths to sell at a school function. 78 of one pan were sold. How many eighths are left to sell? Show a mathematical representation of the ribbon that is left.

Possible student response: 2 18 = 1 + 1 + 18 = 88 + 88 + 18



Example (4.4.6)

- Luisa needs to know how much bigger her 214 inch piece of cardstock is than her 134 inch piece of cardstock in order to finish her project. o Possible student response: The 214 inch piece is 24 inch bigger than the 134 inch piece.



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STANDARD: 4.NF.B.4

Standards Statement (JUNE 2021):

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. Represent and solve problems in authentic contexts involving multiplication of a fraction by a whole number.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Extend understanding multiplication to include fractions.
    - Understand a fraction a/b as a multiple of 1/b.
    - Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

Boundaries

- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Examples

- MP2 - Use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
- MP3 - Use a visual fraction model to demonstrate 3 × (2/5) is the same as 6 × (1/5), recognizing this product as 6/5. Justify the general idea n × (a/b) = (n × a)/b.

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CLUSTER: 4.NF.C - Understand decimal notation for fractions, and compare decimal fractions.

STANDARD: 4.NF.C.5

Standards Statement (JUNE 2021):

Demonstrate and explain the concept of equivalent fractions with denominators of 10 and 100, using concrete materials and visual models. Add two fractions with denominators of 10 and 100.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should also use mixed numbers and fractions greater than 1.
- Students should express fractions such as 310 as 30100, and add fractions such as 310 + 4100 = 34100.

Boundaries

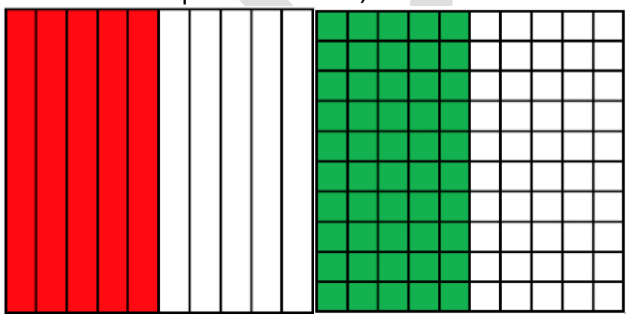
- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Teaching Strategies

- Students should be able to solve contextual, mathematical problems involving the addition of two fractions with denominators of 10 and 100.
- Students should be given multiple opportunities to use visual models to develop part-whole reasoning when building an understanding of equivalent fractions.

Example

- MP7 - Express 3/10 as 30/100 and add 3/10 + 4/100 = 34/100.
- Colin wants to use 510 of a board for a project. He is wondering how he can cut his whole board into pieces that are equivalent to 510. What fraction(s) of the whole board can Colin cut the board that are equivalent to 510 ? Use visual models to support your reasoning.
- Possible student response: I know that 510 is the same as 50100 because they both take up the same amount of space in the decimal squares below. So, 510= 50100. I also notice that half of each square is shaded, so I think that 510= 50100= 12.



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STANDARD: 4.NF.C.6

Standards Statement (JUNE 2021):

Use and interpret decimal notation for fractions with denominators 10 or 100.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Represent decimal number values on a place value chart.

Boundaries

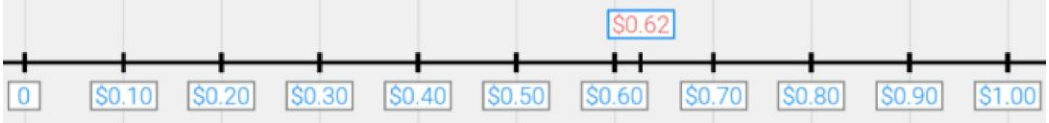
- Students are not expected to write word names of decimal numbers at this grade level.
- To the hundredths place

Teaching Strategies

- Concrete materials could include base ten block where the "flat" or hundred square is considered one whole or a ten frame where the whole frame is considered one whole.

Example

- Rewrite 0.62 as 62100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.
    - Eric overpaid his medical bill by $0.62. When businesses write refund checks, they often write the cents as a fraction. What fraction will the doctor's office use to represent the $0.62 on the check?
    - Possible student response: I wrote 62 cents ($0.62) as 62100 because $0.62 is sixty-two hundredths of a dollar. If I place $0.62 on a number line, it would be between $0.50 and $0.75.



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STANDARD: 4.NF.C.7

Standards Statement (JUNE 2021):

Use decimal notation for fractions with denominators 10 or 100. Compare two decimals to hundredths place by reasoning about their size, and record the comparison using the symbols >, =, or <.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Recognize that comparisons are valid only when the two decimal numbers refer to the same whole.
- Students should be able to order up to 5 whole numbers less than 1,000,000 through the hundred-thousands place.

Teaching Strategies

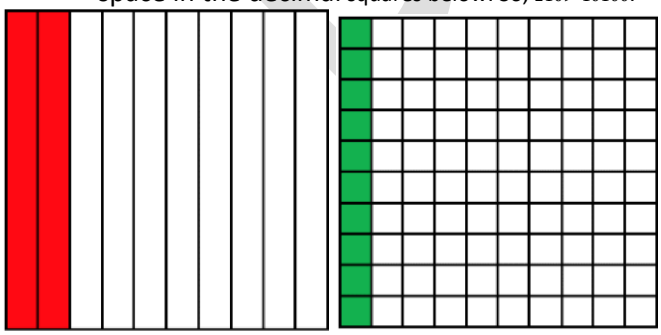
- Decimal quantities should be presented within a context.
- Students should be given multiple opportunities to use visual models to develop part-whole reasoning when comparing decimal numbers.
- Students should be able to determine and explain, through investigation, the relationship between decimal numbers, using a variety of tools (e.g., concrete materials, drawings, number lines) and strategies.

Boundaries

- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Students are not expected to use more than two inequality symbols when recording comparisons (< or >).
- To the hundredths place

Example

- MP2 - Recognize that comparisons are valid only when decimals refer to the same whole.
- MP6 - Record with comparison symbols >, =, or <.
- MP3 - Justify conclusions using visual models.
- What do you notice about the fractions 210 and 10100? Write a comparison statement about the two fractions and use visual models to support your reasoning.
- Possible student response: I know that 210 is greater than 10100 because 210 takes up more space in the decimal squares below. So, $210 > 10100$.



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4.GM - Geometric and Measurement Reasoning

CLUSTER: 4.GM.A. - Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

STANDARD: 4.GM.A.1

Standards Statement (JUNE 2021):

Explore, investigate, and draw points, lines, line segments, rays, angles, and perpendicular and parallel lines. Identify these in two-dimensional figures.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular lines.
- Expectation that drawing and identifying right, acute, and obtuse angles are included in this standard.

Terminology

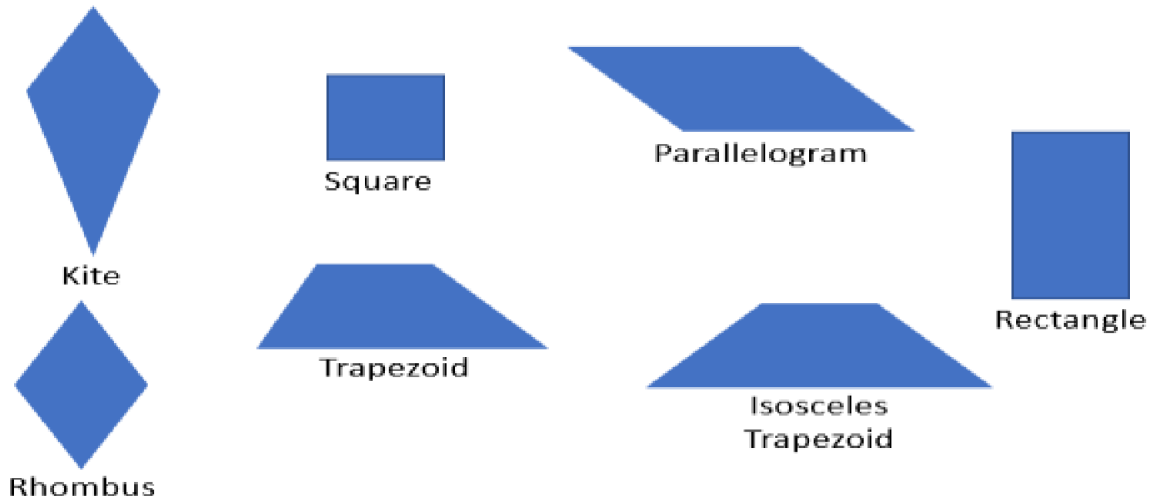
- Right angle – An angle measuring exactly 90°.
- Acute angle – An angle larger than 0° and smaller than 90°.
- Obtuse angle – An angle larger than 90° and smaller than 180°.
- Perpendicular lines – Two lines that meet to form an intersection at a right angle

Teaching Strategies

- Students should investigate lines of symmetry in two dimensional figures as a property. This is an extension from work in third grade.

Examples

- How many lines of symmetry do each of the quadrilaterals below have?



STANDARD: 4.GM.A.2

Standards Statement (JUNE 2021):

Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Right angles should be indicated with a square symbol.
- Polygons should include triangles, quadrilaterals including kites, trapezoids, rectangles, squares, rhombuses, and other parallelograms, and pentagons.

Terminology

- A polygon is a closed figure with at least three straight sides and angles; a polygon is regular only when all sides are equal and all angles are equal; and a polygon is irregular when all sides are not equal or all angles are not equal.
- Isosceles triangle – A triangle containing at least two equal length sides and two equal interior angle measures. Sub- class includes equilateral triangles.
- Equilateral triangle – A triangle with three equal-length sides and three 60-degree interior angles. Also known as an equiangular triangle.
- Scalene triangle – A triangle containing three unequal side lengths and three unequal angle measures.
- Right triangle – a triangle with one right angle.
- Acute triangle – a triangle containing three acute angles.
- Obtuse triangle – a triangle containing one obtuse angle.

Boundaries

- The intent of this learning object is for students to classify shapes based on specific properties such as perpendicular line segments, lines of symmetry, congruent angles or sides, or a lack of these attributes. The focus should not be on having students memorize terminology.
- This objective does not require students to create a hierarchy.

Teaching Strategies

- Students should investigate lines of symmetry in two dimensional figures as a property. This is an extension from work in third grade.
- Recognize and identify right triangles as a category. (MP.6)

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STANDARD: 4.GM.A.3

Standards Statement (JUNE 2021):

Recognize and draw a line of symmetry for a two dimensional figure.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- A line of symmetry is a line across the figure such that the figure can be folded along the line into matching parts.

Teaching Strategies

- Students should be provided multiple opportunities to investigate symmetry through paper folding and/or the use of mirrors.
- Students should develop an understanding of what a line of symmetry is through exploration with real-world objects.
- Identify or create line-symmetric figures by drawing and testing proposed lines of symmetry and sketching the second half of a symmetrical figure. (MP.7)

Example

- Identify lines of symmetry seen in real-life objects, such as a butterfly, stop sign, flower, or dragonfly. Identify lines of symmetry seen and how they connect to the object.

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CLUSTER: 4.GM.B - Solve problems involving measurement and conversion of measurements.

STANDARD: 4.GM.B.4

Standards Statement (JUNE 2021):

Know relative sizes of measurement units and express measurements in a larger unit in terms of a smaller unit.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students can convert within a single system of measurement, express measurements in a larger unit in terms of a smaller unit.
- Students can record measurement equivalents in a two-column table.

Boundaries

- Measurement units within one system a student should be familiar with include km, m, cm, kg, g, lb, oz, l, hr, min, sec.

Connections

- Justify conversions using understanding that larger units can be partitioned into smaller equal sized units (MP.3)

Example

- Know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 inches. Generate a conversion table for feet and inches listed as number pairs (1, 12), (2, 24), (3, 36), ….

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STANDARD: 4.GM.B.5

Standards Statement (JUNE 2021):

Apply knowledge of the four operations and relative size of measurement units to solve problems in authentic contexts that could include simple fractions or decimals.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should express larger units in terms of smaller units within the same measurement system
- Students should express smaller units in terms of larger units within the same measurement system.
- Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
- Student should experience problems involving distances, intervals of time, liquid volumes, masses of objects, and money

Terminology

- Metric measurement units include weight (grams and kilograms) capacity (milliliters and liters), length (centimeter, meter, and kilometer).

Boundaries

- Word problems should involve simple fractions or decimals and expressing measurements given in a larger unit in terms of a smaller unit.
- Fractions should be limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Multiplication and division of fractions is not a requirement of this grade level.

Teaching Strategies

- Represent measurement quantities using number line diagrams that feature a measurement scale.
- Students should reason about the relative sizes of measurement units within the metric system.
- Students should be able to accurately record measurement equivalents in a two-column table.

Connections

- Contexts include distance, intervals of time, liquid volumes, mass and money.
- Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (MP.3)

Example

- What time does Eric have to leave his house to get to the concert by quarter after nine, if the trip takes 90 minutes?
- If you have a prescription for 5,000 mg of medicine, and upon getting it filled, the dosage reads 5 g of medicine, did the pharmacist make a mistake?

DRAFT

STANDARD: 4.GM.B.6

Standards Statement (JUNE 2021):

Apply the area and perimeter formulas for rectangles in authentic contexts and mathematical problems.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should express their answers in linear (perimeter) and square (area) units. Students are not expected to use the 1 cm^2 notation.

Boundaries

- Students should not be expected to find unknown side lengths when exploring composite rectangles.

Example

- Find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

DRAFT

CLUSTER: 4.GM.C - Geometric measurement: understand concepts of angle and measure angles.

STANDARD: 4.GM.C.7

Standards Statement (JUNE 2021):

Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. Understand and apply concepts of angle measurement.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- An angle can be viewed as a wedge of a circle or a turn through a circular arc where 1/360 of the wedge or turn is one-degree.
- An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
- Draw right, acute, and obtuse angles

Teaching Strategies

- Students should also be able to explore this learning objective in the context of angles within circles.
- Angle measurement should be introduced with non- standard tools such as pattern blocks, unit angles, and/or wedges prior to introducing protractors. 360-degree protractors would make an explicit connection to the degrees of a circle and builds conceptual understanding of angles.

Example

- The student can place four squares around the center of a circle. Since there are 360 degrees in a circle, 360 ÷ 4 = 90, so each square has 90-degree angles.

DRAFT

STANDARD: 4.GM.C.8

Standards Statement (JUNE 2021):

Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

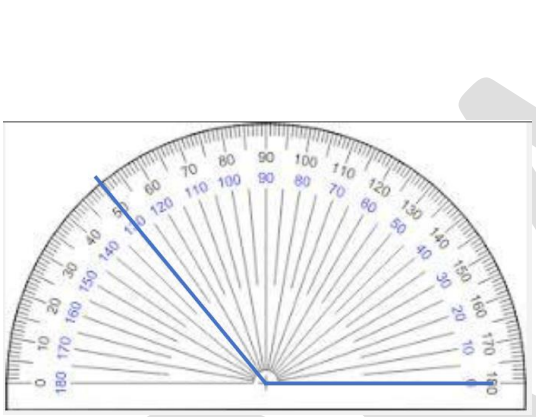
- To understand measurement, students should measure in non-standard units, such as unit angles or wedges, before being introduced to tools with abstract units such as degrees.

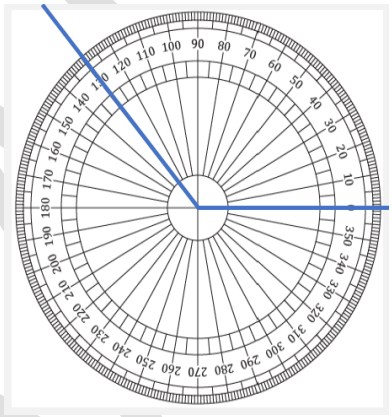
Teaching Strategies

- Use angle measurement tools that help students connect non-standard units (wedges, unit angles, etc.) to standard units of angle measurement (degrees).

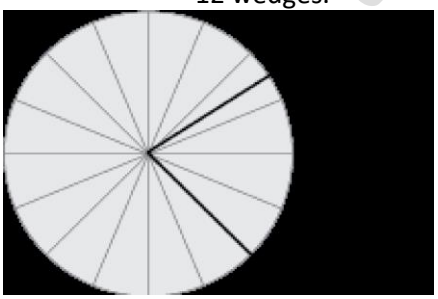
Examples

- Students may be given angles to find precise measurements of angles. Here are some examples of how students may use protractors and measurement reasoning to determine precise angle measurements.





    - Sample student response: The angle is an obtuse angle because it is open more than a square corner angle. It measures 130 degrees.
- Fold a circle of patty paper or waxed paper in half four times to create an angle measuring tool with 16 wedges. This protractor can be used to determine the number of units (wedges) in an angle.
    - What type of angle is shown below? Use your protractor to determine its measurement. o The angle is an acute angle because it is smaller than a square corner and it is about 3 12 wedges.



STANDARD: 4.GM.C.9

Standards Statement (JUNE 2021):

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Expectation includes solving addition and subtraction problems to find unknown angles on a diagram in authentic contexts and mathematical problems such as by using an equation with a symbol for the unknown angle measure.

Example

- Using an equation with a symbol for the unknown angle measure.

DRAFT

4.DR – Data Reasoning

CLUSTER: 4.DR.A - Pose investigative questions and collect/consider data.

STANDARD: 4.DR.A.1

Standards Statement (JUNE 2021):

Generate questions to investigate situations within the classroom, school or community. Determine strategies for collecting or considering data involving addition and subtraction of fractions that can naturally answer questions by using information presented in line plots.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.
- Students should be given opportunities to generate questions about things they notice and wonder from a real-life situation.

Terminology

- A statistical investigative question is one that requires data that will vary.

Teaching Strategies

- Students should be able to use rulers to measure to the nearest 1/8.
- By measuring repeatedly students learn that measurements can vary.
- Based on the posed question, create a plan that determines the appropriate population to survey and how to collect that data.

Connections

- Students should be able to measure objects found in everyday life to collect data.
- Developing strategies for collecting data include students collaborating to determine ways to collect data.
- Data can be gathered from a variety of sources to answer the statistical investigative question posed.

Examples

- “How tall are the tomato plants in the class garden?” is a statistical investigative question because it anticipates variability in the lengths of the tomato plants.
- “How tall is the tomato plant right here?” is a question used to collect data to answer the investigative question.

DRAFT

CLUSTER: 4.DR.B – Analyze, represent, and interpret data.

STANDARD: 4.DR.B.2

Standards Statement (JUNE 2021):

Analyze line plots to display a distribution of numerical measurement data, which include displays of data sets of fractional measurements with the same denominator. Interpret information presented to answer investigative questions.

DRAFT Standards Guidance (JUNE 2021):

Fundamentals

- Students should be able to determine the appropriate representation for the type of data to be collected based on the statistical investigative question.
- Students should have opportunities to determine the difference between representations for categorical data and numerical data presented.
- Representations for data should include bar graphs, pictographs, and dot plots (line plots).

Terminology

- Dot plots and line plots can be used interchangeably.
- Numerical data: A data type expressed in numbers rather than natural language descriptions. This is sometimes called quantitative data.

Boundaries

- Fractional measurements can include 1/2, 1/4, 1/8 units.
- Students should record observations they notice about the shape of the distribution using informal language such as spread out and/or grouped.

Example

- Based on a class survey, the students determined each student's favorite flavor of ice cream. The student is able to determine that the best representation for the data would be a bar graph since the data are categorical.
- When determining the lengths of specimens in an insect collection, the student is able to determine that the best representation for the data would be a dot plot (line plot).
- How long are the specimens in an insect collection? From a dot plot, find and interpret the difference in length between the longest and shortest specimens in an insect collection.

DRAFT

Grade 5 – Mathematics Standards and Guidance

5.OA - Algebraic Reasoning: Operations

CLUSTER: 5.OA.A - Write and interpret numerical expressions.

STANDARD: 5.OA.A.1

Standards Statement (JUNE 2021):

Write and evaluate simple numerical expressions that include parentheses.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- The expressions should be no more complex than the expressions one finds in a simple application of the associative and distributive properties.
- Simple expressions should only include two operations.
- Grouping symbols used in expressions may include parentheses, brackets, or braces.

Teaching Strategies

- Use of nested parentheses should be used in favor of brackets or braces in numerical expressions.
- Students should begin with concrete models. Concrete models may include color tiles or base ten blocks for constructing area models and rods for representing numerical values.

Example

- Express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7).
- If this expression were tripled, then it could represented as 3 x (2 × (8 + 7)), where the number of open parentheses is the same as the number of closed parentheses.
- Karl brought 3 ten-packs of juice boxes to the class party. Joshua brought 4 six-packs of soda to the party. How many drinks did they bring altogether?
    - (3 × 10) + (4 × 6)

DRAFT

STANDARD: 5.OA.A.2

Standards Statement (JUNE 2021):

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- The expressions should be no more complex than the expressions one finds in a simple application of the associative and distributive properties.
- Simple expressions should only include two operations.
- Grouping symbols used in expressions may include parentheses, brackets, or braces.
- Nested grouping symbols (more than one grouping symbol used within another grouping symbol in an expression) could also be used within expressions at this grade level.

Teaching Strategies

- Expressions included should contain numbers, operations, and grouping symbols.
- Students should begin with concrete models. Concrete models may include color tiles or base ten blocks for constructing area models and rods for representing numerical values.

Example

- Recognize that 3 × (18,932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.
- Karl brought 3 ten-packs of juice boxes to the class party. Joshua brought 4 six-packs of soda to the party. How many drinks did they bring altogether?
    - (3 × 10) + (4 × 6)
- Express the calculation “Add 8 and 7, then multiply by 2” as 2×(8+7). Recognize that 12×(7+91) is twelve times as large as 7+91, without having to calculate the indicated sum or product.

DRAFT

CLUSTER: 5.OA.B - Analyze patterns and relationships.

STANDARD: 5.OA.B.3

Standards Statement (JUNE 2021):

Generate two numerical patterns using two given rules. Identify and analyze relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph them on a coordinate plane.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- This standard extends the work from fourth grade, where students generate numerical patterns when they are given one rule. In Fifth Grade, students are given two rules and generate two numerical patterns.

