OREGON
DEPARTMENT OF EDUCATION

2021 Oregon Draft Mathematics Standards

High School Core Mathematics

January 2021 Draft for Public Review

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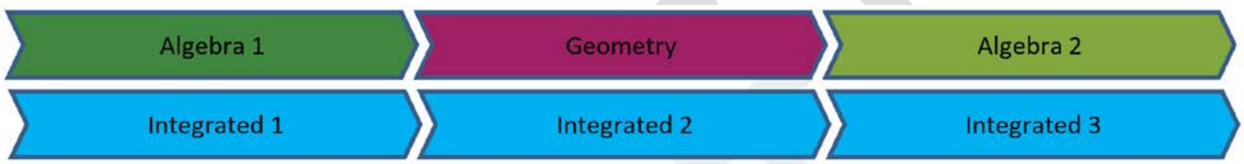
SECTION ONE: Introduction

1A: Common Historical Course Sequences

In Oregon, districts and schools need to plan courses that give the opportunity for students to have access to the adopted high school standards by the end of a three credit sequence or sooner. State law does not prescribe any specific high school math course sequence, only that students have access to the content identified in the adopted state standards. Traditionally, three credits of high school course content have been arranged into either an Algebra, Geometry, Algebra 2 sequence (AGA), or an integrated equivalent.

Figure 1: Common High School Course Options



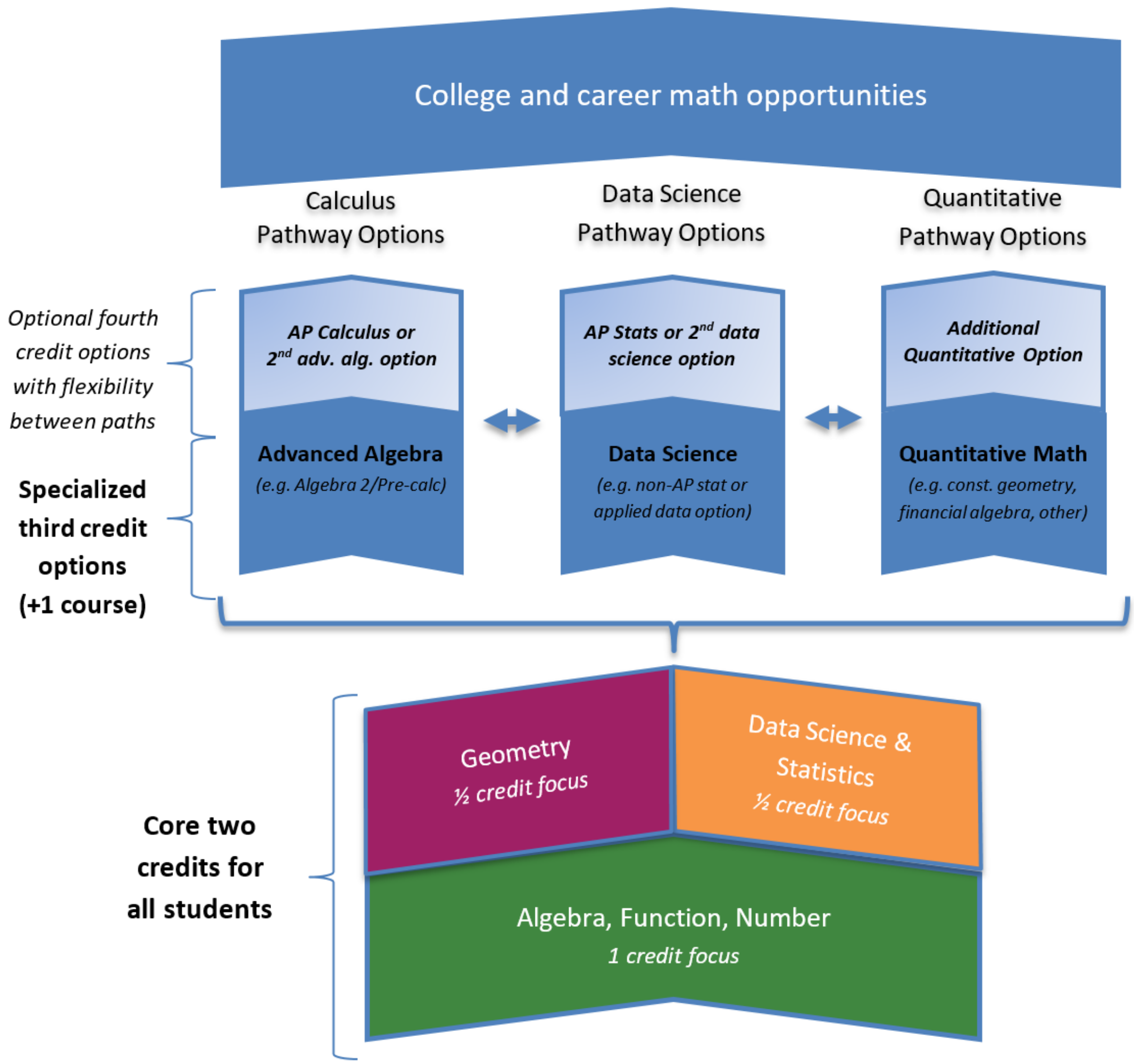
In 2010, Oregon adopted high school math standards based on the Common Core State Standards (CCSS) which currently identify 111 non-advanced standards for all students. In addition, there are 36 optional advanced (+) standards that could either be included in the core three credit sequence or as additional study. Content is divided into six domains of Algebra, Functions, Modeling, Geometry, and Statistics & Probability.