Boundaries

- Generating numerical patterns is a fourth grade standard, therefore is also an expectation for 5th grade.
- This learning objective is limited to patterns involving whole numbers.

Example

- Given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences.
    - Identify and explain why the terms in one sequence are twice the value of the terms in the corresponding sequence.
- Sam and Terri live by a lake and enjoy going fishing together every day for five days. Sam catches 2 fish every day, and Terri catches 4 fish every day. Make a chart (table) to represent the number of fish that Sam and Terri catch.

DRAFT

5.NBT - Numeric Reasoning: Base Ten Arithmetic

CLUSTER: 5.NBT.A - Understand the place value system.

STANDARD: 5.NBT.A.1

Standards Statement (JUNE 2021):

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should identify the value of a digit up 100 times greater or 11000 of the value of a digit.
- Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Examples

- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.
    - For example, 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
    - 700 is 10 times as much as 70, and 70 is 1/10 of 700.
- Mara has a digital scale. He placed one playing card on the scale and it read 1.3 grams. How much would you expect 10 playing cards to weigh?
- Chris took the cards off the scale and then placed 10 pennies on the scale and the scale read 24 grams. How much would you expect one penny to weigh.

DRAFT

STANDARD: 5.NBT.A.2

Standards Statement (JUNE 2021):

Use whole number exponents to denote powers of 10 and explain the patterns in placement of digits that occur when multiplying and/or dividing whole numbers and decimals by powers of 10.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should explain what happens to the value of a digit as it shifts to the left or right and discover the decimal point remains between the ones and tenths place as the digits shift.
- Use whole-number exponents to denote powers of 10, up to 10^3.

Content Boundaries

- Work with exponents at this grade is limited to powers of 10.

Example

- Observe and explain the patterns in the number of zeros of a product when multiplying a whole number by a power of 10, and the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

DRAFT

STANDARD: 5.NBT.A.3

Standards Statement (JUNE 2021):

Read, write, and compare decimals to thousandths.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Read and write decimals to thousandths using standard form, expanded form, and word from.
- Compare two decimals to thousandths based on meanings of the digits in each place, and record the results of the comparisons using >,=, and <.

Boundaries

- Students should be provided opportunities to simultaneously compare decimals and fractions, including equivalent fractions and decimals, on both single and double number lines.
- Base-ten numerals should range between millions and thousandths.
- Students are not expected to write decimal numbers in word form.
- Exponents and decimal numbers should not be included in expanded form notation.
- The decimal fractions used in Grade 5 should be limited to those for which the equivalent fraction can be written as a fraction where the denominator is a power of ten.

Teaching Strategies

- Students should be presented with decimal number comparisons from contextual, mathematical situations.
- Students should have opportunities to determine and explain comparisons using a variety of tools such as concrete materials, drawings, number lines, other visual representations, and strategies.

Example

- Use >, =, and < symbols to record comparisons of two decimals. For example:
    - 347.392 =
    - = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
    - =three hundred forty-seven and three hundred ninety-two thousandths
- 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (110) + 9 × (1100) + 2 × (11000)
- Which is greater 0.13 or 0.031? Explain. Use a visual representation to illustrate your explanation.
    - I think 0.13 is greater because it fills up more of the whole square than 0.031 does.

DRAFT

STANDARD: 5.NBT.A.4

Standards Statement (JUNE 2021):

Use place value understanding to round decimals to any place.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Work with decimals at this grade is limited to decimals up to the thousandths.

Teaching Strategies

- Students should round decimal numbers to the hundredths place in contextual, mathematical problems using visual aids, such as a number line.

DRAFT

CLUSTER: 5.NBT.B - Perform operations with multi-digit whole numbers and with decimals to hundredths.

STANDARD: 5.NBT.B.5

Standards Statement (JUNE 2021):

Fluently multiply multi-digit whole numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- The National Council of Teachers of Mathematics provides the following definition of procedural fluency:
    - "Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another.

Boundaries

- Students may use but are not limited to partial products (area model).
- Students may also use a standard algorithm by making connections from previous part-whole strategies.
- Students should choose a strategy that makes sense to them based on the context of the problem. The focus should always be on efficiency.

Teaching Strategies

- Students should be presented with contextual, real-life situations involving multiplication of multi-digit whole numbers.
- Students should fluently (flexibly, accurately, and efficiently) multiply to solve contextual, mathematical problems using efficient strategies that are based on knowledge of place value and properties of operations.
- Examples of different strategies and representations can be found within the Computational Strategies for Whole Numbers document found in the appendices.

DRAFT

STANDARD: 5.NBT.B.6

Standards Statement (JUNE 2021):

Use a variety of representations and strategies to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
- Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models

Boundaries

- Students should divide multi-digit whole numbers up to 4- digit dividends and 2-digit divisors no greater than 25.
- Students may use but are not limited to partial quotients (area model).
- Students should choose a strategy that makes sense to them based on the context of the problem. The focus should always be on efficiency.

Teaching Strategies

- Students should be presented with contextual, real-life situations involving the division of multi-digit whole numbers.
- Students should fluently (flexibly, accurately, and efficiently) divide, to solve contextual, mathematical problems using an efficient algorithm and flexible strategies, based on knowledge of place value and properties of operations.
- Examples of different strategies and representations can be found within the Computational Strategies for Whole Numbers document found in the appendices.

DRAFT

STANDARD: 5.NBT.B.7

Standards Statement (JUNE 2021):

Use a variety of representations and strategies to add, subtract, multiply, and divide decimals to hundredths. Relate the strategy to a written method and explain the reasoning used.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- As part of this standard, students must be able to use concrete models, visual drawings and strategies based on place value, properties of operations, and the relationship between addition and subtraction.

Boundaries

- Fluency with operations with decimals is part of the 6th grade standards.
- Students should be given the choice of which strategy they can use.

Teaching Strategies

- Students should be presented with a variety of contextual, real-life situations involving addition and subtraction of decimal numbers to the hundredths place.
- Students should add and subtract decimal numbers to hundredths, using concrete models, drawings, strategies based on place value, properties of operations, and the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

DRAFT

5.NF - Numeric Reasoning: Fractions

CLUSTER: 5.NF.A - Use equivalent fractions as a strategy to add and subtract fractions.

STANDARD: 5.NF.A.1

Standards Statement (JUNE 2021):

Add and subtract fractions with unlike denominators, including common fractions larger than one and mixed numbers.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- A common fraction is a fraction in which numerator and denominator are both integers, as opposed to fractions. Fractions such as 4/3, or 14/5 should be thought of as common fractions greater than one, which could also be written using mixed numbers as 1-1/3 and 2-4/5 respectively.
- Use of the term "improper fraction" should be avoided.

Boundaries

- Work with fractions at grade 5 should focus on fractions with denominators 2-10, 12, 16, 20, 25, 50, 100 and 1000.

Example

- Include replacing given fractions with equivalent fractions to produce an equivalent sum or difference.
    - 2/3 + 5/4 = 8/12 + 15/12 = 23/12 or 1 11/12.

DRAFT

STANDARD: 5.NF.A.2

Standards Statement (JUNE 2021):

Solve problems in authentic contexts involving addition and subtraction of fractions with unlike denominators, including common fractions larger than one and mixed numbers.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Use visual fraction models or equations to represent the problem.
- Use benchmark fractions and number sense of fractions to estimate and assess the reasonableness of answers.
- Students should use benchmark fractions and number sense of fractions to estimate and assess the reasonableness of answers as an introduction to addition and subtraction.

Boundaries

- Work with fractions at grade 5 should focus on fractions with denominators 2-10, 12, 16, 25, 100 and 1000.

Teaching Strategies

- Students should use numerical reasoning to add and subtract fractions and mixed numbers with unlike denominators in contextual, mathematical problems by finding a common denominator and equivalent fractions to produce like denominators using a variety of tools and strategies.

Example

- Tom is baking a cake. He added 12 teaspoon of vanilla extract to the cake mix. He tasted the batter and determined he needed more, so he added another 34 teaspoon of vanilla extract. How much total vanilla extract did he add to the cake mix?
- Possible student response: A student may decompose one of the fractions to a make a benchmark number (12):
    - 1/2 + 3/4
    - = 1/2 + (2/4 + 1/4)
    - = (1/2 + 2/4) + 1/4
    - = 1 ¼

DRAFT

CLUSTER: 5.NF.B - Apply and extend previous understandings of multiplication and division.

STANDARD: 5.NF.B.3

Standards Statement (JUNE 2021):

Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve problems in authentic contexts involving division of whole numbers that result in answers that are common fractions or mixed numbers.

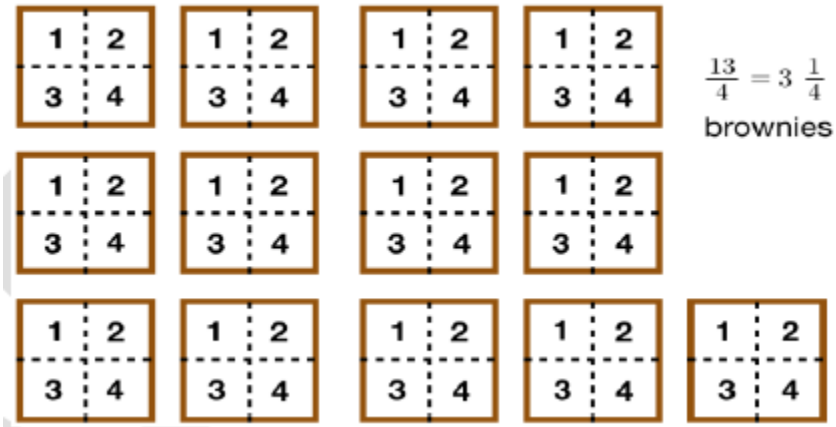
DRAFT Standards Guidance (JUNE 2021):

Boundaries

- As part of this standard, students should have opportunities to use visual models or equations to represent and solve problems.

Example

- If 3 cookies are shared equally among 4 people each person receives ¾ of a cookie.
    - Sample Tasks:
    - If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get?
    - Between what two whole numbers does your answer lie?
- Four children want to share 13 brownies so each child gets the same amount. How many does each child get? Possible solution:



STANDARD: 5.NF.B.4

Standards Statement (JUNE 2021):

Apply and extend previous understanding and strategies of multiplication to multiply a fraction or whole number by a fraction. Multiply fractional side lengths to find areas of rectangles, and represent fractional products as rectangular areas.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Students should explain the meaning of a fraction ab as a multiple of $1b$.
- Students should be exposed to fractions less than 1, equal to 1, and greater than 1.

Teaching Strategies

- Interpret the product of the fraction a/b and a whole number (q) as
    - partitioning the whole number into b parts and counting a parts
    - Repeating the fraction a/b q number of times.
- Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths
- Students should be presented with a variety of real-life, mathematical problems involving multiplication of a fraction and a whole number.
- Students should use their understanding of equivalency to flexibly reason with equivalent fractions based on the context of the problem. Simplifying fractions is not an expectation of this grade level.

Examples

- Understand that ⅔ x 4 can be seen as partitioning 4 into 3 equal parts as well as counting 2 of the 3 (4/3 x 2) parts or as iterating ⅔ four times [(2 x 4)/3]. In general, a/b x q = q/b x a = (a x q)/b.
- Use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15.
- Each cupcake takes 14 cup of frosting. If Betty wants to make 20 cupcakes for a party, how much frosting will she need?
- Mr. Rogers need to make peanut butter and jelly sandwiches for 12 children. He wants to make 34 of a sandwich for each child. How many sandwiches does he need to make?

DRAFT

STANDARD: 5.NF.B.5

Standards Statement (JUNE 2021):

Apply and extend previous understandings of multiplication and division to represent and calculate multiplication and division of fractions. Interpret multiplication as scaling (resizing) by comparing the size of products of two factors.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- As part of this standard, students must be able to
    - Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
    - Explain that multiplying a given number by a fraction greater than 1 results in a product greater than the given number.
    - Explain that multiplying a given number by a fraction equivalent to 1 (such as 4/4) results in the same product as multiplying by 1.
    - Explain that multiplying a given number by a fraction less than 1 results in a product smaller than the given number.

Teaching Strategies

- Students should be presented with a variety of real-life, mathematical situations involving multiplication as scaling (resizing) that include fractions and whole numbers.

Example

- Mrs. Cole needs to make lunch for 12 children at a day care. Each child gets 1/2 of a sandwich. How many whole sandwiches does Mrs. Cole need to make? NOTE: The student should be able to recognize that the solution to 12 x 1/2 will be less than 12 because each child only gets half of a sandwich.

DRAFT

STANDARD: 5.NF.B.6

Standards Statement (JUNE 2021):

Solve problems in authentic contexts involving multiplication of common fractions and mixed numbers.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should be given opportunities to use both visual fraction models and equations to represent and solve problems.
- Students should be given opportunities to use both visual fraction models and equations to represent and solve problems.

Connections

- Solve applied problems involving multiplication of fractions and mixed numbers by using visual fraction models and/or equations to represent the problem.

DRAFT

STANDARD: 5.NF.B.7

Standards Statement (JAN 2021):

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions, including solving problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Division of a fraction by a fraction is not a requirement at this grade. However, students who are able to multiply fractions can develop strategies to divide a fraction by a fraction by reasoning about the relationship between multiplication and division.

Teaching Strategies

- Students should be given opportunities to use both visual fraction models and equations to represent and solve problems.
- Students should begin with modeling for deeper understanding.
- Students should be presented with a variety of contextual, real-life problems involving division of a whole number by a unit fraction and division of a unit fraction by a whole number.

Example

- Create a story context for (1/3) ÷ 4 and use a visual fraction model to show the quotient.
- Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.
- Create a story context for 4 ÷ (1/5) and use a visual fraction model to show the quotient.
- Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.
- How much chocolate will each person get if 3 people share ½ lb of chocolate equally?
- How many ⅓-cup servings are in 2 cups of raisins?
- Knowing the number of groups/shares and finding how many/much in each group/share Four students sitting at a table were given 1/3 of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally? The diagram shows the 1/3 pan divided into 4 equal shares with each share equaling 1/12 of the pan.

DRAFT

5.GM - Geometric Reasoning and Measurement

CLUSTER: 5.GM.A. - Graph points on the coordinate plane to solve real-world and mathematical problems.

STANDARD: 5.GM.A.1

Standards Statement (JUNE 2021):

Graph and name coordinate points in the first quadrant using the standard x, y notation. Understand the coordinate points values represent the distance traveled along the horizontal x-axis and vertical y-axis.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- This is students first formalized introduction to the conventions of coordinate graphing:
    - The first number indicates the distance from the origin on the x-axis.
    - The second number indicates the distance from the origin on the y-axis.
    - The names of the two axes and coordinates (or ordered pairs) correspond (x-axis and x-coordinate, y-axis and y-coordinate).
- In addition to whole numbers, ordered pairs should include the decimal and fractional values of halves and fourths.

Boundaries

- Graphing beyond the first quadrant is not a requirement at this grade.
- All four quadrants of the coordinate plane can be displayed, but students will only plot and label within the first quadrant.

Teaching Strategies

- Students should be provided with a variety of real-life, mathematical problems involving graphing points in the first quadrant.
- Students should interpret coordinate values of points in the context of the problem or situation.

DRAFT

STANDARD: 5.GM.A.2

Standards Statement (JUNE 2021):

Represent authentic contexts and mathematical problems by graphing points in the first quadrant of the coordinate plane. Interpret the meaning of the coordinate values based on the context of a given situation.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be given ample experience with organizing, representing, and analyzing data from real-life contexts.
- Data should not be limited to numerical data collected from linear measurements.
- Students should continue to create dot plots (line plots)
- 1 1 1
- with measurements in fractions of a unit (, ,).
- 2 4 8

Terminology

- Distribution refers to how the data is spread across the graph.
- Dot plots and line plots can be used interchangeably.
- Dot plots should be used for numerical data representation on a number line.
- Numerical data is data that expressed in numbers rather than natural language. An example of numerical data that could be collected is the number of people who attended the movie theater over the course of a month.
- Categorical data is a type of data that is used to group information with similar characteristics. Examples of categorical data that could be collected might be marital status, favorite sport, or favorite type of movie.

Examples

- The coordinate (1,1.5) or (1,1½) means that in the first year, a person grew 1.5 or 1 ½ inches.
- Numerical variable(s): number of pets; categorical variable(s): type of pets, (e.g., cats, dogs, hamsters)"

DRAFT

CLUSTER: 5.GM.B. - Classify two-dimensional figures into categories based on their properties.

STANDARD: 5.GM.B.3

Standards Statement (JUNE 2021):

Classify two-dimensional figures within a hierarchy based on their geometrical properties, and explain the relationship across and within different categories of these figures.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should explore, compare, and contrast polygons based on properties.
- Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

Boundaries

- This objective does not require students to create a hierarchy.
- In Georgia resources and assessments, the inclusive definitions for the classification of shapes are used.

Teaching Strategies

- Polygons should include triangles, quadrilaterals including kites and trapezoids (rectangles, squares, rhombuses, and other parallelograms), pentagons, hexagons, and octagons.
- Properties may include angles, side lengths, symmetry, congruence, and the presence or absence of parallel or perpendicular lines.

Example

- Explain that since all rectangles have four right angles, and squares are rectangles, then all squares have four right angles. Explain that parallelograms and trapezoids are both quadrilaterals, and both have at least one set of parallel sides, but that they differ in that trapezoids have exactly one set and parallelograms have exactly two sets.
- All rectangles have four right angles and squares are rectangles, so all squares have four right angles.

DRAFT

CLUSTER: 5.GM.C - Convert like measurement units within a given measurement system.

STANDARD: 5.GM.C.4

Standards Statement (JUNE 2021):

Convert between different-sized standard measurement units within a given measurement system. Use these conversions in solving multi-step problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Fifth grade is the first time students are expected to convert between different units within the same measurement system.
- Conversion chart should be provided.
- Students should be presented with contextual problems involving distance, weight, volume, and time that are practical and relevant to their everyday lives.
- Students should have opportunities to solve problems involving conversions within both metric and customary systems
    - Customary measurement units include weight (oz., lbs., tons) capacity (fl. oz, cups, pints, quarts, gallons), distance (in., ft., yds., miles).
    - Common metric units include weight (grams), capacity (liters), distance (meters)
        - Common metric conversions include Kilo- (1000), centi- (1/100), & milli- (1/1000)
- Students do not need to convert between customary and metric systems.

Example

- Convert 5 cm to 0.05 m
- Convert 1 gallon = 4 quarts = 8 pints = 16 cups.

DRAFT

CLUSTER: 5.GM.D - Geometric measurement: understand concepts of volume.

STANDARD: 5.GM.D.5

Standards Statement (JUNE 2021):

Recognize that volume is a measurable attribute of solid figures.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
- A solid figure that can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

DRAFT

STANDARD: 5.GM.D.6

Standards Statement (JUNE 2021):

Measure the volume of a rectangular prism by counting unit cubes using standard and nonstandard units.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should have opportunities to use metric, customary and improvised units.

DRAFT

STANDARD: 5.GM.D.7

Standards Statement (JUNE 2021):

Relate volume of rectangular prisms to the operations of multiplication and addition. Solve problems in authentic contexts involving volume using a variety of strategies.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should recognize volume as an attribute of solid figures.
- Students should explore the dimensions of all possible rectangular prisms given a total number of cubic units.

Terminology

- Total volume is defined as the total number of units that fill the space.
    - A solid figure packed with n unit cubes is said to have a volume of n cubic units.
- The dimensions of a rectangular prism can be referred to as length, width, and height.
- A cube with side length 1 unit, called "a unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume (e.g., cubic cm, cubic m, cubic in, cubic ft).

Boundaries

- Work with volume at fifth grade is limited to whole number edge lengths.
- If students are provided with an image of a right rectangular prism, the unit cubes should be visible.

Teaching Strategies

- Students should be provided opportunities to use a variety of strategies including counting cubes, addition and multiplication, and applying the formula.
- Students should investigate real-life problems involving volume to make sense of this concept.
- Students should explore the volume of solid figures from real-life contexts by packing them with unit cubes with no gaps or overlaps.

Examples

- Find the volume of a rectangular prism with whole-number side lengths by packing it with unit cubes.
  Show that the volume is the same as would be found by multiplying the edge lengths or by multiplying the height by the area of the base.
- Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping rectangular prisms by adding the volumes of the non-overlapping parts.
- Given the volume and 2 side lengths, determine the missing side length.

DRAFT

5.DR – Data Reasoning

CLUSTER: 5.DR.A - Pose investigative questions and collect/consider data.

STANDARD: 5.DR.A.1

Standards Statement (JUNE 2021):

Generate questions to investigate situations within the classroom, school or community. Determine strategies for collecting or considering data involving operations with fractions for this grade that can naturally answer questions by using information presented in line plots.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students can generate questions about things they notice and wonder from a real-life situation.
- Based on the posed question, create a plan that determines the appropriate population to survey and how to collect that data.

Terminology

- A statistical investigative question is one that requires data that will vary.

Boundaries

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.

Teaching Strategies

- Students should be provided with learning experiences to collect and analyze both numerical data and categorical data.
- Developing strategies for collecting data include students collaborating to determine ways to collect data.
- Data can be gathered from a variety of sources to answer the statistical investigative question posed.

Example

- Survey question: “How many pets do you have at home?” and “What grade are you in?” to make sure that the sample included only 5th grade students.

DRAFT

CLUSTER: 5.DR.B – Analyze, represent, and interpret data.

STANDARD: 5.DR.B.2

Standards Statement (JUNE 2021):

Analyze graphical representations and describe the distribution of the numerical data through line plots or categorical data through bar graphs. Interpret information presented to answer investigative questions.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be given ample experience with organizing, representing, and analyzing data from real-life contexts.
- Data should not be limited to numerical data collected from linear measurements.
- Students should continue to create dot plots (line plots) with measurements in fractions of a unit (1/2, 1/4, 1/8).

Terminology

- Distribution refers to how the data is spread across the graph.
- Dot plots and line plots can be used interchangeably.
- Dot plots should be used for numerical data representation on a number line.
- Numerical data is data that expressed in numbers rather than natural language. An example of numerical data that could be collected is the number of people who attended the movie theater over the course of a month.
- Categorical data is a type of data that is used to group information with similar characteristics. Examples of categorical data that could be collected might be marital status, favorite sport, or favorite type of movie.

Boundaries

- The mean formula is not an expectation in 5th grade. This concept should be explored visually and conceptually.
- This is the beginning of the progression of the concept of measures of center and will continue to be developed in 6th grade.

Teaching Strategies

- Students should be provided opportunities to read and interpret information presented in line plots.
- Students should be given the opportunity to use manipulatives such as: snap cubes, tiles, etc...to model equal share value.

Example

- Numerical variable(s): number of pets; categorical variable(s): type of pets, (e.g., cats, dogs, hamsters)
- Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.
- “If we combined all of the 5th grade students’ candies and shared them equally with each student so everyone has the same number of candies.” (This is the mean or equal share value.)

DRAFT

Grade 6 – Mathematics Standards and Guidance

6.RP - Proportional Reasoning: Ratios

CLUSTER: 6.RP.A - Understand ratio concepts and use ratio reasoning to solve problems.

STANDARD: 6.RP.A.1

Standards Statement (MAY 2021):

Understand the concept of a ratio in authentic contexts, and use ratio language to describe a ratio relationship between two quantities.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Students should be able to explain the concept of a ratio, such as using part-to-part or part-to-whole.
- Students should be able to fluently use ratio language to describe a ratio relationship between two quantities.
- Students should be able to identify standard fractional notation to compare.

Teaching Strategies

- Students should be able to solve problems involving ratios found in real-life situations.
- Students should be given the opportunity to represent and explain the concept of a ratio and the relationship between two quantities using concrete materials, drawings, tape diagrams (bar models), double number line diagrams, equations, and standard fractional notation

Example

- The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak
- For every vote candidate A received, candidate C received nearly three votes.
- Describe a ratio as a multiplicative relationship between two quantities.
- Model a ratio relationship using a variety of representations.

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STANDARD: 6.RP.A.2

Standards Statement (MAY 2021):

Understand the concept of a unit rate in authentic contexts and use rate language in the context of a ratio relationship.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- When asked contextual, mathematical questions, should demonstrate an understanding of simple multiplicative relationships involving unit rates.
- Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

Terminology

- Students should understand a unit rate as a relationship of a:b where b = 1 (ab associated with a ratio a: b with b ≠ 0 (b not equal to zero), and use rate language).

Teaching Strategies

- Students should create a table of values displaying the ratio relationships to graph ordered pairs of distances and times.
- Students should write equations to represent the relationship between distance and time where the unit rate is the simple multiplicative relationship.
- Students should be able to determine the independent and dependent relationship of rate relationships within contextual, mathematical situations.

Examples

- This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.
- We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.
- In a problem involving motion at a constant speed, list and graph ordered pairs of distances and times, and write an equation such as d = 65t to represent the relationship between distance and time. In this example, 65 is the unit rate or simple multiplicative relationship.

DRAFT

STANDARD: 6.RP.A.3

Standards Statement (MAY 2021):

Use ratio and rate reasoning to solve problems in authentic contexts that use equivalent ratios, unit rates, percents, and/or measurement units.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Students should have opportunities to explore the concept of percents and recognize the connection between fractions, decimal numbers, and percents, such as, 25% of a quantity means 25/100 or .25 times the quantity.
- Students should be able to convert fractions with denominators of 2, 4, 5 and 10 to the decimal notation.
- Students should be able to calculate the percent of a number using proportional reasoning developed through working with ratios and rates.
- Students should be able to solve contextual problems involving finding the whole given a part and the part given the whole.
- Students should determine what percent one number is of another number to solve contextual, mathematical problems.

Teaching Strategies

- Students should be given opportunities to utilize student-selected strategies to solve contextual, mathematical problems involving proportions.
- Students should be given the opportunity to use concrete materials, drawings, tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and equations when solving problems.
- Students can choose a strategy from a variety of strategies developed to solve a specific problem depending on the context.

Connections

- Students should be able to use flexible, strategic thinking to manipulate and transform units appropriately when multiplying or dividing quantities to solve contextual, mathematical problems.
- Students should be able to convert measurement units when given a conversion factor within one system of measurement and between two systems of measurement (customary and metric) using proportional reasoning developed through working with ratios and rates.

Example

- Create and use a table to compare ratios and plotting the pairs of values on the coordinate plane.
- Find missing values in the tables.
- Use unit rates to solve problems, including problems involving unit pricing and constant speed.
- Convert and manipulate measurements using given ratios.
- If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
- Given 1 in. = 2.54 cm, how many centimeters are in 6 inches?

DRAFT

6.NS - Numeric Reasoning: Number Systems

CLUSTER: 6.NS.A - Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

STANDARD: 6.NS.A.1

Standards Statement (MAY 2021):

Represent, interpret, and compute quotients of fractions to solve problems in authentic contexts involving division of fractions by fractions.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Students should use their understanding of equivalency to flexibly reason with equivalent fractions based on the context of the problem. Simplifying fractions is not an expectation of this grade level.
- Students should be able to use the meanings of fractions, multiplication, division and the inverse relationship between multiplication and division to make sense of multiplying and dividing fractions.

Terminology

- Fraction quotients can be represented visually by fraction diagram, concretely with manipulatives, or symbolically with equations to represent the problem.

Teaching Strategies

- Students should be able to utilize fractions with denominators including 2, 3, 4, 5, 6, 8, 10, and 12.
- Students should be able to use numerical reasoning to interpret contextual, mathematical situations involving fractions.
- Students can use a variety of strategies, including but not limited to concrete models, visual fraction models, student-generated strategies, a standard algorithm, or other strategies based on numerical reasoning to represent and solve problems.
- Students should be given the opportunity to apply reasoning strategies and use written methods that make sense to them.
- Students should use flexible, accurate, and efficient written methods to express computational thinking based on numerical reasoning and sense-making developed from learning experiences that focus on the numbers as quantities.

Example

- Reason and solve problems with quotients of fractions using both the measurement and partition models of division (based on what is most appropriate for the fractions in the quotient).
- Describe a context for a given division problem with common fractions, including both measurement and partition contexts.
- Use visual fraction models to represent and solve division problems with common fractions. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient.

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- Use equations and the relationship between multiplication and division to represent and solve a given division problem with fractions.
- For example, (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.)
- How many 3/4-cup servings are in 2/3 of a cup of yogurt?
- How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

DRAFT

CLUSTER: 6.NS.B - Compute fluently with multi-digit numbers and find common factors and multiples.

STANDARD: 6.NS.B.2

Standards Statement (MAY 2021):

Fluently divide multi-digit numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Fluently/Fluency – Students choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Terminology

- Decimal number – a number whose whole number part and fractional part are separated by a decimal point.

Teaching Strategies

- Students should be able to use a variety of part- whole strategies to compute efficiently (area model, partial product, partial quotient).
- The part-whole strategies used should be flexible and extend from previous computation strategies and future work with computation.
- Students should use models and student-selected strategies as an efficient written method of demonstrating place value understanding for each operation (addition, subtraction, multiplication, and division).

DRAFT

STANDARD: 6.NS.B.3

Standards Statement (MAY 2021):

Fluently add, subtract, multiply, and divide positive rational numbers using accurate, efficient, and flexible strategies and algorithms.

DRAFT Standards Guidance (MAY 2021):

Terminology

- Positive rational numbers includes numbers that can be represented by a ratio a/b where a is positive whole number greater than or equal to zero, and b is a non-zero whole number. Such numbers include whole numbers, fractions, and decimals greater than or equal to zero.
- Fluently/Fluency – Students choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Boundaries

- Students should be allowed to choose an appropriate strategy to demonstrate fluency.

Teaching Strategies

- Students should be able to use numerical reasoning to interpret contextual, mathematical situations involving fractions.
- Students should be given the opportunity to apply reasoning strategies while solving problems.

DRAFT

STANDARD: 6.NS.B.4

Standards Statement (MAY 2021):

Determine greatest common factors and least common multiples using a variety of strategies. Apply the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

DRAFT Standards Guidance (MAY 2021):

Clarification

- Students should also be able to apply the least common multiple of two whole numbers less than or equal to 12 to solve contextual, mathematical problems.
- Students should be able to determine the greatest common factor of 2 whole numbers (from 1-100) and use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factors (GCF).

Boundaries

- Find the greatest common factor of two whole numbers less than or equal to 100
- Find the least common multiple of two whole numbers less than or equal to 12.

Teaching Strategies

- Investigate the distributive property using sums and its use in adding numbers 1-100 with a common factor.
- Students should apply these strategies to solve real- life, mathematical problems.
- Note GCF & LCM support use of distributive property.

Example

- Express 36 + 8 as 4 (9 + 2).
- Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.
- Hotdogs come in a package of 8 and buns in a package of 12. How many packages of hot dogs and packages of buns would you need to purchase to have an equal number of hot dogs and buns?

DRAFT

CLUSTER: 6.NS.C - Apply and extend previous understandings of numbers to the system of rational numbers.

STANDARD: 6.NS.C.5

Standards Statement (MAY 2021):

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in authentic contexts, explaining the meaning of zero in each situation.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Students should be able to explain that zero is its own opposite.
- Students should be able to explain that the sign of an integer represents its position relative to zero on a number line.
- Students should be able to show and explain why –(–a) = a. Which is read as, "The opposite of the opposite of a is the same as a."

Terminology

- Rational numbers are numbers that can be written as a fraction where the numerator and denominator are integers.

Teaching Strategies

- Students should be able to use numerical reasoning to interpret and explain the meaning of numerical statements of inequality as the relative position of two integers positioned on a number line.
- Students are introduced to rational numbers. Students should connect their understanding of fractions and integers to comprehend rational numbers as numbers that can be written as a fraction where the numerator and denominator are integers.

Connections

- Temperature above/below zero;
- Elevation above/below sea level, Debits/credit;
- Positive/negative electric charge.

Examples

- Write –3 degrees Celsius > –7 degrees Celsius to express the fact that –3 degree Celsius is warmer than –7 degrees Celsius.
- Interpret –8.3 > –12.3 as a statement that –8.3 is located to the right of –12.3 on a number line oriented from left to right.

DRAFT

STANDARD: 6.NS.C.6

Standards Statement (MAY 2021):

Represent a rational number as a point on the number line. Extend number line diagrams and coordinate axes to represent points on the line and in the coordinate plane with negative number coordinates.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line.
- Recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane.
- Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- Students should use numerical and graphical reasoning to plot points in all four quadrants on the coordinate plane.

Teaching Strategies

- Students should have opportunities to explore this concept using visual models to develop a deeper understanding.
- Number lines should be indicated both vertically and horizontally.
- Students should use numerical and graphical reasoning to show and explain the relationship between ordered pairs and location in quadrants of the coordinate plane.
- Students should extend understanding of number lines and coordinate axes from previous grades to represent points on the line and in the plane with negative number coordinates.

Example

- Find and position integers and other rational numbers on a horizontal or vertical number line diagram.
- Find and position pairs of integers and other rational numbers on a coordinate plane.
- Students should be able to recognize that -a is the same distance from zero as a, and therefore, are opposites of each other.

DRAFT

STANDARD: 6.NS.C.7

Standards Statement (MAY 2021):

Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. Write, interpret, and explain statements of order for rational numbers and absolute value in authentic applications.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation
- Distinguish comparisons of absolute value from statements about order.
- Students should be introduced to the absolute value symbol with this learning objective, i.e., |−3/4|.
- Students should conclude through exploration that absolute value and distance are always expressed as a positive value.

Terminology

- Absolute value is a number's distance from zero (0) on a number line.

Example

- Interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.
- Write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.
- For an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.
- Recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.
- For an account balance of –51.25 dollars, write |– 51.25| = 51.25 to describe the size of the debt in dollars.

DRAFT

STANDARD: 6.NS.C.8

Standards Statement (MAY 2021):

Graph points in all four quadrants of the coordinate plane to solve problems in authentic contexts. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

DRAFT Standards Guidance (MAY 2021):

Teaching Strategies

- Students should be expected to solve problems within the context of a graph only.
- Understand that the slope of these lines is undefined or 0.

Connections

- Students should be able to solve contextual, mathematical problems when graphing points.

Example

- Rectangle RSTU has vertices at (−4,3), SS(−4, −2), TT(5, −2) and UU(5,3).
- Plot the rectangle on a coordinate plane and find the perimeter of the figure.

DRAFT

6.AEE - Algebraic Reasoning: Expressions and Equations

CLUSTER: 6.AEE.A - Apply and extend previous understandings of arithmetic to algebraic expressions.

STANDARD: 6.AEE.A.1

Standards Statement (MAY 2021):

Write and evaluate numerical expressions involving whole-number bases and exponents.

DRAFT Standards Guidance (MAY 2021):

Teaching Strategies

- Extend previous understanding by using brackets and parentheses and order of operations and exponents.
- Students should interpret real-life, mathematical situations to write and evaluate numerical expressions.

DRAFT

STANDARD: 6.AEE.A.2

Standards Statement (MAY 2021):

Write, read, and evaluate expressions in which letters stand for numbers. Apply knowledge of common mathematical terms to move between the verbal and mathematical forms of an expression including expressions that arise from authentic contexts.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Students should write expressions that record operations with numbers and with letters standing for numbers.
- Students should evaluate algebraic expressions for a given value of a variable, using the order of operations.

Boundaries

- Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in authentic problems.
- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
- Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Teaching Strategies

- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
- Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
- Students should identify parts of an expression using mathematical terms (sum, difference, term, product, factor, quotient, coefficient, variable, constant); view one or more parts of an expression as a single entity.
- Students should translate from a word form into variable expression.
- Students should understand letters called variables represent unknown numbers and the same rules apply in operations with numbers also apply in operations with variables.

Example

- Express the calculation subtract y from 5 as 5 – y.
- Describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
- Use the formulas V = s^3 and A = 6 s^2 to find the volume and surface area of a cube with sides of length s = 1/2.
- Express the calculation “Subtract x from 9” as 9 – x.
- Describe the expression 2(8+7) as a product of two factors; view (8+7) as both a single entity and a sum of two terms.
- Some of the students at Georgia Middle School like to walk to and from school. They always walk unless it rains. Let d be the distance in miles from a student's home to the school.

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- Write two different expressions that represent how far a student travels by walking in a two-week period if there is one rainy day each week.
    - Possible Solution: The distance to school, and therefore home, is d. Thus, the student rides (d + d) miles in one day. Equivalently, she rides (2d) miles in one day. Repeatedly adding the distance traveled in one day for each school day of the week, we find that in one week the student travels (2d + 2d + 2d + 2d + 2d) miles. Equivalently, she travels 5(2d) or (10d) miles in a normal, rain free week.

DRAFT

STANDARD: 6.AEE.A.3

Standards Statement (MAY 2021):

Apply the properties of operations to generate equivalent expressions and to determine when two expressions are equivalent.

DRAFT Standards Guidance (MAY 2021):

Clarification

- Identify when two expressions are equivalent such as when the two expressions name the same number regardless of which value is substituted into them.

Boundaries

- This standard includes distributive property and combining like terms.

Example

- Apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x.
- Apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y)
- Apply properties of operation to the expression y + y + y to produce the equivalent expression 3y and know they are equivalent because they name the same number regardless of which number y stands for.

DRAFT

CLUSTER: 6.AEE.B - Reason about and solve one-variable equations and inequalities.

STANDARD: 6.AEE.B.4

Standards Statement (MAY 2021):

Understand solving an equation or inequality as a process of answering which values from a specified set, if any, make the equation or inequality true. Use substitution to determine which number(s) in a given set make an equation or inequality true.

DRAFT Standards Guidance (MAY 2021):

Teaching Strategies

- Students should be able to use algebraic reasoning to solve an equation as a process of answering a contextual question and explain their reasoning.
- When solving an equation or inequality as a process of answering a question, students should be able to explain why specific values from a specified set, if any, make the equation or inequality true.
- Students should use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Example

- Use an inequality of the form $x > c$ or $x < c$.

DRAFT

STANDARD: 6.AEE.B.5

Standards Statement (MAY 2021):

Use variables to represent numbers and write expressions when solving problems in authentic contexts.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

DRAFT

STANDARD: 6.AEE.B.6

Standards Statement (MAY 2021):

Write and solve equations of the form x + p = q and px = q in problems that arise from authentic contexts for cases in which p, q and x are all nonnegative rational numbers.

DRAFT Standards Guidance (MAY 2021):

Teaching Strategies

- p, x, and q include non-whole numbers. Students should be able to solve equations of this form using strategies such as related equations, fact families, inverse operations, and visual models.
- Students should have opportunities to use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction and multiplication and division when solving one-step equations.
- Students should be able to solve equations presented in contextual, mathematical problems involving positive rational numbers using number sense, properties of arithmetic and the idea of maintaining equality on both sides of the equation.
- Students should be able to interpret a solution in the original context and assess the reasonableness of results.

DRAFT

STANDARD: 6.AEE.B.7

Standards Statement (MAY 2021):

Write inequalities of the form x > c and x < c to represent constraints or conditions to solve problems in authentic contexts. Describe and graph on a number line solutions of inequalities of the form x > c and x < c.

DRAFT Standards Guidance (MAY 2021):

Clarification

- Recognize that inequalities of the form $x>c$ or $x<c$ have infinitely many solutions.
- Represent solutions of such inequalities on number line diagrams.

Teaching Strategies

- Students should represent contextual, mathematical situations using inequalities involving variables.
- Students should be able to create contextual, mathematical situations corresponding to specific inequalities.
- This objective includes the use of the symbols, < , > , = , ≤ , ≥.

DRAFT

CLUSTER: 6.AEE.C - Represent and analyze quantitative relationships between dependent and independent variables.

STANDARD: 6.AEE.C.8

Standards Statement (MAY 2021):

Use variables to represent and analyze two quantities to solve problems in authentic contexts. Including those that change in relationship to one another; write an equation to express one quantity in terms of the other quantity.

DRAFT Standards Guidance (MAY 2021):

Boundaries

- Students should be able to represent equations involving positive variables and rational numbers.
- Students should have opportunities to solve contextual, mathematical problems.

Teaching Strategies

- Students should have an opportunity to solve problem situations with variables in all positions.
- Students should be able to explain that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.

Example

- Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example:
    - In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

DRAFT

6.GM - Geometric Reasoning and Measurement

CLUSTER: 6.GM.A - Solve real-world and mathematical problems involving area, surface area, and volume.

STANDARD: 6.GM.A.1

Standards Statement (MAY 2021):

Find the area of triangles, quadrilaterals, and other polygons by composing into rectangles or decomposing into triangles and other shapes. Apply these techniques to solve problems in authentic contexts.

DRAFT Standards Guidance (MAY 2021):

Terminology

- A polygon is a closed figure with at least three straight sides and angles;
- A regular polygon is when all sides are equal and all angles are equal
- An irregluar polygon is when all sides are not equal or all angles are not equal.

Teaching Strategies

- Apply these techniques in the context of solving authentic mathematical problems.
- Students should be able to use knowledge of area of a rectangle to determine the area of a triangle. Students should have opportunities to find the area of a triangle by decomposing the rectangle into two triangles.
- Students should conclude the area of the triangle is half the area of the rectangle and the area of the rectangle is twice the area of the triangle.
- Therefore, the formula for the area of a triangle is 12 x base x height or (basexheight)/2.
- Students should be able to use geometric and spatial reasoning to calculate the area of a triangle, quadrilateral, and regular polygon by composing or decomposing into triangles, rectangles, and other shapes.
- Students should be presented with mathematical problems found in the real world.
- Students should be able to decompose regular and irregular polygons into triangles and quadrilaterals in a way that makes sense from their perspective.

DRAFT

STANDARD: 6.GM.A.2

Standards Statement (MAY 2021):

Find the volume of a right rectangular prism with fractional edge lengths by filling it with unit cubes of appropriate unit fraction edge lengths. Connect and apply to the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths to solve problems in authentic contexts.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Students should make the connection between (length) x (width) and the area of the base to connect this formula to other three-dimensional volume formulas.
- Show that the volume is the same as would be found by multiplying the edge lengths of the prism.
-

Teaching Strategies

- Apply these techniques in the context of solving authentic mathematical problems.
- Students should be able to calculate the volume of a right rectangular prism with fractional edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism.
- Students should apply the formula for the volume of a right rectangular prism in the context of solving contextual, mathematical problems to meet this learning objective.

DRAFT

STANDARD: 6.GM.A.3

Standards Statement (MAY 2021):

Draw polygons in the 4-quadrant coordinate plane given coordinates for the vertices and find the length of a side. Apply these techniques to solve problems in authentic contexts.

DRAFT Standards Guidance (MAY 2021):

Boundaries

- Measurements of between vertices can be done using standardized measurement tools on graph paper. Formal use of the distance formula is not an expectation at this grade level.

Teaching Strategies

- Students should be able to solve problems with polygons when given coordinate pairs with or without a coordinate grid.

Connections

- Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.
- Students should apply the techniques of graphing in the coordinate plane in the context of solving contextual problems involving the application of algebra through geometry.

DRAFT

STANDARD: 6.GM.A.4

Standards Statement (MAY 2021):

Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures, including those from authentic contexts.

DRAFT Standards Guidance (MAY 2021):

Teaching Strategies

- Apply these techniques in the context of solving authentic mathematical problems.
- Students should use various tools and strategies including a picture or physical model of a net to measure the surface area of three-dimensional figures that are composed of rectangular and triangular faces when solving contextual, mathematical problems.
- Students should be provided the net of three- dimensional figures to ensure grade level appropriateness.

DRAFT

6.DR – Data Reasoning

CLUSTER: 6.DR.A. - Formulate Statistical Investigative Questions.

STANDARD: 6.DR.A.1

Standards Statement (MAY 2021):

Formulate and recognize statistical investigative questions as those that anticipate changes in descriptive data related to the question and account for it in the answers.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Students can generate questions about things they notice and wonder from a real-life situation.
- Students should be able to generate their own statistical questions.

Terminology

- A statistical question is one that requires data that vary.
- A statistical investigative question is one that allows for exploration through statistical inquiry and reasoning.