1B: Standards Revision and Future High School Pathways

Oregon State Standards are reviewed and revised on a regular basis. In mathematics, the planned review by educators occurred in the 2019-20 school year and prepared for adoption by the State Board of Education in the 2020-21 school year. This original timeline included preparing for a state review of instructional materials in the summer of 2022 and district updating of materials in the 2022-23 school year. From the student perspective, statewide implantation in classrooms would occur by the fall of the 2023-24 school year. It is possible the school closures could delay this timeline, but no formal decisions have been made as of July 2020. Updated timelines will be reflected in any future guidance provided.

Pending future State Board of Education approval, the work completed to date includes a comprehensive review and revision of our high school math standards. The working assumption of the high school review was to shift to a new course pathway model with two credits of core content for all students, and create third credit pathway options that align to student interests and goals. This model is referred to as the 2+1 course model. Core content would be balanced between approximately one credit of algebra content, ½ credit of geometry, and ½ credit of data science and statistics. For third credit options, Oregon high school staff are invited to innovate by offering new specialized courses within three general paths: (1) a pathway to calculus; (2) a pathway to data science; (3) and a pathway to quantitative mathematics. Figure 2 helps visualize the long term goal of what high school math pathways in the 2+1 model could look like as early as the 2023-24 school year.

Figure 2: Long Term (2023-24) Course Pathway Options



1C: Centering on Equitable High School Mathematics

In June 2020, the Oregon Department of Education released the first version of Ready Schools, Safe Learners with the goal of providing clear statewide requirements and recommendations for health, safety, equity, and instruction. Students in mathematics have inequitable access to grade level content, and unfinished learning can accumulate over time creating a system where students find themselves trapped in tracks repeating K-8 mathematics within high school courses.

The absence of in-person instruction in spring 2019 amplified the issue of access to grade level content in mathematics. Accelerating, rather than remediating, content from prior coursework is needed to achieve the central goal of maximizing access to grade level content for all students. ODE has published a math acceleration summary in support of the call to accelerate learning in mathematics, which is consistent with national guidance from the TNTP Acceleration Guide and the NCTM/NCSM COVID-19 Joint Position Statement.

Ensuring students have access to high school content will take innovation and creativity to re-imagine what course experiences could look like for Oregon students. The intent of this document is to provide additional guidance specific to planning high school math courses not only for the next school year, but to set each and every Oregon student up for equitable access to course options.

Additional information and examples can be found in the Oregon Math Project Practice Brief: Promoting Equity

1D: Detracking Mathematics and Creating Pathways in High School

The Oregon Department of Education supports national calls to detrack math experiences for our students and teachers. Detracking the first two credits of core math content is a long-term goal for Oregon, and can begin as soon as this next school year. At the same time, ODE supports efforts to create grade 11-14 math pathways that could include specialized third credit course options, such as construction geometry or financial algebra, that align to student goals and aspirations. For the 2020-21 school year, it is likely that existing courses such as Algebra 1, Geometry, and Algebra 2 would still be provided with a focus on prioritized content.

For the purposes of ODE guidance, the term “tracking” will refer to the practice of creating different levels of the same course that group students by perceived abilities. Detracking high school courses would ensure that all students have access to the same content and experiences for any given course.