Teaching Strategies

- Students should be able to use the statistical process to formulate questions. The statistical process involves asking a statistical investigative question, collecting the data, analyzing the data, and interpreting the results.

Example

- "How old are the students in my school?" is a statistical question because it anticipates variability in students' ages.
- "How old am I?" is a question used to collect data to answer the investigative question.

DRAFT

CLUSTER: 6.DR.B. – Collect and Consider Data.

STANDARD: 6.DR.B.2

Standards Statement (MAY 2021):

Collect and record data with technology to identify and describe the characteristics of numerical data sets using quantitative measures of center and variability.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (6.SP.A.2)
- Students should understand the concept of outliers.
- Students should be able to describe the nature of the statistical attribute under investigation, including how it was measured and its units of measurement.

Boundaries

- A set of data collected to answer a statistical question has a distribution, which can be described by
    - Measures of center - mean, median, mode
    - Measures of variation (spread) - range, interquartile range, and/or mean absolute deviation
    - Descriptions of overall shape - symmetrical vs non- symmetrical
- The focus of mean absolute deviation (MAD) is visualizing deviations from the mean as a measure of variability as opposed to a focus on calculating MAD.
    - In sixth grade, students should explore the conceptual idea of MAD – not the formula.
    - Students should be able to apply their understanding of absolute value (rather than use operations on negative integers) in the context of MAD.

Teaching Strategies

- Students should be able to analyze the shape of a data distribution and determine which measure of center and variability best describes the data based on the shape of the data and the context in which the data was gathered.

Example

- Arthur and Aaron are on the same 6th grade basketball team. Both players have scored an average of ten points over the past ten games. Here are the students' number of points scored during each of the last ten games.
    - Arthur: 9, 10, 10, 11, 11, 9, 10, 10, 10, 10
    - Aaron: 16, 18, 4, 3, 5, 13, 18, 3, 13, 7
    - Which student is more consistent?
    - Possible Student Response/Solution: Arthur is more consistent because his MAD is smaller than Aaron's MAD; Arthur has less variability than Aaron.

Note: Draft content merged OR review with reviews from GA and ID 2021 draft reviews. Alignment to OR expectations is expected to occur in Fall 2021 and finalized by early 2022. Comments, edits, and suggestions can be sent to ODE.MathProject@ode.state.or.us for review.

CLUSTER: 6.DR.C. – Analyze, summarize, and describe data.

STANDARD: 6.DR.C.3

Standards Statement (MAY 2021):

Analyze data representations and describe measures of center and variability of quantitative data using appropriate displays.

DRAFT Standards Guidance (MAY 2021):

Clarification

- Recognize that a measure of center for a numerical data set is a single number that summarizes all of the values in the data set, while a measure of variation is a single number that describes how the values in the data set vary from one another. (6.SP.A.3)
- Display numerical data in plots on a number line, including dot plots and histograms. (6.SP.A.4)
- Students have experience with displaying categorical data using bar graphs from elementary grades. In sixth grade, students are extending their understanding of analyzing categorical data displayed on histograms.

Boundaries

- Sixth grade students should be able to create dot plots and box plots to analyze the results of a statistical investigation.
- Sixth grade students should focus on describing and interpreting data displayed.
- Students should be able to identify that each quartile presented in a box plot represents 25% of the data set.

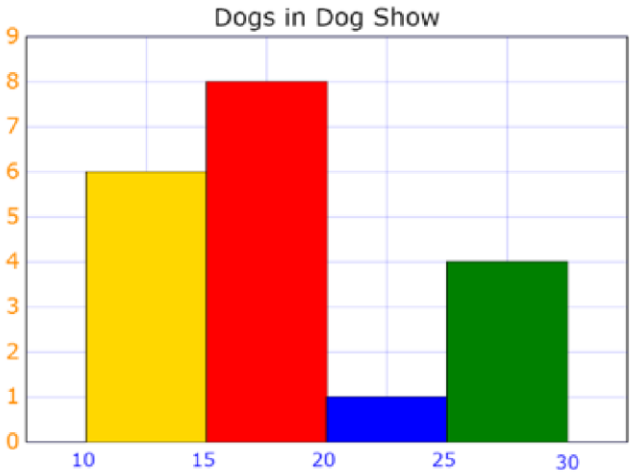
Teaching Strategies

- Students should be able to analyze the shape of a data distribution and determine the impact single data points have on the data set represented visually.
- Describe the impact that inserting or deleting a data point has on the mean, median, and mode of a data set.
- As a result of an investigation, students should summarize categorical and quantitative (numerical) data sets in relation to the context.
- Students should be able to describe the nature of the statistical attribute under investigation, including how it was measured and its units of measurement.

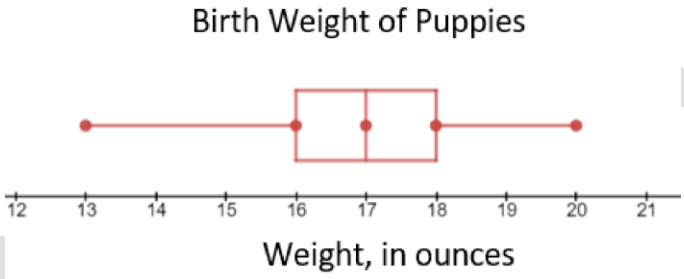
Examples

- Categorical Example:

DRAFT



-
    - What could be the weight of the smallest dog? The largest?
- Quantitative (numerical) Example:
    - Here are the birth weights, in ounces, of all the puppies born at a kennel in the past month.



    -
- What do you notice and wonder about the distribution of the puppy weights?

CLUSTER: 6.DR.D. – Interpret data and answer investigative questions.

STANDARD: 6.DR.D.4

Standards Statement (MAY 2021):

Interpret quantitative measures of center to describe differences between groups from data collected to answer investigative questions.

DRAFT Standards Guidance (MAY 2021):

Clarifications

- Identify, describe, and interpret the characteristics of numerical data sets using quantitative measures of center and variability.
- Additional descriptions include number of observations, measurement attributes, and shape of distribution. (6.SP.A.5)

Terminology

- In sixth grade, students should explore the conceptual idea of MAD – not the formula.
- Students should be able to apply their understanding of absolute value (rather than use operations on negative integers) in the context of MAD.

Boundaries

- Students should be able to determine the number of observations from a context or diagram.
- Students should be able to describe the distribution of a quantitative (numerical) variable collected to answer a statistical investigative question, including its center (median, mean), variability (interquartile range (IQR), mean absolute deviation (MAD), and range), and overall shape (symmetrical vs non-symmetrical).
- Students should be able to describe the nature of the statistical attribute under investigation, including how it was measured and its units of measurement.

Teaching Strategies

- Identification and description of data characteristics related to their context includes:
    - Reporting the number of observations.
    - Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    - Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
- Students should explore conceptually the measures of center (mean, median) and variability (interquartile range and range) for a set of numerical data gathered from contextual, mathematical situations and use these measures to describe the shape of the data presented in various forms.

DRAFT

Grade 7 – Mathematics Standards and Guidance

7.RP - Proportional Reasoning: Ratios and Probability

CLUSTER: 7.RP.A - Analyze proportional relationships and use them to solve mathematical problems in authentic contexts.

STANDARD: 7.RP.A.1

Standards Statement (JUNE 2021):

Solve problems in authentic contexts involving unit rates associated with ratios of fractions.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Ratios of fractions refers to complex fractions where the numerator and/or denominator of a ratio includes a fraction, such as ¼ ÷ ½ is also the ratio of (¼)/(½)

Teaching Strategies

- This includes ratios of lengths, areas and other quantities measured in like or different units.
- Students should have opportunities to create visual representations to solve complex ratio problems.
- Students should build upon their understanding of fractions as a form of division.
- Students should build upon their fluency in division of fractions.
- Students should be able to solve problems involving unit rate presented in practical, real-life situations.

Example

- For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour.

DRAFT

STANDARD: 7.RP.A.2

Standards Statement (JUNE 2021):

Recognize and represent proportional relationships between quantities in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Identify the constant of proportionality (unit rate) within various representations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should demonstrate a conceptual understanding of slope.
- Students should recognize equations in the form y = mx are proportional.
- Students should know that a graph with a straight line through the origin is proportional.
- Explain what a point (x,) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0,0) and (1,r) where r is the unit rate.
- This standard builds on students' understanding of unit rates from 6th grade.

Boundaries

- In seventh grade, students are expected to understand that unit rate and constant of proportionality are the same.

Teaching Strategies

- Represent proportional relationships using equations
- Identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Recognize the constant of proportionality as both the unit rate and as the multiplicative comparison between two quantities.
- Decide whether two quantities are in a proportional relationship,

Examples

- If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
- Jennifer rides on a train for 6 hours and travels 360 miles. How many miles per hour does she travel?
- Mary deposits $115 into her bank account every month, represented by the equation d = 115m. Identify the unit rate from this situation.
- Erik feeds stray cats near his house. A graph shows different amounts of cat food he puts out based on the number of cats near his house. Erik graphs point P to represent the unit rate. What does point P mean in terms of the situation? Cups of cat food per cat.
- Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
- Mark was looking to fertilize his lawn, which is 432 sq. ft. He read the packages of 2 different fertilizer bags to see how much should be used. Bag A stated 2 ounces per 4 square feet and Bag B can be represented using the table below:
    - Ounces 2 4 12 Square Feet 3 6 18
    - What is the unit rate for each bag? Which bag should Mark purchase for his lawn? Why?

STANDARD: 7.RP.A.3

Standards Statement (JUNE 2021):

Use proportional relationships to solve ratio and percent problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Simple interest – a quick and easy method of calculating the interest charge on a loan. Simple interest is determined by multiplying the daily interest rate by the principal by the number of days that elapse between payments. Simple Interest = (principal) * (rate) * (# of periods)
- Tax – is money that people must pay to the government.
- Markups and markdowns - increase and decrease in the amount of a quantity
- Gratuities - a tip given to a waiter, taxicab driver, etc.
- Commissions - a fee paid to an agent as compensation for completing a transaction.

Boundaries

- This includes solving multi step problems involving simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.

Teaching Strategies

- Student should be able to identify, represent, and use proportional relationships between quantities using verbal descriptions, tables of values, equations, and graphs to model contextual, mathematical problems: translate from one representation to another.
- Students should be able to model contextual, mathematical relationships involving constant rates where the initial condition starts at 0 using tables of values and graphs.
- Students should be able to represent proportional relationships using equations.
- Students should be able to analyze and make decisions about relationships using proportional reasoning strategies, which may include but not limited to graphing on a coordinate plane and/or observing whether a graph is a straight line passing through the origin
- Students may use flexible strategies such as a + 0.05a = 1.05a with the understanding that adding a 5% tax to a total is the same as multiplying the total by 1.05.

Example

- If the total cost, t, is proportional to the number, n, of items purchased at a constant price, p, the relationship between the total cost and the number of items can be expressed as t = np.
- If Tina uses 2 eggs to make 6 pancakes and Allison uses 4 eggs to make 12 pancakes, is this proportional?
- Jane runs 12 miles in 2.5 hours. Sarah runs 14 miles 3.5 hours. Are Jane and Sarah running at the same rate? Justify your answer.

DRAFT

CLUSTER: 7.RP.B- Investigate chance processes and develop, use, and evaluate probability models.

STANDARD: 7.RP.B.4

Standards Statement (JUNE 2021):

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Represent probabilities as fractions, decimals, and percents.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Descriptions may include impossible, unlikely, equally likely, likely, and certain.
- Know that a probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is equally likely, and a probability near 1 indicates a likely event.

Teaching Strategies

- Students should be able to represent the probability as a fraction, decimal numbers, or percent.

DRAFT

STANDARD: 7.RP.B.5

Standards Statement (JUNE 2021):

Use experimental data and theoretical probability to make predictions. Understand the probability predictions may not be exact.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Approximate the (theoretical) probability of a chance event by collecting data and observing its long-run relative frequency (experimental probability). Predict the approximate relative frequency given the (theoretical) probability.

Teaching Strategies

- Students should draw upon understanding of proportional relationships to make predictions.
- Students should be able to predict the approximate, relative frequency given the theoretical probability.

Example

- When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
- When drawing chips out of a bag containing an unknown number of red and white chips, estimate the probability of selecting a particular chip color given 50 draws.

DRAFT

STANDARD: 7.RP.B.6

Standards Statement (JUNE 2021):

Develop a probability model and use it to find probabilities of events. Compare theoretical and experimental probabilities and explain possible sources of discrepancy if any exists.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Teaching Strategies

- Probability models may include various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips.
- Students should have multiple opportunities to collect data using physical objects, graphing calculators, or web-based simulations.

Example

- If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
- Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
- Kim calculates the probability of landing on heads when tossing a coin to be 50%. She uses this to predict that when Tiffany tosses a coin 20 times, the coin will land on heads 10 times. When Tiffany performed the experiment, the coin landed on heads 7 times. Explain possible reasons why Kim's prediction and Tiffany's results do not match.

DRAFT

STANDARD: 7.RP.B.7

Standards Statement (JAN 2021):

Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

DRAFT Standards Guidance (JAN 2021):

Clarifications

- Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Teaching Strategies

- Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
- Design and use simulations to generate experimental frequency data for compound events.

Example

- Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?
- Determine the probability of "rolling double sixes"

DRAFT

7.NS - Numeric Reasoning: Number Systems

CLUSTER: 7.NS.A - Apply and extend previous understandings of operations with fractions.

STANDARD: 7.NS.A.1

Standards Statement (JUNE 2021):

Apply and extend previous understandings of addition, subtraction and absolute value to add and subtract rational numbers in authentic contexts. Understand subtraction as adding the additive inverse, $p – q = p + (–q)$.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be allowed to explore the signs of integers and what they really mean to discover integer rules.

Terminology

- Part-whole reasoning refers to how numbers can be split into parts to add and subtract numbers more efficiently.
- In the equation $3 + −3 = 0$, 3 and −3 are additive inverses of each other.
- Students should represent a variety of types of rational numbers on a number line diagram presented both horizontally and vertically.

Teaching Strategies

- Represent operations with rational numbers both visually and numerically, including number line diagrams.
- Solve authentic mathematical problems involving adding and subtracting rational numbers.
- Interpret operations of rational numbers by describing authentic contexts.
- Apply properties of operations such as identity, inverse, distributive, associative and commutative properties.
- Students should be able to use the Commutative and Associative properties to combine more than two rational numbers flexibly.
- Students should be able to add and subtract integers and other rational numbers presented within contextual, mathematical problems, using strategic thinking and a variety of tools.

Example

- $(–8) + 5 + (–2)$ may be solved as $(–8) +(–2) + 5$ to first make –10 by using the Commutative Property.
- Your bank account balance is − $25.00. You deposit $25.00 into your account. The net balance is $0.00.
- $6 + (–4)$ is 4 units to the left of 6 on a horizontal number line or 4 units down from 6 on a vertical number line.
- Find the distance between a submarine submerged at a depth of 27 3/4 feet below sea level and an airplane flying at an altitude of 1262 1/2 feet above sea level.
- $−12 – (–2)$ is the same expression as $−12 + – (–2)$, which is 2 units to the right of −1/2 on a horizontal number line or 2 units up from −1/2 on a vertical number line.

DRAFT

STANDARD: 7.NS.A.2

Standards Statement (JUNE 2021):

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. Interpret operations of rational numbers solving problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be allowed to explore the signs of integers and what they really mean to discover integer rules.
- If p and q are integers ($q \neq 0$), then $–(-p/q)=(-p)/q= p/(-q)$
- Students should be allowed to explore the signs of integers and what they really mean to discover integer rules.
- Students should be able to reason about direction on a number line when representing multiplication and division using the tool.

Boundaries

- Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number.
- Apply properties of operations as strategies to multiply and divide rational numbers.
- Convert a rational number to a decimal using division; know that the decimal form of a rational number terminates or eventually repeats.

Teaching Strategies

- Represent operations with rational numbers both visually and numerically,
- Students should build upon previous work with number lines for addition and subtraction.
- Solve authentic mathematical problems involving multiplying and dividing rational numbers.
- Interpret operations of rational numbers by describing real-world contexts.
- Apply properties of operations such as identity, inverse, distributive, associative and commutative properties.
- Student should have opportunities to use concepts of repeated addition and the meaning of a negative sign as the “opposite of,” with both models and representations, leading to deriving the rules for multiplying signed numbers.
- Models may include, but are not limited to, number lines and counters.
- Students can represent multiplication and division using number lines, counters, etc.
- Students should be able to use the Commutative and Associative properties to combine more than two rational numbers flexibly.

Examples

- $–(20/5) = –4$ is the same as $(−20)/5= –4$ and $20/(−5) = – 4$
- Create a model and context for each of the products. Write and model the family of equations related to $2 \times 3 = 6$.
- $(– 8) * 2 * (–5)$ may be solved as $(– 8) * (2*(–5))$ to multiply by negative ten, using the Associative Property.

DRAFT

STANDARD: 7.NS.A.3

Standards Statement (JUNE 2021):

Understand that equivalent rational numbers can be written as fractions, decimals and percents.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should build upon their understanding of percents as a ratio comparison to 100.
- This is an extension of previous understanding from 6th grade of writing common fractions as decimal numbers and percents.

Boundaries

- Use long division to convert fractions to decimals.
- Students should know that every rational number can be written as the ratio of two integers, terminating decimal numbers, or repeating decimal numbers.

Example

- A water well drilling rig has dug to a height of −60 feet after one full day of continuous use. If the rig has been running constantly and is currently at a height of −143.6 feet, for how long has the rig been running? (Modified from Illustrative Mathematics)
- Identify whether the decimal form of a rational number is a terminating or repeating decimal.
- Convert terminating decimals to fractions.
- If Sara makes $25 an hour gets a 10% raise, she will make an additional 110 of her salary an hour, or $2.50, for a new salary of $27.50.

DRAFT

7.AEE - Algebraic Reasoning: Expressions and Equations

CLUSTER: 7.AEE.A - Use properties of operations to generate equivalent expressions.

STANDARD: 7.AEE.A.1

Standards Statement (JUNE 2021):

Identify and write equivalent expressions with rational numbers by applying associative, commutative, and distributive properties.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Identify like terms and combine like terms to create equivalent expressions.
- Apply the distributive property to factor and expand linear expressions.
- Use numerical substitution to identify equivalent expressions.

Example

- $4x+2=2(2x+1)$ and $-3(x-5/3)=-3x+5$
- If Massey and Brenda both get paid a wage of $11 per hour, but Massey was paid an additional $55 for overtime, the expression 11(M+B) + 55 may be more clearly interpreted as 11M+55+11B for purposes of understanding Brenda's pay separated from Massey's pay.

DRAFT

STANDARD: 7.AEE.A.2

Standards Statement (JUNE 2021):

Understand that rewriting an expression in different forms in a contextual problem can show how quantities are related.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Building on work in Grade 6, where students used conventions about the order of operations to rewrite simple expressions such as 2(3 + 8x) as 6 +16x and 10p-2 as 2(5p-1), students now encounter linear expressions with more operations that require an understanding of integers, such as 7 - 2(3 - 8x).

Examples

- For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."
- For example, 3 friends each buy a drink for x dollars and popcorn for y dollars. The total cost could be expressed by "x + x + x +y + y + y", "3x +3y" and "3(x+y)"
- A shirt at a clothing store is on sale for 20% off the regular price, p. The discount can be expressed as $0.2p$. The new price for the shirt can be expressed as $p-0.2p$ or $0.8p$.
- A rectangle is twice as long as it is wide. One way to write an expression to find the perimeter would be w + w + 2w + 2w. Write the expression in two other ways.
- Write an equivalent expression for 9 – 7(2x + 4).

DRAFT

CLUSTER: 7.AEE.B - Solve mathematical problems in authentic contexts using numerical and algebraic expressions and equations.

STANDARD: 7.AEE.B.3

Standards Statement (JUNE 2021):

Write and solve problems in authentic contexts using expressions and equations with positive and negative rational numbers in any form. Contexts can be limited to those that can be solved with one or two-step linear equations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to fluently solve equations of the specified forms presented in the learning objective.
- Students should use the properties of equality to solve for the value of a variable.

Terminology

- Fluently/Fluency – Students choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Boundaries

- Continue to build on 6th grade objectives of writing and solving one-step equations from a problem situation to multi-step problem situations. This is also another context for students to practice using rational numbers including: integers, and positive and negative fractions and decimal numbers.

Teaching Strategies

- Solve problems in authentic contexts leading to equations such as form $px + q = r$ and $p(x + q) = r$, in which p, q, and r are specific rational numbers in any form (whole numbers, fractions, and decimals).
- Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- Apply properties of operations as strategies to calculate with numbers in any form; convert between forms as appropriate; Assess the reasonableness of answers using mental computation and estimation strategies.
- Students should be able to represent relationships in various contextual, mathematical situations with equations involving variables and positive and negative rational numbers and explain the meaning of the solution based on the context.
- Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Examples

- For example, “The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?” This can be solved with the equation $6x = 54$.
- Vicky and Bob went to a store to buy school supplies. Vicky spent a total of $22 on school supplies. She spent $13 on a book and spent the rest of the money on notebooks. The store sells notebooks for $1.50 each. Without using a variable, determine the number of notebooks Vicky bought.

DRAFT

- Write an equation that can be used to find the number of notebooks Vicky bought. Use the variable v for the number of notebooks. Solve the equation. Explain the similarities and differences between finding the number of notebooks Vicky bought with and without a variable, paying attention to the sequence of your operations.

DRAFT

STANDARD: 7.AEE.B.4

Standards Statement (JUNE 2021):

Use variables to represent quantities in an authentic mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Solve word problems leading to equations of the form $px+q=r$ and $(x+q)=r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- Solve word problems leading to inequalities of the form $px+q>r$ or $px+q<r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Teaching Strategies

- Students should be able to represent relationships in various contextual, mathematical situations with inequalities involving variables and positive and negative rational numbers.
- Students should be able to fluently solve inequalities of the specified forms. To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.
- Students should use the properties of inequality to solve for the value of a variable.
- When identifying a specific value for p, q, and r, any rational number can be used.
- Students should be able to graph and interpret the solution of an inequality used as a model to explain real-life phenomena.

Example

- For example, as a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.
- As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions.

DRAFT

7.GM - Geometric Reasoning and Measurement

CLUSTER: 7.GM.A - Draw construct, and describe geometrical figures and describe the relationships between them.

STANDARD: 7.GM.A.1

Standards Statement (JUNE 2021):

Solve problems involving scale drawings of geometric figures. Reproduce a scale drawing at a different scale and compute actual lengths and areas from a scale drawing.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should understand scale factor as a rate comparison between similar figures and scale drawings.
- Students should build upon their understanding of proportional relationships.

Teaching Strategies

- Students should be given opportunities to use technology and tools to reproduce scale drawings.
- Students should have opportunities to use proportional reasoning to compute unknown lengths by setting up proportions in tables or equations, or they can reason about how the lengths compare multiplicatively.
- Students should be able to determine the dimensions of figures when given a scale and identify the impact of a scale on actual length (one-dimension) and area (two–dimensions). Students should be able to identify the scale factor given two figures.
- Using a given scale drawing, students should be able to reproduce the drawing at a different scale. Students should understand that the lengths will change by a factor equal to the product of the magnitude of the two size transformations.

Connections

- Opportunity to connect to proportional reasoning to explain why the slope, m, is the same between any two distinct points (7.NRP.A.2).
- This can lead to recognizing patterns in perimeter and area of similar geometric figures.

Example

- Mariko has an 1/4 inch scale-drawing (1/4 inch=1 foot) of the floor plan of her house. On the floor plan, the scaled dimensions of her rectangular living room are 4-1/2 inches by 8-3/4 inches. What is the area of her living room in square feet?

DRAFT

STANDARD: 7.GM.A.2

Standards Statement (JUNE 2021):

Draw triangles from three measures of angles or sides. Understand the possible side lengths and angle measures that determine a unique triangle, more than one triangle, or no triangle.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine unique triangles, more than one triangle, or no triangle.

Boundaries

- Know when 3 side lengths will form a triangle.
- Know that the angle measures in a triangle have a sum of 180 degrees.

Teaching Strategies

- Students should be provided opportunities to draw triangles with ruler and protractor, and with technology.

Example

- A triangle with side lengths 3 cm, 4 cm, and 5 cm exists. Use a compass and ruler to draw a triangle with these side lengths. (Modified from Engage NY M6L9)

DRAFT

CLUSTER: 7.GM.B - Solve mathematical problems in authentic contexts involving angle measure, area, surface area, and volume.

STANDARD: 7.GM.B.3

Standards Statement (JUNE 2021):

Understand the relationship between area and circumference of circles. Choose and use the appropriate formula to solve problems with radius, diameter, circumference and area of circles.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Know that a circle is a two-dimensional shape created by connecting all of the points equidistant from a fixed point called the center of the circle.
- Develop an understanding of circle attributes including radius, diameter, circumference, and area and investigate the relationships between each.
- Informally derive and know the formulas for the area and circumference of a circle and use them to solve problems.

Terminology

- Students should know how to write responses in terms of pi.
- Special Note: The terms pi, radius, diameter, and circumference are new academic vocabulary for students.
    - Pi - The ratio of a circle's circumference to its diameter.
    - Radius - The distance from the center to the circumference of a circle.
    - Diameter - The distance from one point on a circle through the center to another point on the circle.
    - Circumference - The distance around the edge of a circle.

Boundaries

- Square roots are an 8th grade expectation.

Teaching Strategies

- Students should use proportional reasoning to explain the relationship between the diameter and circumference of a circle and that the unit rate (constant of proportionality) is π in order to derive the formulas for the circumference and area of a circle.

Example

- The seventh-grade class is building a mini golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might you communicate this information to the salesperson to make sure you receive a piece of carpet that is the correct size?

DRAFT

STANDARD: 7.GM.B.4

Standards Statement (JUNE 2021):

Apply facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to determine an unknown angle in a figure.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- In previous grades, students have studied angles by type according to size: acute, obtuse, and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical, and adjacent angles.
- Students should be able to use relationships to write and solve equations for multi-step problems.

Terminology

- Supplementary angles – two angles add up to 180 degrees
- Complementary angles – two angles add up to 90 degrees
- Vertical angles – angles opposite each other when two lines intersect
- Adjacent angles – Two angles that have a common side and a common vertex (corner point), and do not overlap.

Boundaries

- This includes writing and solving simple equations for an unknown angle in a figure.

Example

- The ratio of the measurement of an angle to its complement is 1:2. Create and solve an equation to find the measurement of the angle and its complement. (From Engage NY M5L1)

DRAFT

STANDARD: 7.GM.B.5

Standards Statement (JUNE 2021):

Solve problems in authentic contexts involving two- and three-dimensional figures. Given formulas, calculate area, volume and surface area.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should understand the formulas for prisms as the general statement of the area of the base times the height. Students may build upon this generalization for volumes of figures in 8th grade.
- Students should relate the formulas for parallelograms, triangles and trapezoids to the formula for a rectangle.

Terminology

- Cylinder – any three-dimensional figure with two congruent, opposite faces called bases connected by adjacent curved or flat faces (bases can include circles, triangles, rectangles, or other shapes). The bases can be connected by two lines that are parallel to each other.
- Right prism – any three-dimensional figure with two polygons for bases that are opposite, congruent, and perpendicular to the adjacent faces
- The inclusive definition of a cylinder classifies prisms as special types of cylinders used to derive formulas that apply to all types of cylinders and prisms alike. (Van de Walle, Karp, & Bay-Williams, 2010)
- All prisms are cylinders, but not all cylinders are prisms. (Van de Walle, Karp, Lovett & Bay-Williams, 2010)

Boundaries

- This includes two- and three-dimensional objects composed of polygons.
- Cylinders explored in Grade 7 should be limited to right circular cylinders. Right circular cylinders are three-dimensional solid figures with two congruent, parallel, circular bases that are connected by a curved face that is perpendicular to each base.
- Students should explore experimentally and conceptually the hierarchy of cylinders and prisms.

Connections

- Students should have opportunities to apply knowledge of the area of triangles, rectangles, and other polygons to solve problems involving surface area of prisms.
- Students should apply their knowledge of area of a circle when finding the volume of a cylinder.

Teaching Strategies

- Students should apply knowledge of cross sections as a strategy for revealing a base of cylinders including right prisms.
- Students should apply reasoning about the volume of rectangular prisms to explore the volume of cylinders and other three-dimensional objects composed of cubes and right prisms.
- Students should use the formula Volume = area of the base times height or V = B x h to find the volume of a cylinder.
- Students should have opportunities to discover the surface area of a cylinder by decomposing the figure into circles and rectangles.

DRAFT

7.DR – Data Reasoning

CLUSTER: 7.DR.A. - Formulate Statistical Investigative Questions

STANDARD: 7.DR.A.1

Standards Statement (JUNE 2021):

Formulate summary or comparative investigative questions to gain information about a population and that a sample is valid only if the sample is representative of that population.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Students can generate questions about things they notice and wonder from an authentic situation.
- Understand that statistics can be used to gain information about a population and that a sample is valid only if the sample is representative of that population. (7.SP.A.1)
- Understand that random sampling tends to produce representative samples and support valid inferences.

Terminology

- A statistical investigative question is one that requires data that will vary.
- Understand that random sampling tends to produce representative samples and support valid inferences.
- A statistical investigative question is one that requires data that will vary.
- Potential limitations may include how the sample was selected and/or how the questions were asked.

Teaching Strategies

- Students should have opportunities to answer statistical investigative questions about a population by collecting data from a representative sample, using random sampling techniques to collect the data.

Example

- “How old are the students in my class?” is a statistical investigative question because it anticipates
- variability in students’ ages. “How old am I?” is a question used to collect data to answer the investigative question.

DRAFT

CLUSTER: 7.DR.B - Collect and Consider Data

STANDARD: 7.DR.B.2

Standards Statement (JUNE 2021):

Collect or consider data from a random sample to compare and draw inferences about a population with an unknown characteristic of interest.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Use data from a random sample to gauge how far off the estimate or prediction might be.
- Students should use sample data collected to draw inferences.

Terminology

- A statistical investigative question is one that requires data that will vary.
- Potential limitations may include how the sample was selected and/or how the questions were asked.

Teaching Strategies

- Students should have opportunities to answer statistical investigative questions about a population by collecting data from a representative sample, using random sampling techniques to collect the data.
- Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
- Students should have opportunities to critique examples of sampling techniques.
- Students should conclude when conditions of sampling methods may be biased, random, and not representative of the population.

Examples

- Estimate the mean word length in a book by randomly sampling words from the book. Gauge how far off the estimate is from the actual mean.
- Predict the winner of a school election based on randomly sampled survey data. Gauge how far off the prediction might be.
- “How old are the students in my class?” is a statistical investigative question because it anticipates variability in students’ ages. “How old am I?” is a question used to collect data to answer the investigative question.

DRAFT

CLUSTER: 7.DR.C - Analyze, summarize, and describe data

STANDARD: 7.DR.C.3

Standards Statement (JUNE 2021):

Assess two data distributions visually to compare multiple measures of center and variability.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Given visual representations of data from dot plots, line graphs, histograms and box-plots, create statements that compare the measures of center and variability between two data sets.
- Students should use side by side bar graphs or segmented bar graphs to compare categorical data distributions of samples from two populations.
- Students should compare data of two samples or populations displayed in box plots and dot plots to make inferences.
- Students should be able to draw inferences using measures of central tendency (mean, median, mode) and/or variability (range, mean absolute deviation and interquartile range) from random samples.
- Conclusions should be made related to a population, using a random sample, by describing a distribution using measures of central tendency (mean, median, mode) and/or variability (range, mean absolute deviation, and interquartile range).
- Students should be given multiple opportunities to compare quantitative data distributions of samples from two populations.

Example

- By comparing distributions, investigate whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.
- The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

DRAFT

CLUSTER: 7.DR.D - Interpret data and answer investigative questions

STANDARD: 7.DR.D.4

Standards Statement (JUNE 2021):

Interpret measures of center and measures of variability for numerical data from random samples to compare between two populations, and to answer investigative questions.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Students should use sample data collected to draw inferences.

Teaching Strategies

- Students should have opportunities to critique examples of sampling techniques.
- Students should conclude when conditions of sampling methods may be biased, random, and not representative of the population.

Examples

- Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.
- Estimate the mean word length in a book by randomly sampling words from the book. Gauge how far off the estimate is from the actual mean.
- Predict the winner of a school election based on randomly sampled survey data. Gauge how far off the prediction might be.

DRAFT

Grade 8 – Mathematics Standards and Guidance

8.NS - Numeric Reasoning: Number Systems

CLUSTER: 8.NS.A - Know that there are numbers that are not rational, and approximate them by rational numbers.

STANDARD: 8.NS.A.1

Standards Statement (JUNE 2021):

Know that real numbers that are not rational are called irrational.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Rational numbers are numbers that can be represented by a ratio a/b where "a" is an integer, and "b" is a non-zero whole number (e.g. natural number set).
- Rational numbers have decimal expansions that terminate in zeros or eventually repeat.
- Irrational numbers cannot be represented by a ration a/b and would include non-terminating, non- repeating decimals.

Content Boundaries

- This specific example is limited to the tenths place; however, the concept for this grade level extends to the hundredths place.

Teaching Strategies

- Students should be provided with experiences to use numerical reasoning when describing decimal expansions.
- Students should be able to classify real numbers as rational or irrational.
- Students should know that when a square root of a positive integer is not an integer, then it is irrational.
- Students should use prior knowledge about converting fractions to decimals learned in 6th and 7th grade to connect changing decimal expansion of a repeating decimal into a fraction and a fraction into a repeating decimal.
- Emphasis is placed on how all rational numbers can be written as an equivalent decimal. The end behavior of the decimal determines the classification of the number.

Example

- Understand that every number has a decimal expansion.
- For rational numbers show that the decimal expansion terminates or repeats eventually.
- Convert a decimal expansion which terminates or repeats eventually into a rational number expressed as a fraction.

DRAFT

STANDARD: 8.NS.A.2

Standards Statement (JUNE 2021):

Use rational approximations of irrational numbers to compare size and locate on a number line.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should use visual models and numerical reasoning to approximate irrational numbers.

Boundaries

- Locate them approximate location of irrational numbers on a number line diagram, and estimate the value of expressions.

Example

- Compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of square roots. For example,
    - Start with locating the nearest perfect squares and obtain closer and closer successive decimal approximations.
- Using successive approximations, estimate the decimal expansion of √17 , such as by showing that √17 is between 4 and 5, then closer to 4 (between 4.0 and 4.5) on a number line.
- Estimate the value of √2.
    - By truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

DRAFT

8.AEE - Algebraic Reasoning: Expressions and Equations

CLUSTER: 8.AEE.A - Expressions and Equations Work with radicals and integer exponents.

STANDARD: 8.AEE.A.1

Standards Statement (JUNE 2021):

Apply the properties of integer exponents using powers of 10 to generate equivalent numerical expressions.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should use numerical reasoning to identify patterns associated with properties of integer exponents.
- The following properties should be addressed: product rule, quotient rule, power rule, power of product rule, power of a quotient rule, zero exponent rule, and negative exponent rule.

Example

- Generate equivalent numerical expressions. For example, 10^2 × 10^(–5) = 10^(–3) = 1/(10^3) = 1/1000.
- $3^2 \times 3^{-5} = 3^{-3} = 1/(3^3) = 1/27$

DRAFT

STANDARD: 8.AEE.A.2

Standards Statement (JUNE 2021):

Represent solutions to equations using square root and cube root symbols.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Equations should include rational numbers such as x2 = 14.

Boundaries

- Use square root and cube root symbols to represent solutions to equations of the form $x^2=p$ and $x^3=p$, where p is a positive rational number.
- Evaluate square roots of small perfect squares up to 225 and cube roots of small perfect cubes up to 1000.
- Know irrational numbers include square roots of non-perfect squares, such as √2, and cube roots of non-perfect cubes.

Teaching Strategies

- Students should be able to find patterns within the list of square numbers and then with cube numbers.
- Students should be able to recognize that squaring a number and taking the square root of a number are inverse operations; likewise, cubing a number and taking the cube root are inverse operations.

Example

- $\sqrt{64} = \left(\sqrt{8}\right)^2 = 8$ and $\sqrt[3]{125} = 5$
    - Since $\sqrt{p}$ is defined to mean the positive solution to the equation $x^2 = p$ (when it exists). It is not mathematically correct to say √64 = ±8 (as is a common misconception).
    - In describing the solutions to $x^2 = 64$, students should write x = ± √64 = ±8.

DRAFT

STANDARD: 8.AEE.A.3

Standards Statement (JUNE 2021):

Estimate very large or very small quantities using scientific notation with a single digit times an integer power of ten.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should use place value reasoning which supports the understanding of digits shifting to the left or right when multiplied by a power of 10.
- Product and quotient rules for powers is relevant at 8th grade, and only for powers of 10

Teaching Strategies

- Students should use the magnitude of quantities to compare numbers written in scientific notation to determine how many times larger (or smaller) one number written in scientific notation is than another.
- Students should have opportunities to compare numbers written in scientific notation in contextual problems.

Example

- Compare two quantities written in this format. For example, estimate the population of the United States as 3 × 10^8 and the population of the world as 7 × 10^9, and determine that the world population is more than 20 times larger.
- Estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109 and determine that the world population is more than 20 times larger.

DRAFT

STANDARD: 8.AEE.A.4

Standards Statement (JUNE 2021):

Perform operations with numbers expressed in scientific notation.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.
- Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.
- Interpret scientific notation that has been generated by technology.

Teaching Strategies

- Students should use place value reasoning which supports the understanding of digits shifting to the left or right when multiplied by a power of 10.
- Students combine knowledge of integer exponent rules and scientific notation to perform operations with numbers expressed in scientific notation.
- Students should solve problems involving real-life contexts.

Examples

- Include authentic contexts where both standard and scientific notation are used. Use scientific notation to choose units of appropriate size for measurements of very large or very small quantities.
- Use millimeters per year for seafloor spreading.
- Interpret scientific notation that has been generated by technology such as 1.2E6.

DRAFT

CLUSTER: 8.AEE.B - Understand the connections between proportional relationships, lines, and linear equations.

STANDARD: 8.AEE.B.5

Standards Statement (JUNE 2021):

Graph proportional relationships in authentic contexts. Interpret the unit rate as the slope of the graph, and compare two different proportional relationships represented in different ways.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Various forms of linear functions include standard and slope-intercept forms.
- Key features include rate of change (slope), intercepts, strictly increasing or strictly decreasing, positive, negative, and end behavior.

Teaching Strategies

- Use verbal descriptions, tables and graphs created by hand and/or using technology.

Examples

- Interpret the unit rate as the slope of the graph. Compare one or more proportional relationships represented in different ways.
- For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

DRAFT

STANDARD: 8.AEE.B.6

Standards Statement (JUNE 2021):

Write the equation for a line in slope intercept form y = mx + b, where m and b are rational numbers, and explain in context why the slope m is the same between any two distinct points.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane
- Derive the equation $y=mx$ for a line through the origin and the equation $y=mx+b$ for a line intercepting the vertical axis at b.

Terminology

- Forms of linear equations: standard and slope-intercept forms.

Teaching Strategies

- Students should be able to rewrite linear equations written in different forms depending on the given context.

Examples

- Know that the slope m is the same between any two distinct points on a non-vertical line and be able to explain or demonstrate why.
- Derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

DRAFT

CLUSTER: 8.AEE.C - Analyze and solve linear equations and pairs of simultaneous linear equations.

STANDARD: 8.AEE.C.7

Standards Statement (JUNE 2021):

Solve linear equations with one variable including equations with rational number coefficients, with the variable on both sides, or whose solutions require using the distributive property and/or combining like terms.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.
- Students should rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Interpret and explain the results.

Terminology

- Parts of an expression include terms, factors, coefficients, and operations.

Boundaries

- This standard also includes solving or giving examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.
- Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Teaching Strategies

- Students should use algebraic reasoning in their descriptions of the solutions to linear equations.
- Building upon skills from grade 7, students combine like terms on the same side of the equal sign and use the distributive property to simplify the equation when solving. Emphasis in this standard is also on using rational coefficients. Solutions of certain equations may elicit infinitely many or no solutions.Include linear equations and inequalities with rational number coefficients and whose solutions require expanding expressions using the distributive property and collecting like terms.
- Students should use algebraic reasoning to solve linear equations and inequalities in one variable.

Connections

- Problems should be practical and contextual providing a purpose for analyzing equivalent forms of an expression.
- Rewrite a function expressed in standard form to slope-intercept form to make sense of a contextual situation.

Example

- Find the radius given the formula $V=\pi r^2 h$ by rearranging the equation to solve for the radius, r.
- Given $ax + 3 = 7$, solve for x.

DRAFT

STANDARD: 8.AEE.C.8

Standards Statement (JUNE 2021):

Find, analyze, and interpret solutions to pairs of simultaneous linear equations using graphs or tables.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs.
- Estimate solutions by graphing the equations; solve simple cases by inspection, or by using tables.

Connections

- Students should be provided with opportunities to explore systems of equations represented on using technology such as interactive graphs to analyze and interpret the solutions to the systems.
- Students should have the opportunity to explore visual graphs of equations that are parallel, perpendicular or neither parallel nor perpendicular to develop a deep, conceptual understanding.

Teaching Strategies

- Include mathematical problems in authentic contexts leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
- Students should be able to analyze and solutions to systems of equations presented numerically, algebraically, and graphically.

Example

- Given coordinates for two pairs of points, a student can determine whether the line through the first pair of points intersects the line through the second pair.
- A student can graph two linear equations that represent a culturally relevant problem using digital graphing tools (e.g., Desmos, graphing calculators, or other) and visually make sense of the graphed lines in context. A student can provide a verbal or written explanation of their reasoning.
- A student can recognize that there is no solution to the system of equations formed by $3x + 2y = 5$ and $3x + 2y = 6$ because the lines are parallel and $3x + 2y$ cannot simultaneously be 5 and 6.
- A trampoline park that you frequently go to is $9 per visit. You have the option to purchase a monthly membership for $40 and then pay $4 for each visit. Explain whether you will buy the membership, and why.
    - Option A: y=$9x
    - Option B: y= $30 + $4x
        - Anya is traveling from out of town. This is the only time she will visit this trampoline park. Which option should she choose?
        - Jin plans on going to the trampoline park seven times this month. Which option should he choose?
- What does the point of intersection of the graphs represent?

DRAFT

8.AFN - Algebraic Reasoning: Functions

CLUSTER: 8.AFN.A - Define, evaluate, and compare functions.

STANDARD: 8.AFN.A.1

Standards Statement (JUNE 2021):

Understand in authentic contexts, that the graph of a function is the set of ordered pairs consisting of an input and a corresponding output.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Understanding that a function is a rule that assigns exactly one output to each input.

Boundaries

- Use of function notation is not required in Grade 8.

Teaching Strategies

- Students should be able to use algebraic reasoning when formulating an explanation or justification regarding whether or not a relationship is a function or not a function.
- Describe the graph of a function as the set of ordered pairs consisting of an input and the corresponding output.

Example

- If a function gives the number of hours it takes a person to assemble n engines in a factory, then the set of positive integers would be an appropriate domain for the function.

DRAFT

STANDARD: 8.AFN.A.2

Standards Statement (JUNE 2021):

Compare the properties of two functions represented algebraically, graphically, numerically in tables, or verbally by description.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

Example

- Given a linear function represented by a table of values and a linear function represented by an algebraic equation, determine which function has the greater rate of change.

DRAFT

STANDARD: 8.AFN.A.3

Standards Statement (JUNE 2021):

Understand and identify linear functions, whose graph is a straight line, and identify examples of functions that are not linear.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be given opportunities to explore how an equation in the form y = mx + b is a translation of the equation y = mx.
- In Grade 7, students had multiple opportunities to build a conceptual understanding of slope as they made connections to unit rate and analyzed the constant of proportionality for proportional relationships.
- Students should be given opportunities to explore and generalize that two lines with the same slope but different intercepts, are also translations of each other.
- Students should be encouraged to attend to precision when discussing and defining b (i.e., b is not the intercept; rather, b is the y-coordinate of the y- intercept). Students must understand that the x- coordinate of the y-intercept is always 0.

Teaching Strategies

- Students should be given the opportunity to explore and discover the effects on a graph as the value of the slope and y-intercept changes using technology.
- Students should be able to model contextual situations using graphs and interpret graphs based the contextual situations.
- Students should model functions that are nonlinear and explain, using precise mathematical language, how to tell the difference between linear (functions that graph into a straight line) and nonlinear functions (functions that do not graph into a straight line).
- Students should analyze a graph by determining whether the function is increasing or decreasing, linear or non-linear.
- Students should have the opportunity to explore a variety of graphs including time/distance graphs and time/velocity graphs.
-

Example

- For example, A) determine if an equation represents a linear function and give examples of both linear and non-linear functions and B) show that the function A = s^2 is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.
- The business model for a company selling a service with no flat cost charges $3 per hour. What would the equation be as a proportional equation? If the company later decides to charge a flat rate of $10 for each transaction with the same per hour cost, what would be the new equation? How do these two equations compare when analyzed graphically?
    - What is the same? What is different? Why?
- The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.
- Examples such as this can be used to help students learn that graphs can tell stories.

DRAFT

CLUSTER: 8.AFN.B - Use functions to model relationships between quantities.

STANDARD: 8.AFN.B.4

Standards Statement (JUNE 2021):

Construct a function to model a linear relationship in authentic contexts between two quantities.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Teaching Strategies

- This learning objective also includes verbal descriptions and scenarios of equations, tables, and graphs.

DRAFT

STANDARD: 8.AFN.B.5

Standards Statement (JUNE 2021):

Describe qualitatively the functional relationship between two quantities in authentic contexts by analyzing a graph.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Identify where the function is increasing or decreasing, linear or nonlinear.
- Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Teaching Strategies

- Students should use algebraic reasoning to show and explain that the graph of an equation represents the set of all its solutions.
- Students continue to build upon their understanding of proportional relationships, using the idea that one variable is conditioned on another.
- Students should relate graphical representations to contextual situations.
- Students should use tables to relate solution sets to graphical representations on the coordinate plane.

DRAFT

8.GM - Geometric Reasoning and Measurement

CLUSTER: 8.GM.A - Understand congruence and similarity using physical models, transparencies, or geometry software.

STANDARD: 8.GM.A.1

Standards Statement (JUNE 2021):

Verify experimentally the properties of rotations, reflections, and translations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Understand that:
  - Lines are taken to lines, and line segments to line segments of the same length.
  - Angles are taken to angles of the same measure.
  - Parallel lines are taken to parallel lines.

Boundaries

- Rotations can be limited to 90, 180, ~~and~~ 270 and 360 degrees around the origin
- Reflections can be limited to reflection over horizontal and vertical lines

Examples

- Show these properties using physical models, transparencies, and/or geometry software.
- Include here--other technology

DRAFT

STANDARD: 8.GM.A.2

Standards Statement (JUNE 2021):

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Students describe a series of rigid transformations that map a two dimensional figure onto its image.

Terminology

- Rigid transformations include translations (slides), reflections (flips), rotations (turns), or glide reflections.

Example

- Given two congruent figures, describe a sequence of transformations that demonstrates the congruence between them.

DRAFT

STANDARD: 8.GM.A.3

Standards Statement (JUNE 2021):

Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.

DRAFT Standards Guidance (JUNE 2021):

Examples

- Given a triangle with given coordinates, give the new coordinates after a prescribed transformation.
- The image of Triangle ABC with A=(−3,0), B=(−3,−2) and C=(4,−2) would have coordinates A'=(−3−3,0+2)=(−6,2), B'=(−3−3,−2+2)=(−6,0), and C'=(4−3,−2+2)=(1,0) following a translation 3 units to the left and 2 units up.
- The center of dilation should be limited to a) the origin on the coordinate plane or b) one vertex of a figure such as a triangle.

DRAFT

STANDARD: 8.GM.A.4

Standards Statement (JUNE 2021):

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and/or dilations.

DRAFT Standards Guidance (JUNE 2021):

Example

- Given two similar two-dimensional figures, describe a sequence of transformations that demonstrates the similarity between them.

DRAFT

STANDARD: 8.GM.A.5

Standards Statement (JUNE 2021):

Use informal arguments to establish facts about interior and exterior angles of triangles and angles formed by parallel lines cut with a transversal.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Including identify alternate exterior angles, alternate interior angles, linear pairs, same side interior angles, same side exterior angles, and corresponding angles.

Boundaries

- This standard includes using the properties of the angle sum of the interior angles of a triangle, exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles to find missing angle measures.

Teaching Strategies

- Use informal (visual) construction with tools (patty paper, protractor, etc.) to discover the angle relationships between angles formed when two lines are cut by a transversal.
- When using more than one transversal, tie into similar triangles and to set up problems using triangle sum relationships (angle sum).

Example

- Arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so.

DRAFT

CLUSTER: 8.GM.B - Understand and apply the Pythagorean Theorem.

STANDARD: 8.GM.B.6

Standards Statement (JUNE 2021):

Distinguish between applications of the Pythagorean Theorem and its Converse in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Analyze and justification can be done using a variety of representation including use of pictures, diagrams, narratives, or models.

Terminology

- The Pythagorean Theorem states that the area of the square whose side is the hypotenuse is equal to the sum of the areas of the squares on the other two sides.
- The converse of the Pythagorean Theorem states that if a triangle has sides of length a, b, and c and if $a^2+b^2=c^2$ then the angle opposite the side of length c is a right angle.

Teaching Strategies

- Students should have the opportunity to explore one or more proofs of the Pythagorean Theorem, but are not required to prove the Theorem.
- Geometric and spatial reasoning should be used when explaining the Pythagorean Theorem.

Example

- Many ancient cultures used simple Pythagorean triples such as (3,4,5) in order to accurately construct right angles: if a triangle has sides of lengths 3, 4, and 5 units, respectively, then the angle opposite the side of length 5 units is a right angle.
    - The Pythagorean Theorem tells us that a certain relation holds amongst the side lengths of a right triangle. These ancient architects, however, do not have a right triangle but rather want to *produce* a right triangle. The converse of the Pythagorean Theorem enables them to do just this: they can conclude that an angle is a right angle provided a certain relationship holds between side lengths of a triangle.

DRAFT

STANDARD: 8.GM.B.7

Standards Statement (JUNE 2021):

Apply the Pythagorean Theorem in authentic contexts to determine unknown side lengths in right triangles.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

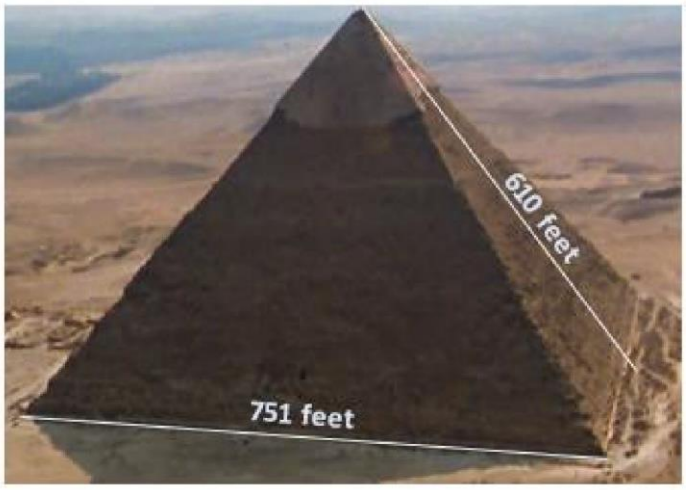
- Geometric and spatial reasoning should be used to solve problems involving the Pythagorean theorem.
- Models and drawings may be useful as students solve contextual problems in two- and three-dimensions.

Boundaries

- Include authentic mathematical problems in two and three dimensions.

Example

- How tall is the Great Pyramid of Giza?



DRAFT

STANDARD: 8.GM.B.8

Standards Statement (JUNE 2021):

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

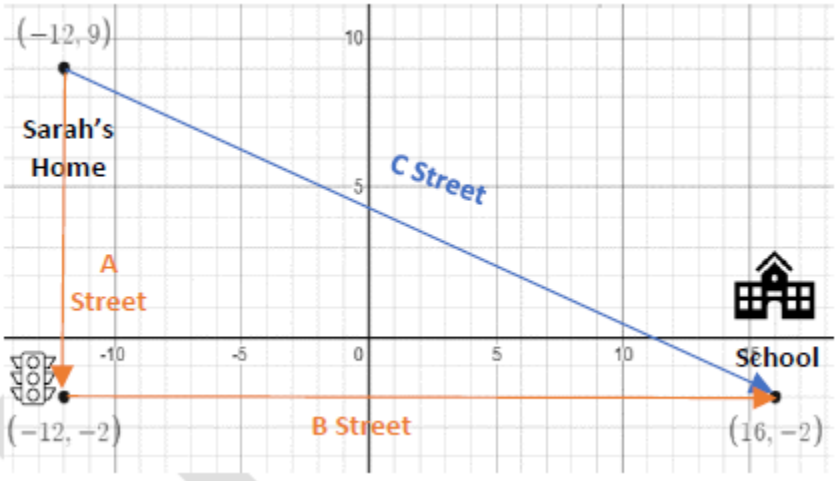
- The Distance Formula is NOT included in the 8th grade standard.
- Students should apply their understanding of the Pythagorean Theorem to find the distance. Use of the distance formula is not an expectation for this grade level.

Teaching Strategies

- Students should be provided opportunities to solve problems using a variety of strategies.

Example

- There are two paths that Sarah can take when walking to school. One path is to take is to take A Street from home to the traffic light and then walk on B street from the traffic light to the school, and the other way is for her to take C street directly to the school. How much shorter is the direct path along C Street?
    - To answer this question, students may use what they learned in 6th grade to find the distance between (-12,9) and (-12, -2) representing A street and the distance between (-12, -2) and (16, -9) representing B street. Then, students could use those two distances to find the sum of the distances for the first path. Then, students can apply the Pythagorean theorem to determine the distance between the final two points, (-12,9) and (16,-2) to determine the answer to the question.



CLUSTER: 8.GM.C - Solve mathematical problems in authentic contexts involving volume of cylinders, cones, and spheres.

STANDARD: 8.GM.C.9

Standards Statement (JUNE 2021):

Choose and use the appropriate formula for the volume of cones, cylinders, and spheres to solve problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Given the volume, solve for an unknown dimension of the figure. Students will need to be able to express the answer in terms of pi and as a decimal approximation.

Boundaries

- Memorizing the formulas is NOT included in this standard.

DRAFT

8.DR – Data Reasoning

CLUSTER: 8.DR.A. - Formulate Statistical Investigative Questions

STANDARD: 8.DR.A.1

Standards Statement (JUNE 2021):

Formulate statistical investigative questions to articulate research topics and uncover patterns of association seen in bivariate categorical data.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.
- Students can generate questions about things they notice and wonder from a real-life situation.

Terminology

- A statistical investigative question is one that requires data that will vary.

Teaching Strategies

- Students should be able to use statistical reasoning to anticipate patterns of association, such as clustering, outliers, positive or negative association, linear association, and nonlinear association

Examples

- Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

DRAFT

CLUSTER: 8.DR.B - Collect and Consider Data

STANDARD: 8.DR.B.2

Standards Statement (JUNE 2021):

Collect or consider data using surveys and measurements to capture patterns of association, and critically analyze data collection methods.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Know that straight lines are widely used to model relationships between two quantitative variables.

Terminology

- The line of best fit shows the linear relationship between two variables in a data set.
- It is important to indicate 'predicted' to indicate this is a probabilistic interpretation in context, and not deterministic.

Teaching Strategies

- Students should be able to use statistical reasoning to consider patterns of association, such as clustering, outliers, positive or negative association, linear association, and nonlinear association through the analysis of data presented in multiple ways.
- Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
- Use relative frequencies calculated for rows or columns to describe possible association between the two variables.
- Students should discover the line of best fit as the one that comes closest to most of the data points.

DRAFT

CLUSTER: 8.DR.C - Analyze, summarize, and describe data

STANDARD: 8.DR.C.3

Standards Statement (JUNE 2021):

Analyze patterns of association between two quantitative or categorical variables and reason about distributions to compare groups.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Construct and interpret scatter plots for bivariate data to investigate patterns of association between two quantities.
- Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- Students should be given opportunities to analyze the data distribution displayed graphically to answer the statistical investigative question generated from a real-life situation.

Terminology

- Bivariate data are data for two variables (usually two types of related data), such as height and weight.

Teaching Strategies

- Students should be able to use statistical reasoning to describe patterns of association, such as clustering, outliers, positive or negative association, linear association, and nonlinear association through the analysis of data presented in multiple ways.
- Create a scatter plot for bivariate data and, if appropriate, informally fit a straight line and use the line to predict values. Informally assess the model fit by judging the closeness of the data points to the line.

Example

- Given a set of data points, a student creates a scatter plot (see below), approximates a line of best fit, and writes the equation for the approximated line.

DRAFT

CLUSTER: 8.DR.D - Interpret data and answer investigative questions

STANDARD: 8.DR.D.4

Standards Statement (JUNE 2021):

Interpret scatter plots for bivariate quantitative data to investigate patterns of association between two quantities to answer investigative questions.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Interpret the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Terminology

- A linear model shows the relationship between two variables in a data set, such as lines of best fit.
- Bivariate data are data for two variables (usually two types of related data), such as height and weight.
- It is important to indicate 'predicted' to indicate this is a probabilistic interpretation in context, and not deterministic.

Teaching Strategies

- Students should interpret contextual linear problems involving situations using bivariate quantitative data.

Example

- In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

DRAFT

High School – Mathematics Standards and Guidance

HS.NQ - Numeric Reasoning: Number and Quantity (HS.NQ)

CLUSTER: HS.NQ.A - Represent all points on the number line using a complete real number system that includes both rational and irrational numbers.

STANDARD: HS.NQ.A.1

Standards Statement (JUNE 2021):

Establish properties of positive integer exponents. Use these properties to extend the definition of exponentiation to include negative and rational exponents.

DRAFT Standards Guidance (JUNE 2021):

Connections

- Students should be able to use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots and cube roots.
- MP8: generalizing patterns

Catalyzing Change Connection

- Determine precise calculations using rational and irrational numbers to make comparisons and solve problems.
- Use estimation and approximation of calculations to make comparisons and solve problems.

DRAFT

CLUSTER: HS.NQ.B - Attend to units of measurement needed to solve problems through quantitative reasoning and mathematical modeling.

STANDARD: HS.NQ.B.2

Standards Statement (JUNE 2021):

Choose and interpret units consistently in formulas, graphs, and data displays, as a way to understand problems and to guide the solution of multi-step problems.*

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Identify, use, and record appropriate units of measure within context, within data displays, and on graphs.
- Convert units and rates using proportional reasoning given a conversion factor;
- Use units within multi-step problems and formulas and interpret units of input and resulting units of output

Boundaries

- This standard applies universally in modeling situations.
- Use units of measure (linear, area, capacity, rates, and time) as a way to make sense of conceptual problems
- This includes authentic applications that require changing units to understand a given context.

Teaching Strategies

- Dimensional analysis may be used when converting units and rates.

Examples

- Units of measure may include linear, area, capacity, rates, and time.
- MP2: quantitative and abstract reasoning
- MP4: mathematical modeling

DRAFT

STANDARD: HS.NQ.B.3

Standards Statement (JUNE 2021):

Define and manipulate appropriate quantities using real numbers to authentically model situations and justify these choices.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Given a situation, context, or problem, students should be able to determine, identify, and use appropriate quantities for representing the situation.
- In some cases, students should be translating between different real number representations (for example, if something is to be cut to square root of 31 inches, what is that on a standard ruler, in sixteenths?). Students should draw on fluency with real arithmetic and estimation.

Boundaries

- This standard applies universally in modeling situations.
- Quantities are real number quantities

Connections

- Use length, area, and volume measurements to solve applied problems.
- Use properties of congruence and similarity to solve applied problems.
- Use graphs and coordinates to solve applied problems.

DRAFT

STANDARD: HS.NQ.B.4

Standards Statement (JUNE 2021):

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in modeling situations.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Note: This standard applies universally in modeling situations.
- MP4: mathematical modeling

DRAFT

HS.AEE - Algebraic Reasoning: Expressions and Equations (HS.AEE)

CLUSTER: HS.AEE.A - Rewrite expressions in equivalent forms by using algebraic properties to make different characteristics or features visible.

STANDARD: HS.AEE.A.1

Standards Statement (JUNE 2021):

Interpret an expression which models a quantity by viewing one or more of its parts as a single entity and reasoning about how changes in these parts impact the whole, and vice versa.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to interpret parts of an expression, such as terms, factors, leading coefficient, coefficients, constant and degree in context.
- Given contextual situations that utilize formulas or expressions with multiple terms and/or factors, students should be able to interpret the meaning in context of individual terms or factors.

Terminology

- Parts include terms, factors, coefficients, exponents, numerators and denominators.

Examples

- MP4: mathematical modeling
- MP7: using structure

DRAFT

STANDARD: HS.AEE.A.2

Standards Statement (JUNE 2021):

Create and recognize an equivalent form of an expression to understand the quantity represented in an authentic context.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Equivalent forms are found through application of algebraic properties including properties of exponents, combining like terms, and distributive property.

Boundaries

- Algebraic manipulation for its own sake should be avoided.

Teaching Strategies

- Students should be able to use interactive graphing technologies to make sense of equivalent expressions in context.
- Students should be able to move fluently (flexibly, accurately, efficiently) between equivalent forms of an expression.

Connections

- MP2, 7 & 8: quantitative & abstract reasoning, using structure & generalizing --Equivalent forms are found through application of algebraic properties including properties of exponents, combining like terms, and distributive property.
- Exponential equations are limited to those containing like bases, or exponential equations that could easily be transferred to like bases with linear operations.

DRAFT

STANDARD: HS.AEE.A.3

Standards Statement (JUNE 2021):

Rearrange formulas and equations to solve for different variables.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Represent constraints using systems of equations and/or inequalities and interpret solutions as viable or non- viable options in a modeling context

Boundaries

- Full proficiency in rearranging linear equations and developing proficiency with exponential (solved via roots, not logs) is expected.
- Opportunities with simple quadratic and rational situations when called for by context are also included.
- MP - MP2: quantitative & abstract reasoning

Example

- Rearrange Ohm's law V = IR to highlight resistance R.

DRAFT

CLUSTER: HS.AEE.B - Find and verify solutions to an equation, inequality, or system of equations or inequalities.

STANDARD: HS.AEE.B.4

Standards Statement (JUNE 2021):

Define variables and create equations with two or more variables to represent relationships between quantities in order to solve problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to define variables to represent the quantities and write equations to show the relationship
- Students should have the opportunity to use graphs to show a visual representation of the relationship while adhering to appropriate labels and scales.
- Full proficiency in creating equations arising from linear situations and developing proficiency for exponential situations is expected

Boundaries

- Focus of the standard is creating equations to solve problems in authentic contexts.
- Representations include expressions, equations, and graphing equations on coordinate axes with labels and scales.
- Opportunities to explore simple quadratic and rational equations when called for by context are also included.

Teaching Strategies

- When necessary, students should be able to rewrite the inequality in various forms, such as slope-intercept form, for graphing.
- Students should be given opportunities to solve contextual linear inequalities graphically and algebraically.

Examples

- Four people may be seated at one rectangular table. If two rectangular tables are placed together end-to-end, 6 people may be seated at the table. If 10 tables are placed together end-to-end, how many people can be seated? How many tables are needed for n people?
- The cost of parking in the parking garage is $2.00 for the first hour and $1.00 for every hour after that. Write an equation in terms of x and y that shows the total cost for parking, y, for x hours. Use the equation to calculate the cost for parking in the garage for 5 hours.

DRAFT

STANDARD: HS.AEE.B.5

Standards Statement (JUNE 2021):

Define variables and create inequalities with one or more variables and use them to solve problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Students should be given the opportunity to explore the difference between solid lines and dashed lines through exploration on an interactive graph.
- Students should have had opportunities to create and solve linear equations and inequalities throughout middle school mathematics.
- Students should recognize that the graph of a linear inequality in two variables is a half- plane.
- Full proficiency in creating inequalities arising from linear situations and developing proficiency for exponential situations is expected.

Boundaries

- Focus of the standard is creating inequalities to solve problems in authentic contexts.
- Representations include expressions, inequalities, and graphing equations on coordinate axes with labels and scales.

DRAFT

STANDARD: HS.AEE.B.6

Standards Statement (JUNE 2021):

Solve systems of linear equations or inequalities through algebraic means for simple systems and strategically using technology when needed.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Ensure constraints are represented.
- Students in Grade 8 mathematics modeled with and solved systems of linear equations to solve real-life problems.

Terminology

- Simple system of linear equations would include solving on pairs of linear equations in two variables represented by the intersection of two lines graphically.

Boundaries

- Simple systems of equations are those that could easily be solved by hand, including whole number coefficients and/or rational number solutions.
- Full proficiency with pairs of linear equations in two variables is expected. Opportunities with non-linear systems when called for by context are also included.
- MP5: Using graphing technology

Teaching Strategies

- Students should be provided opportunities to use technology tools to solve systems of linear inequalities graphically.

Example

- A school club is selling hats and t-shirts for a fundraiser. The group expects to sell a total of 50 items. They make a profit of 15 dollars for each t-shirt sold and 5 dollars for each hat sold. How many hats and t-shirts will the school club need to sell to make a profit of $300?

DRAFT

CLUSTER: HS.AEE.C - Analyze the structure of an equation or inequality to determine an efficient strategy to find and justify a solution.

STANDARD: HS.AEE.C.7

Standards Statement (JUNE 2021):

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities; interpret solutions as viable or nonviable options in a modeling context.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- Possible data points are solutions to the inequality or inequalities; data points that are not possible are non-solutions to the inequality or inequalities.

Boundaries

- Full proficiency in creating and interpreting equations or inequalities arising from linear situations is expected.
- Opportunities to explore exponentials, simple quadratic and rational situations when called for by context are also included.
- MP4: Mathematical Modeling

Example

- Represent inequalities describing nutritional and cost constraints on combinations of different foods.

DRAFT

STANDARD: HS.AEE.C.8

Standards Statement (JUNE 2021):

Construct a viable argument to justify a method for solving a simple equation.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to interpret parts of an expression, such as terms, factors, leading coefficient, coefficients, constant and degree in context.
- Given contextual situations which utilize formulas or expressions with multiple terms and/or factors, students should be able to interpret the meaning in context of individual terms or factors.

Boundaries

- Limit to real number solutions
- Viable arguments must give reasoning for important steps. Supporting this work may also involve justifying algebraic rules through models.
- Full proficiency in solving linear equations, quadratics which are solvable without factoring, completing the square or quadratic formula, exponentials solvable without logarithms and simple rational equations are expected.
- MP3: Construct arguments & critique the reasoning of others

Teaching Strategies

- Students should be able to move fluently (flexibly, accurately, efficiently) between different equivalent forms of an expression. Students should be able to analyze and explain what the zeros describe in context.

DRAFT

CLUSTER: HS.AEE.D - Make predictions in different applications using expressions, equations, and inequalities to analyze authentic contexts.

STANDARD: HS.AEE.D.9

Standards Statement (JUNE 2021):

Understand that the solutions to an equation in two variables is a set of points in the coordinate plane that form a curve, which could be a line.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Common graphs include lines, parabolas, circles, and exponential curves.
- Students can show that any point on the graph of an equation in two variables is a solution to the equation.

DRAFT

STANDARD: HS.AEE.D.10

Standards Statement (JUNE 2021):

Recognize and explain why the point(s) of intersection of the graphs of f(x) and g(x) are solutions to the equation f(x)=g(x). Interpret the meaning of the coordinates of these points.*

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Use technology to graph the functions, make tables of values, or find successive approximations.
- Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- *MP4: mathematical modeling
- MP5: using graphing technology

DRAFT

STANDARD: HS.AEE.D.11

Standards Statement (JUNE 2021):

Graph and explain why the points in a half plane are solutions to a linear inequality and the solutions to a system of inequalities are the points in the intersection of corresponding half planes. Interpret the meaning of the coordinates of these points in context.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Graphs can be created by hand in simple cases but in general with technology to allow the emphasis on the interpretations of solutions.
- MP4: mathematical modeling
- MP5: using graphing technology

DRAFT

HS.AFN - Algebraic Reasoning: Functions

CLUSTER: HS.AFN.A - Describe functions by using both symbolic and graphical representations.

STANDARD: HS.AFN.A.1

Standards Statement (JUNE 2021):

Understand a function as a rule that assigns a unique output for every input and that functions model situations where one quantity determines another.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Functions are often represented by tables, expressions or graphs. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
- Modeling examples should include both contexts where only one quantity can be considered the independent variable as well as contexts where both quantities could.

Boundaries

- Standard included expectation students connect the concept of a function to use of notation where if f is a function and x is an element of its domain, then (x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y=(x).
- Concept of a function introduced in grade 8, but formal use of function notation is not an expectation until high school.

DRAFT

STANDARD: HS.AFN.A.2

Standards Statement (JUNE 2021):

Use function notation and interpret statements that use function notation in terms of the context and the relationship it describes.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Student should develop a deep understanding of function notation to build, evaluate, and interpret linear functions; this understanding will be applied to other functions studied hereafter.
- Students should be able to interpret the domain when given a function expressed numerically, algebraically, and graphically.
- Students should apply their understanding of function notation from their work with linear functions to build, evaluate, and interpret quadratic functions using function notation.
- Students should apply their understanding of function notation from their work with non-linear functions when needed to build, evaluate, and interpret functions in authentic contexts.

Connections

- MP4: mathematical modeling

DRAFT

STANDARD: HS.AFN.A.3

Standards Statement (JUNE 2021):

Calculate and interpret the average rate of change of a function over a specified interval.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be given opportunities to estimate the rate of change from a graph.
- Students should be able to show that linear functions grow by equal differences over equal intervals and recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
    - Students should be able to compare this behavior to that of the average rate of change of quadratic functions. This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals.

Boundaries

- Work with functions presented as graphs, tables or symbolically.
- Students should choose specified intervals for analysis of functions with substantially varying rates of change.
- Interpreting also includes estimates of the rate of change from a graph.
- MP6: precision
- MP7: structural thinking

Teaching Strategies

- Functions can be presented symbolically, as a graph, or as a table.

DRAFT

CLUSTER: HS.AFN.B - Distinguish functions as member of the same family by using attributes common to all functions writing a given category.

STANDARD: HS.AFN.B.4

Standards Statement (JUNE 2021):

Compare properties of two functions using multiple representations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to compare key characteristics of exponential functions with the key characteristics of linear and quadratic function.
- Students should be able to observe using graphs and tables that a quantity is increasing .

Boundaries

- Functions can be represented algebraically, graphically, numerically in tables, or by verbal descriptions.

Example

- Given a graph of one function and an algebraic expression for another, determine which has the larger y-intercept.
- Given a graph of one quadratic function and an algebraic equation for another, students should be able to determine which has the larger maximum.
- Given a graph of one function and an algebraic equation for another, students should be able to determine which has the larger y-intercept.

DRAFT

STANDARD: HS.AFN.B.5

Standards Statement (JUNE 2021):

Relate the domain of a function to its graph and to its context.

DRAFT Standards Guidance (JUNE 2021):

Boundaries

- Contexts can demand discrete vs. continuous and domain restrictions.
- MP4: mathematical model
- MP6: precision

Terminology

- Use symbolic notation to represent the domain and range of a linear function, considering the specific context.
    - $(-\infty,\infty)$
    - $[3, \infty)$
    - D: $\{x| x \epsilon R\}$
    - D: $\{x| x > 0\}$
    - D: $\{x| x = 1,2,3,4,5,\ldots\}$
    - R: $\{y| y = 10,20,30,\ldots\}$

Examples

- If the function h(n) gives the number of hours it takes a person to assemble n engines in a factory, then the set of positive integers would be an appropriate domain for the function.
- If the function h(t) gives the path of a projectile over time, t, then the set of non- negative real numbers would be an appropriate domain for the function because time does not include negative values.
- A bird is building a nest in a tree 36 feet above the ground. The bird drops a stick from the nest. The function f(x) = -16x2 + 36 describes the height of the stick in feet after x seconds. Graph this function. Identify the domain and range of this function. (A student should be able to determine that the appropriate values for the domain and range of this graph are $0 \leq x \leq 1.5$ and $0 \leq y \leq 36$, respectively.)

DRAFT

CLUSTER: HS.AFN.C - Represent functions graphically and interpret key features in terms of the equivalent symbolic representation.

STANDARD: HS.AFN.C.6

Standards Statement (JUNE 2021):

Interpret key features of functions, from multiple representations, and conversely predict features of functions from knowledge of context. (★)

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to express characteristics in interval and set notation with linear functions.
- Students should be able to interpret the key characteristics of the graph in a contextual situation.

Boundaries

- Key features include: domain, range, discrete, continuous, intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums.
- Representations include: graphs, tables, spreadsheet representations, as well as symbolic.

Teaching Strategies

- Students should be able to use graphs created by hand and with technology, verbal descriptions, tables, and function notation when analyzing linear functions in context.
- Students should be given opportunities to use interactive graphing technologies to explore and analyze key characteristics of linear functions, including domain, range, intercepts, intervals where the function is increasing or decreasing, positive or negative, maximums and minimums over a specified interval, and end behavior.

DRAFT

STANDARD: HS.AFN.C.7

Standards Statement (JUNE 2021):

Graph functions using technology to show key features.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to sketch a graph showing key features including domain, range, and intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; asymptotes; end behavior.
- Key characteristics of the quadratic functions should be expressed in interval and set- builder notation using inequalities.

Boundaries

- Key features include: specific values when context demands; domain and range; discrete or continuous; intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima.
- Use technology to graph functions expressed symbolically or in tables, with intentional choices of window and scale. In some simple cases, graphing functions could by hand or for approximations.
    - Graph linear and quadratic functions and show intercepts, maxima, and minima.★
    - Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.★
    - Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.★
    - Graph exponential and logarithmic functions, showing intercepts and end behavior.★

Teaching Strategies

- Students should be able to use verbal descriptions, tables, and graphs created using interactive technology tools.

Examples

- If the function, h(n), gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
- The function can be presented symbolically, as a graph, or as a table.
- Students should be able to estimate the rate of change from a graph.

DRAFT

CLUSTER: HS.AFN.D - Model a wide variety of authentic situations using functions through the process of making and changing assumptions, assigning variables, and finding solutions to contextual problems.

STANDARD: HS.AFN.D.8

Standards Statement (JUNE 2021):

Model situations involving arithmetic and geometric sequences. Use a variety of representations including an explicit formula for the sequence, and translate between the forms.*

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to:
  - make connections between linear functions and arithmetic sequences presented in contextual situations.
  - build and interpret arithmetic sequences as functions presented graphically and algebraically.
- Sequences can be defined explicitly.
- Connections should be made between exponential functions and geometric sequences.
- The focus of this learning objective is on building and interpreting geometric sequences.

Example

- By graphing or calculating terms, students should be able to show how the arithmetic sequence in recursive form a1=7, an=an-1 +2; the arithmetic sequence in explicit form an = 2(n-1) + 7; and the function f(x) = 2x + 5 (when x is a natural number) all define the same sequence.
- By graphing or calculating terms, students should be able to show how the geometric sequence in recursive form a1=8, an=2an-1; the geometric sequence in explicit form sn = 8(2)n - 1; and the function f(x) = 4(2)x (when x is a natural number) all define the same sequence.
- MP2: quantitative and abstract reasoning
- MP4: mathematical modeling

DRAFT

STANDARD: HS.AFN.D.9

Standards Statement (JUNE 2021):

Identify and interpret the effect on the graph of a function when the equation has been transformed.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should be given opportunities to experiment with cases and illustrate an explanation of the effects on the graph using technology.

Boundaries

- Transformations include translations (f(x)+k, and f(x-h)), reflections (e.g. -f(x) and f(-x), and dilations (e.g. a*f(x)). Interpretations include accounting for different choices of variables, such as initial values or units.
- Full proficiency with linear functions and developing proficiency with exponential functions is expected. Technology provides opportunities for exploration with non-linear functions.
- MP4: mathematical modeling
- MP5: using graphing technology

DRAFT

STANDARD: HS.AFN.D.10

Standards Statement (JUNE 2021):

Explain why a situation can be modeled with a linear function, an exponential function, or neither.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be provided with opportunities to learn mathematics in the context of real-life problems.
- Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).

Terminology

- Linear functions grow by equal differences over equal intervals.
- Exponential functions grow by equal factors over equal intervals.

Boundaries

- Identify situations in which one quantity changes at a constant rate per unit interval relative to another.
- Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Teaching Strategies

- Students should be able to fluently navigate between mathematical representations that are presented numerically, algebraically, and graphically.
- For graphical representations, students should be given opportunities to analyze graphs using interactive graphing technologies.

Connections

- Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
- MP4: Mathematical Modeling

DRAFT

HS.GM - High School Geometric & Measurement Reasoning

CLUSTER: HS.GM.A - Apply geometric transformations to figures through the concept of functions and through the analysis of graphs of functions as geometric figures.

STANDARD: HS.GM.A.1

Standards Statement (JUNE 2021):

Apply definitions of rotations, reflections, and translations to transform a figure or map between two figures in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to determine congruency by identifying the rigid transformation(s) that produced the image of a figure.
- Opportunities should be provided for students to write statements of congruency.
- Given two polygons , students should be able to use the definition of congruence in terms of rigid motions to verify congruence if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- Students should be able to use function notation to represent transformations in the coordinate plane.

Terminology

- A rigid transformation that preserves size and shape.

Boundaries

- Draw the transformation (rotation, reflection, or translation) for a given geometric figure.
- Students should be able to apply definitions of reflections across any line in context or on a coordinate grid.
- Students should be able to apply definitions of rotations around any point of any degree in context or on a coordinate grid.

Teaching Strategies

- Students should have ample opportunities to use geometric tools and/or technology to explore figures created from translations, reflections, and rotations.
- Students should be able to determine that translations, reflections, and rotations produce

DRAFT

STANDARD: HS.GM.A.2

Standards Statement (JUNE 2021):

Verify experimentally the properties of a dilation given a center and a scale factor. Solve problems in authentic contexts involving similar triangles or dilations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- Students should be able to identify dilation as reduction or enlargement depending on scale factor.
- Students should be given opportunities to draw a dilated image given any center and scale factor in context or on a coordinate grid.
- Students should be able to describe properties of dilations, such as center, scale factor, angle measure, parallelism, and collinearity.

Terminology

- A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Teaching Strategies

- Triangles can be shown to be similar using transformations and triangle similarity theorems. Apply theorems of AA similarity, SSS similarity, and SAS similarity to prove that two given triangles are similar.
- Dilations should be limited to those centered at the origin.

Connections

- Model with mathematics to use similarity to solve authentic problems to measure lengths and distances indirectly.
- Use the properties of similarity transformations could be used to establish the Angle-Angle (AA) criterion for two triangles to be similar.

Example

- A high school student visits a giant cedar tree near the town of Elk River, Idaho and the end of his shadow lines up with the end of the tree's shadow. The student is 6 feet tall and his shadow is 8 feet long. The cedar tree's shadow is 228 feet long. How tall is the cedar tree?

DRAFT

STANDARD: HS.GM.A.3

Standards Statement (JUNE 2021):

Use the slopes of segments and the coordinates of the vertices of triangles, parallelograms, and trapezoids to solve problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should have opportunities to analyze and apply theorems about lines and angles from the context of parallel lines cut by a transversal to make sense of relationships between lines and angles in quadrilaterals and triangles.
- Students should be familiar with triangle congruence theorems (SSS, SAS, ASA, AAS, or HL) to solve problems and to prove relationships in geometric figures by applying geometric and algebraic reasoning.

Connections

- Possible applications include using slopes to determine parallel sides in parallelograms and trapezoids, perpendicular diagonals in rhombuses, perpendicular sides in a rectangle
- Use slope and coordinates to verify mid-segment properties in triangles and trapezoids.
- Use coordinates of vertices for lengths of sides and diagonals to classify quadrilaterals and triangles.

DRAFT

STANDARD: HS.GM.A.4

Standards Statement (JUNE 2021):

Use definitions of transformations and symmetry relationships to justify the solutions of problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to define and identify figures as preimages and images.
- Students use definitions to identify lines of symmetry and angles of rotation to map a figure onto itself.
- Students use definitions to identify angles of rotation, lines of reflection, and directions of translations to map a preimage onto its image.
- Students use definitions to experiment with transformations represented on and off the coordinate plane.

Terminology

- Definitions of geometric figures and geometric relationships could include definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Boundaries

- Definitions should include angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

DRAFT

CLUSTER: HS.GM.B - Construct and communicate geometric arguments through use of proofs, logical reasoning, and geometric technology.

STANDARD: HS.GM.B.5

Standards Statement (JUNE 2021):

Apply and justify triangle congruence and similarity theorems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure;
- Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- Students should be able to apply properties of congruence to solve problems with missing values involving corresponding parts. Opportunities should also be available for students to understand when conditions do not result in congruence.

Boundaries

- The focus here is to develop an understanding of techniques for proving that two triangles are congruent.
- Advanced courses could include explanations for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions (HSG.CO.B.8).

Terminology

- Logic statements include conditional, converse, inverse, and contrapositive statements.

Teaching Strategies

- Use of triangle congruence theorems (SSS, SAS, ASA, AAS, or HL) should be used to solve problems in authentic contexts.
- Students' ways of communicating triangle congruence could possibly include formal methods such as: logic statements, two-column proofs, paragraph proofs, and flow proofs.

Example

- Construct viable arguments and critique the reasoning of others when showing that two triangular roof trusses must be congruent.

DRAFT

STANDARD: HS.GM.B.6

Standards Statement (JUNE 2021):

Justify theorems of line relationships, angles, triangles, and parallelograms; and use them to solve problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Students should be given opportunities to explore using visual tools in order to precisely prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.

Boundaries

- Angle and line relationship theorems include:
  - when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and conversely use to justify lines are parallel;
  - points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
  - vertical angles are congruent;
- Triangle Theorems include:
  - Sum of interior angles 180 degrees
  - Properties of special triangles (isosceles, equilateral, and right).
  - Angle sums formed by polygons
- Parallelogram theorems include:
  - Properties of special quadrilaterals (sides, angles, and diagonals), and
  - Properties of special triangles (isosceles, equilateral, and right).

Clarifications

- Justification should require a precise chain of reasoning that verifies the validity of a mathematical theorem.
- Students should be provided opportunities to build a conceptual understanding of a point, line, line segment, plane, arc, and angle through modeling and exploration of authentic phenomena.
- Students should use symbolic notation for point, line, plane, line segment, angle, circle, arc, perpendicular line, and parallel line.

Connections

- Construct viable arguments and critique the reasoning of others when justifying the congruence of diagonals in a rectangle that is built by a contractor installing a rectangular window.

DRAFT

STANDARD: HS.GM.B.7

Standards Statement (JUNE 2021):

Perform geometric constructions with a variety of tools and methods.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Student should be able to:
  - Copy a segment and angle.
  - Bisect a segment and angle.
  - Construct perpendicular lines, including the perpendicular bisector of a line segment.
  - Construct a line parallel to a given line through a point not on the line.

Teaching Strategies

- Tools to include compass and straightedge, string, reflective devices, paper folding, and/or dynamic geometric software.
- Constructions to include copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Connections

- Use appropriate tools strategically when choosing the physical method and appropriate procedures for performing a construction

DRAFT

CLUSTER: HS.GM.C - Solve problems and interpret solutions of area and volume of shapes by applying concepts of congruence, similarity, symmetry in authentic contexts.

STANDARD: HS.GM.C.8

Standards Statement (JUNE 2021):

Solve authentic modeling problems using area formulas for triangles, parallelograms, trapezoids, regular polygons, and circles.*

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should give informal arguments for area formulas, and combine them to solve problems with composite figures.
- Students should be able to choose the appropriate geometric solid to approximate volumes of irregular objects.

Example

- Model with Mathematics can be used here to solve a variety of problems involving area.
- Modeling a tree trunk or a human torso as a cylinder to estimate surface area.

DRAFT

STANDARD: HS.GM.C.9

Standards Statement (JUNE 2021):

Use volume and surface area formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems and apply to authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should be able to choose the appropriate geometric figure or solid to approximate density of irregular objects in a geometric situation.
- Students should give informal arguments for area and volume formulas, and combine them to solve problems with composite figures. This standard is limited to right solids.

Example

- Make sense of problems and persevere in solving them when finding the volume of prisms and pyramids with regular polygon bases (possibly using trigonometry)
- Persons per square mile, fish per cubic feet of a fish tank

DRAFT

STANDARD: HS.GM.C.10

Standards Statement (JUNE 2021):

Use geometric shapes, their measures, and their properties to describe real world objects, and solve related authentic modeling and design problems.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- This includes the use of volume formulas for prisms, cylinders, pyramids, cones, and spheres.
- Students should be able to verify experimentally the formulas for the volume of a cylinder, pyramid, sphere, prism and cone; emphasize volume as the product of the area of the base and the height for both prisms and cylinders.
- Students should find the volume of solids and composite solids to explain real-life phenomena.

Terminology

- Prism – a solid figure that has the same cross section all along its length

Example

- Model with Mathematics can be used here to solve a variety of problems such as designing a real world object with CAD design tools for 3D printing or CNC machining.

DRAFT

STANDARD: HS.GM.C.11

Standards Statement (JUNE 2021):

Apply concepts of density based on area and volume in authentic modeling situations.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- The focus is on geometric probability and proportional reasoning.
- This should include an understanding of the ratios of areas (area ratio = (scale factor)^2) and volumes (volume ratio = (scale factor)^3) of similar figures.

Example

- Model with Mathematics to compute persons per square miles, BTUs per cubic foot, or specimens per acre.

DRAFT

CLUSTER: HS.GM.D - Apply concepts of right triangle trigonometry in authentic contexts to solve problems and interpret solutions.

STANDARD: HS.GM.D.12

Standards Statement (JUNE 2021):

Apply sine, cosine, and tangent ratios, and the Pythagorean Theorem, to solve problems in authentic contexts.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- In seventh grade, students write and solve equations using supplementary, complementary, vertical, and adjacent angles.
- Explain and use the relationship between the sine and cosine of complementary angles.

Teaching Strategies

- Demonstrate understanding that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- Students should be able to use sine, cosine, and tangent to solve real-life problems that require them to find missing side and angle measurements.

Connections

- Applications should involve finding angle and side measures of right triangles.

DRAFT

STANDARD: HS.GM.D.13

Standards Statement (JUNE 2021):

Apply the Pythagorean Theorem in authentic contexts, and develop the standard form for the equation of a circle.

DRAFT Standards Guidance (JUNE 2021):

Terminology

- The standard form of the equation for a circle is $(x-h)^2 + (y-k)^2 = r^2$.

Clarifications

- Students should be able to identify the center and radius of a circle from an equation in standard form or from the graph of a circle.
- Students should be able to write the equation of a circle in standard form given the graph of the circle.
- Students should be able to graph a circle from the standard form equation of a circle.

Teaching Strategies

- Given the coordinates of the center and length of the radius, write the equation of the circle in standard form.
- Given the equation of a circle in standard form, determine the coordinates of its center and the length of its radius.

Connections

- Use the Pythagorean Theorem to develop and apply the distance formula
- Look for and make use of structure to make connections to the Pythagorean Theorem and distance formula.

DRAFT

STANDARD: HS.GM.D.14

Standards Statement (JUNE 2021):

Use the coordinate plane to determine parallel and perpendicular relationships, and the distance between points.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to classify quadrilaterals as parallelograms (including rectangles, rhombi, and squares) using sides and diagonals.
- Students should be familiar with the distance formula when calculating the area and perimeter of quadrilaterals and triangles.

Terminology

- Cartesian coordinates refer to (x,y) system on a rectangular grid with the x-coordinate representing horizontal distance from the origin, and the y-coordinate representing vertical distance from the origin.

Boundaries

- Course level expectation is limited to use of a rectangular (Cartesian) coordinate system.

Teaching Strategies

- Applications include the use of coordinates to compute perimeters of polygons and areas of triangles and rectangles. The distance formula will play an important role in these applications.
- Students apply their understanding of linear relationships to derive definitions and to solve problems related to distance, midpoint, slope, area, and perimeter.

Connections

- Use slope and length of line segments to classify quadrilaterals in the coordinate plane.
- Calculate the area and perimeter of parallelograms, triangles, and regular polygons in the coordinate plane.

Example

- Use appropriate tools strategically to choose between tools such as the slope formula, distance formula, midpoint formula, or Pythagorean Theorem.
- Find the length of a line segment plotted on the coordinate plane.

DRAFT

HS.DR - High School Data Reasoning

CLUSTER: HS.DR.A - Formulate Statistical Investigative Questions

STANDARD: HS.DR.A.1

Standards Statement (JUNE 2021):

Formulate multivariable statistical investigative questions and determine how data from samples can be collected and analyzed to provide an answer.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Focus of standard is supporting students to understand and ask questions about how data could be collected.
- As students engage in multivariable thinking, the types of statistical investigative questions should expand to include questions concerning association and prediction.
- Students pose statistical investigative questions for a particular sample to determine any association of the variables of interest for that sample.

Terminology

- A statistical investigative question is one that requires data that will vary.
    - Statistical questions are set in a context where one wants to know something; are based in variability or uncertainty; are always data based; and are approximations or estimates that emerge from data analysis.
    - Deterministic questions are based upon exact calculations or theoretical deductions elicited from prior certain knowledge.
- A sample is a subset of a population.
- Samples are taken when examining the entire population is not possible or feasible.

Teaching Strategies

- This is an opportunity for students to create a survey, collect data, and use graphical displays, sample statistics or two way tables to help estimate population parameters which are unknown values.
- It is important to understand samples used on social media or in the news.

Connections

- CCSS - (HSS.IC.A.1) Understand the process of statistical reasoning, formulate questions, collect, analyze, and interpret data to answer statistical investigative questions.
- GAISE II - (1.C.1) Formulate multivariable statistical investigative questions and determine how data can be collected and analyzed to provide an answer

Example

- Students can distinguish between situations where a small group (e.g., a classroom) is the entire population (census) and when it is a sample from a larger population (e.g., the classroom is used to answer a question about an entire grade level in a school).
- "Given a list of the arm spans of 9th grade students, what can be predicted about the heights of those students?"

DRAFT

STANDARD: HS.DR.A.2

Standards Statement (JUNE 2021):

Formulate summative, comparative, and associative statistical investigative questions for surveys, observational studies, and experiments using primary or secondary data.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students will draft statistical questions for which appropriate data can be collected and analyzed to answer the statistical investigative question.
- Students will use appropriate sampling techniques, critique a poorly constructed survey, and make suggestions for good questions.
- Students should understand the advantages and disadvantages of each data collection method for specific statistical questions.
- For experimental studies, students are able to identify, discuss, and explain the aspects of best statistical practice for designing an experimental study, including:
    - the clear identification of the statistical question to be investigated;
    - the variables under investigation; and the random selection of experimental units and/or
    - random assignment of treatments for experimental studies.

Terminology

- Types of data collections could include:
    - Surveys involve the collection of data from a pre-defined group to gain insight and information about the statistical investigative question.
    - Observational studies measure a sample as it is without attempting to influence the results.
    - Experiments involve the use of a treatment to explore the effects of the treatment on a sample.
- Types of data include:
    - Primary data is collected through first-hand sources such as surveys, experiments, and other studies.
    - Secondary data is obtained from previously conducted studies or research.

Connections

- CCSS - (HSS.IC.B.3) Recognize the difference between sample surveys, experiments and observational studies and understand the role of randomization in each.
- GAISE II - (1.C.2) Pose summary, comparative, and association statistical investigative questions for surveys, observational studies, and experiments using primary or secondary data

Examples

- Students should consider features such as whether the population is well-defined, whether the sampling procedure is random or non-random, and whether the objectivity or bias of questions will result in valid/invalid answers.

DRAFT

STANDARD: HS.DR.A.3

Standards Statement (JUNE 2021):

Formulate inferential statistical investigative questions regarding causality and prediction from correlation.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students pose statistical investigative questions for a particular sample to determine any association of the variables of interest for that sample.
- Students should be able to understand the magnitude of a correlation coefficient represents the strength of association; understand and able to calculate a residual; understand that any straight line other than the best fit line (by least squares) will have a larger sum of squared residuals than the best fit line.

Boundaries

- Understand and explain the difference between correlation and causation. It is important for students to discover and understand that strong correlation does not indicate causation.

Connections

- CCSS - (HSS.ID.C.9) Distinguish between correlation and causation.
- GAISE II - (1.C.3) Pose inferential statistical investigative questions regarding causality and prediction.

Example

- Determine if statements of causation seem reasonable or unreasonable and justify reasoning.
- Correlation coefficients of r =-.65 and r = .65 indicate the same strength.

DRAFT

STANDARD: HS.DR.A.4

Standards Statement (JUNE 2021):

Students use mathematical and statistical reasoning to formulate questions about data to evaluate conclusions and assess risks.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Focus of standard is supporting students to evaluate data presented in reports to evaluate conclusions and/or assess risks.
- Understand different ways in which number appear in everyday discussions of government, business, scientific results, and personal activities.
- Apply mathematical and statistical knowledge to inform and make decisions students face or many need to evaluate in society.

Teaching Strategies

- Generate reasonable estimates and use scale to place quantities in context.
- Interpret visual representations of data to assess conclusions and risks
- Locate data to assess validly of claims and conclusions.

Connections

- CCSS – (HSS.IC.B.6) Evaluate reports based on data.
- NCTM Essential Skills - Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.

DRAFT

CLUSTER: HS.DR.B - Collect and Consider Data

STANDARD: HS.DR.B.5

Standards Statement (JUNE 2021):

Articulate what constitutes good practice in designing a sample survey, an experiment, and an observational study.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students are able to identify, discuss, and explain the aspects of best statistical practice for designing an experimental study, including:
    - the clear identification of the statistical question to be investigated;
    - the variables under investigation; and
    - the random selection of experimental units and/or the random assignment of treatments to the experimental units.
- Students should be able to describe the ethical consequences of their experiments and analyses.
- Practices for handling data that enhance reproducibility and ensure ethical use include providing descriptions of alterations to collected data, proper treatment of sensitive information, maintaining the confidentiality of data and experimental units, and using Institutional Review Boards to review study designs.

Teaching Strategies

- Students should be able to design and conduct comparative experiments using random assignment and demonstrate correct methods for planning data collection for comparison of treatments.
- Students should be able to randomly assign treatments to experimental units.
- Students provide or select appropriate interpretations of graphical displays and numerical summaries to compare two or more groups in the context of a study.

Connections

- GAISE II - (2.C.3) Understand what constitutes good practice in designing a sample survey, an experiment, and an observational study
- NCTM Essential Skills –
    - The role of randomization is different in randomly selecting samples and in randomly assigning subjects to experimental treatment groups.
    - The larger the sample size, the less the expected variability in the sampling distribution of a sample statistic.

DRAFT

STANDARD: HS.DR.B.6

Standards Statement (JUNE 2021):

Distinguish between surveys, observational studies, and experiments, and design an appropriate data collection to answer an investigative question of interest.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should understand the advantages and disadvantages of each data collection method for specific statistical questions.
- Students should be able to design and conduct comparative experiments using random assignment, or non-experimental designs when random assignment is not possible, and demonstrate correct methods for planning data collection for comparison of treatments.

Terminology

- Surveys involve the collection of data from a pre-defined group to gain insight and information about the statistical investigative question.
- Observational studies measure a sample as it is without attempting to influence the results.
- Experiments involve the use of a treatment to explore the effects of the treatment on a sample.
    - For experimental designs, students should be able to randomly assign treatments to experimental units.
- Nonexperimental research is research that lacks the manipulation of an independent variable, random assignment of participants to conditions or orders of conditions, or both.
    - Examples of non-experimental research could include case studies, focus groups, interviews, correlational or quasi-experimental research, or qualitative studies.

Boundaries

- Limit to population proportion, graphical representations, and visual overlap.

Connections

- CCSS - (HSS.IC.B.4) Use data from a randomized experiment to compare two treatments to decide if differences between parameters are significant based on the statistics.
- GAISE II - (2.C.2) Distinguish between surveys, observational studies, and experiments.
- NCTM Essential Skills - Study designs are of three main types: sample survey, experiment, and observational study.

DRAFT

STANDARD: HS.DR.B.7

Standards Statement (JUNE 2021):

Apply an appropriate data collection plan when collecting primary data or selecting secondary data for the statistical investigative question of interest.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students will use appropriate sampling techniques, critique a poorly constructed survey, and make suggestions for good questions.
- Students should identify types of displays that are appropriate for categorical data versus quantitative (numerical) data.
- Students should have opportunities to analyze meaningful, real-life data and recognize possible associations and trends in the data.
- Students should understand and apply concepts of sample space to describe categorical data.

Terminology

- Primary data is collected through first-hand sources such as surveys, experiments, and other studies.
- Secondary data is obtained from previously conducted studies or research.

Boundaries

- Students should consider features such as whether the population is well-defined, whether the sampling procedure is random or non-random, and whether the objectivity or bias of questions will result in valid/invalid answers.

Teaching Strategies

- Students may use spreadsheets, graphing calculators, and statistical software to create frequency tables and determine associations or trends in the data.
- Recognize the association between two variables by comparing conditional and marginal percentages.
- Describe patterns observed in the data

Connections

- CCSS - (HSS.ID.B.5) Analyze the association between two categorical variables by using two-way tables and comparative bar graphs.
- GAISE II - (2.C.1) Apply an appropriate data collection plan when collecting primary data or selecting secondary data for the statistical investigative question of interest.
- NCTM Essential Skills - The scope and validity of statistical inferences are dependent on the role of randomization in the study design.

Examples

- Read, interpret and write clear summaries of data displayed in a two-way frequency table.
- Calculate joint, marginal, and conditional relative frequencies.
- Make appropriate displays of joint, marginal, and conditional distributions.

DRAFT

STANDARD: HS.DR.B.8

Standards Statement (JUNE 2021):

Articulate issues of bias and confounding variables in observational studies and their implications for interpretation.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to design and conduct surveys from both non-random and randomly selected participants.
- Students should be able to explain why random samples can provide more unbiased information about a population than other types of samples, such as convenience samples or self-selected samples.
- Samples must be randomly selected from the appropriate population to allow for generalizations that extend beyond the sample from which the data were collected.
- Sampling procedures that are not random do not allow for generalizations to the sampled population because they may be biased.

Connections

- GAISE II - (2.C.6) Understand the issues of bias and confounding variables in observational studies and their implications for interpretation.
- NCTM Essential Skills - Bias, such as sampling, response, or nonresponse bias, may occur in surveys, yielding results that are not representative of the population of interest.

Examples

- Types of bias include wording bias, under coverage, non-response bias, selection bias, and experimenter bias.

DRAFT

CLUSTER: HS.DR.C - Analyze Data

STANDARD: HS.DR.C.8

Standards Statement (JUNE 2021):

Identify appropriate ways to summarize and then represent the distribution of univariate and bivariate data multiple ways with graphs and/or tables.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should identify types of displays that are appropriate for categorical data versus quantitative (numerical) data.
- Students should be able to construct scatterplots, and describe positive, negative or no relationship.
- Strength of association is demonstrated by degree of spread about the line of best fit in a scatterplot.
- Numerical data can be displayed visually with graphs, such as using dot plots, histograms, and box plots, to discover patterns and deviations from patterns.
- Students should use spreadsheets, graphing calculators, and statistical software to analyze the bivariate data.

Terminology

- Univariate data involves describing a single variable, such as the age of a student or the height of a student.
- Bivariate data involves relationships between two variables, such as comparing the age of a student and their height.

Teaching Strategies

- This is an extension of middle school expectations where students display data on dot plots and box plots.
- Opportunity for students to collect and graph their own data and use modeling to fit a function to the data; use a function fitted to data to solve problems in the context of the data. (Emphasize linear models.)
- Students should be able to fluently utilize dot plots, histograms, and box plots to represent data.

Connections

- CCSS –
    - (HSS.ID.A.1) Represent the distribution of data multiple ways with plots on the real number line.
    - (HSS.ID.B.6) Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
- GAISE II – (3.C.2) Identify appropriate ways to summarize quantitative or categorical data using tables, graphical displays, and numerical summary statistics, which includes using standard deviation as a measure of variability and a modified boxplot for identifying outliers.

Examples

- Analyze the strengths and weakness inherent in each type of plot by comparing different plots of the same data.

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- Describe and give simple conclusions and interpretations of a graphical representation of data.
- Fit a linear function for a scatter plot that suggests a linear association.

DRAFT

STANDARD: HS.DR.C.2

Standards Statement (JUNE 2021):

Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should have the opportunity to gain an understanding of this concept through the use of technology tools.
- Students should use the meaning of mean absolute deviation (MAD) learning in sixth grade to interpret the meaning of standard deviation.
- Students were first introduced to the concept of MAD as a tool for comparing variability of multiple data sets in sixth grade mathematics.
- Students should be able to construct scatterplots, and describe positive, negative or no relationship.
- Data may be displayed using histograms, dot plots, or smooth normal curves.

Boundaries

- Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread.
    - Measures of center include the mean, median, and mode.
    - Measures of spread include the range, interquartile range, and standard deviation.
- The shape of a data distribution might be described as symmetric, skewed, uniform, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range).

Connections

- CCSS
    - (HSS.ID.A.2) Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets.
    - (HSS.ID.A.4) Use the mean and standard deviation of an approximately normally distributed data set to estimate population percentages.
    -
- GAISE II –
    - (3.C.6) Describe associations between two categorical variables using measures such as difference in proportions and relative risk
    - (3.C.7) Describe the relationship between two quantitative variables by interpreting Pearson's correlation coefficient and a least-squares regression line
- NCTM Essential Skills - Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.

DRAFT

STANDARD: HS.DR.C.3

Standards Statement (JUNE 2021):

Use data to compare two groups, describe sample variability, and decide if differences between parameters are significant based on the statistics.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to describe how population estimates may be overstated or understated due to the presence of outliers.
- Students should be able to describe how missing or erroneous values can lead to biased or inaccurate estimations.
- Strength of association is demonstrated by degree of spread about the line of best fit in a scatterplot.
- Students should be able to recognize how sampling variability is influenced by sample size.

Teaching Strategies

- Use data from multiple sources to interpret differences in shape, center and spread
- Discuss the effect of outliers on measures of center and spread.
- Use the 1.5 IQR rule to determine the outliers and analyze their effects on the data set.

Connections

- CCSS – (HSS.ID.A.3) Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- GAISE II – (3.C.4) Understand how sampling distributions (developed through simulation) are used to describe the sample-to-sample variability of sample statistics
- NCTM Essential Skills - Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line and finding a correlation coefficient, and differentiating between correlation and causation.

Example

- Students should use spreadsheets, graphing utilities and statistical software to identify outliers and analyze data sets with and without outliers as appropriate.
- Using the 1.5 IQR rule on data set {5,7,8,10,11,12,30}, 30 is determined to be an outlier since it is greater than 19.5, which is the 1.5*IQR +12 (the 3Q).

DRAFT

STANDARD: HS.DR.C.4

Standards Statement (JUNE 2021):

Use technology to subset and filter data sets and transform variables, including smoothing for time series data.

DRAFT Standards Guidance (JUNE 2021):

Connections

- GAISE II – (3.C.1) Use technology to subset and filter data sets and transform variables, including smoothing for time series data.
- NCTM Essential Skills - Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to "clean" and organize data, including very large data sets, into a useful and manageable structure—a first step in any analysis of data.

DRAFT

CLUSTER: HS.DR.D - Interpret Results

STANDARD: HS.DR.D.13

Standards Statement (JUNE 2021):

Use statistical evidence from analyses to answer the statistical investigative questions.

DRAFT Standards Guidance (JUNE 2021):

Clarification

- Identify when data can be generalized to a target population.
    - Samples must be randomly selected from the appropriate population to allow for generalizations that extend beyond the sample from which the data were collected.
    - Sampling procedures that are not random do not allow for generalizations to the sampled population because they may be biased.
- Evidence could be interpreted from data displays such as histograms, dot plots, or smooth normal curves.

Teaching Strategies

- Students should be able to recognize that sample statistics vary with repeated sampling.
- Students should be able to interpret the sampling variability in a summary statistic.
- Students should be able to interpret the sampling variability from simulation studies of statistics.
- Students should be able to recognize how sampling variability is influenced by sample size.
- Recognize that there are data sets for which the empirical rule is not appropriate.

Connections

- GAISE II – (4.C.1) Use statistical evidence from analyses to answer the statistical investigative questions and communicate results through more formal reports and presentations

DRAFT

STANDARD: HS.DR.D.14

Standards Statement (JUNE 2021):

Articulate what it means for an outcome or an estimate of a population characteristic to be plausible or not plausible compared to chance variation.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- Students should be able to decide whether an observed difference is something that would be likely to be observed by chance and whether this difference has any practical meaning.
- Students recognize that significance is demonstrated by a result that is unlikely to occur by chance
- Students recognize that statistical, but not practical, significance is influenced by sample size.

Teaching Strategies

- Students should use spreadsheets, graphing calculators and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals and correlation coefficients.
- Students should be given the opportunity to utilize interactive graphing technologies to interpret the correlation coefficient, r.
    - Students should be able to use the correlation coefficient, r, to make predictions and describe the reasonableness of the prediction in the context of a practical, real-life situation.
    - Explain that the correlation coefficient must be between –1 and 1 inclusive and explain what each of these values means.
    - Determine whether the correlation coefficient shows a weak positive, strong positive, weak negative, strong negative, or no linear correlation. Interpret what the correlation coefficient is telling about the data.

Connections

- CCSS – (HSS.ID.C.8) Compute, using technology, and interpret the correlation coefficient of a linear fit.
- GAISE II – (4.C.3) Understand what it means for an outcome or an estimate of a population characteristic to be plausible or not plausible compared to chance variation
- NCTM Essential Skills - Data-analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.

DRAFT

STANDARD: HS.DR.D.15

Standards Statement (JUNE 2021):

Use multivariate thinking to articulate how variables impact one another, and measure the strength of association using correlation coefficients for regression curves.

DRAFT Standards Guidance (JUNE 2021):

Clarifications

- As students engage in multivariable thinking, the types of statistical investigative questions should expand to include questions concerning association and prediction.
- Students should be able to identify contexts where a change in one attribute may be related to a change in another attribute.
- Students should be able to describe how population estimates may be overstated or understated due to the presence of outliers.
- Students should be able to describe how missing or erroneous values can lead to biased or inaccurate estimations.

Boundaries

- Students should be able to provide a reasonable estimate of the Pearson's correlation coefficient (r) for a scatterplot; identify linear and non-linear relationships in scatterplots; correctly interpret the strength of a linear relationship based on r.
- Students should be able to understand the magnitude of a correlation coefficient represents the strength of association; understand and able to calculate a residual; understand that any straight line other than the best fit line (by least squares) will have a larger sum of squared residuals than the best fit line.

Teaching Strategies

- Opportunity to connect the concept of distinguishing between correlation and causation as students interpret data.
- Understand and explain the difference between correlation and causation. It is important for students to discover and understand that strong correlation does not indicate causation.

Connections

- CCSS – (HSS.ID.C.9) Distinguish between correlation and causation.
- GAISE II – (4.C.6) Use multivariate thinking to understand how variables impact one another.
- NCTM Essential Skills - Making and defending informed data-based decisions is a characteristic of a quantitatively literate person.

Example

- Determine if statements of causation seem reasonable or unreasonable and justify reasoning.
- Correlation coefficients of r =-.65 and r = .65 indicate the same strength.

DRAFT

STANDARD: HS.DR.D.4

Standards Statement (JUNE 2021):

Communicate results of statistical reasoning or informed data-based decisions in a variety of formats (verbal, written, visual).

DRAFT Standards Guidance (JUNE 2021):

Teaching Strategies

- Students should be given the opportunity to utilize interactive graphing technologies to model linear data and make sense of the slope (predicted rate of change) visually.
- Students would use technology to develop an awareness of how outliers might affect the rate of change and the intercept of a given model.
- Students should be able to explain when intercepts might be outside the scope of the model.
- Students should be able to interpret the confidence interval(s) in context.
    - Sampling variability is associated with summary statistics and uses the margin of error to form an interval (confidence interval) to estimate the characteristic.

Boundaries

- Student should have the opportunity to interpret the slope and intercept of a linear model in the context of data collected or considered by students.

Examples

- Students demonstrate interpreting slope in the context of a given situation when examining two variable statistics as being "for each additional known unit increase in an explanatory variable, we expect or predict a known unit increase (or decrease) in the response variable."
- Students demonstrate interpreting intercept in the context of a given situation when examining two variable statistics as being "the predicted known unit of a response variable when the explanatory variable is zero known units."

Connections

- CCSS – (HSS.ID.C.7) Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- GAISE II – (4.C.7) Communicate statistical reasoning and results to others in a variety of formats (verbal, written, visual)

DRAFT

CLUSTER: HS.DR.E – Understand independence and conditional probability and use them to interpret data.

STANDARD: HS.DR.E.17

Standards Statement (JUNE 2021):

Describe the possible outcomes for a situation as subsets of a sample space.

DRAFT Standards Guidance (JUNE 2021):

Connections

- This provides an opportunity for students to engage with finding the outcomes of situations which include words such as **and**, **or**, **not**, **if**, and **all**, and to grammatical constructions that reflect logical connections.

DRAFT

STANDARD: HS.NQ.C.6

Standards Statement (JUNE 2021):

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

DRAFT Standards Guidance (JUNE 2021):

Example

- Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

DRAFT