The term “pathway” refers to a specialized math course sequence that leads to career and college readiness. This could include the traditional advanced algebra pathway to prepare for calculus that all students were locked into, but also include a statistics pathway or quantitative applied pathway that lead to specific career and college options for students after high school graduation.

Further resources to understand this work in Oregon and nationally in support of grade 11-14 math pathways include Branching Out: Designing High School Math Pathways for Equity, Dana Center Launch Years Report, and CBMS High School to College Mathematics Pathways forum which our state was invited to share course pathway work described in this document. The time is right for Oregon educators to think innovatively and lead the nation in re-imagining high school math.

- Additional information and examples can be found in the Oregon Math Project Practice Brief: Tracking

1E: Opportunity to Deepen Equity, Innovation, Care, and Connection

High school pathways described in the 2+1 course model are an innovation that high school faculty can use to create equitable opportunities that connect mathematics to students goals and interests and prioritize anti-racist mathematics instruction. Oregon schools and districts are therefore encouraged to use the 2020-21 and 2021-22 school years to plan a path to create math pathways options for students.

This includes leaning into new and innovative ways to incorporate instructional best practices, such as NCTM's Principles to Action, to create student-centered instructional experiences that continue beyond the current health crisis. Resources and courses created today can lay a strong foundation for high school experiences in the future.

- Additional information and examples can be found in the Oregon Math Project Practice Brief: Classroom Discourse

1F: Continued Focus on Math Practices & Modeling

Reimagining math pathway options that meet the needs of more students will require a focus on content rather than courses that students need for success. It will also require ensuring the Standards of Mathematical Practice are attended to as we accelerate learning. This includes finding new ways to infuse applications through mathematical modeling that supports the natural interconnectedness of math to other disciplines and to community-based problems. Modeling is an opportunity to see mathematics as relevant to students' lives and the questions confronting our world. Now more than ever, we need to find ways to increase student interest and enthusiasm in math by providing more opportunities to engage in interactive, student-centered problems that are based in applied mathematics. Examples of mathematical modeling lessons can be found below with additional examples added over time to the Oregon Open Learning Mathematics Group.

- American Statistical Association - STatistics Education Web (STEW)
- Council for Economic Education - EconEdLink Lessons
- Modeling with Mathematics through Three-Act Tasks

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout core content in this document as indicated by a star symbol (★).

- Additional information and examples can be found in the Oregon Math Project Practice Brief: Mathematical Modeling

1G: Connections to National Conversations

The ideas described in this document have been informed and aligned to the call for re-humanizing school mathematics. The National Council of Teachers of Mathematics (NCTM) recognize the need to focus on high school mathematics in *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* that outlines the need for change, and recommendations to focus math content that lead to specialized pathways in high school.

Additional national conversations are occurring within professional math organizations that recognize the need to create more options for students that are contextual and connected to a student's interest. *The Common Vision Project* is a joint effort of five national math organizations led by the Mathematical Association of America (MAA) calling for a shift in college math instruction to introduce contemporary topics and applications and employ a broad range of examples and applications to motivate students and illustrate how math is used.

1H: Overview of 2019-21 Math Standards Review and Revision

Phase 1 (April-September 2019)

Goal: Ensure the **Commitment of Shared Learning** of the standards and the Oregon Equity Lens for all panelists.

- Primary Task: Participate in online learning opportunities in Spring/Summer 2019 and engage in conversations in a Canvas course set up for the content panel

The first phase of the project included grounding our work in a shared learning experience on the Canvas learning management system so that travel could be minimized by taking advantage of available technologies. Learning sessions were recorded and can be found on the Oregon Math Project YouTube channel.

Phase 2 (October 2019-March 2020)

Goal: Ensure the **Language** of the standards is accessible to a wide audience

- Primary Task: Review standards and draft content into (1) a standards statement, and (2) Clarifying Guidance(s)
- Additional HS Task: Propose core two credit content for all students

Work in Phase 2 was primarily done in grade level teams meeting virtually through video conferences, sharing documents online. Each team was tasked with creating a version 1.0 document by January 2020. This draft was then shared with all groups for feedback and use at the in-person co-chair meeting on March 2-3 in Portland, OR.

Additional guests at the March meeting included **Shebi Cole and Jason Zimba from Student Achievement Partners**, **Robert Berry from the National Council of Teachers of Mathematics**, and **Ted Coe from Achieve**. Guest speakers were on site to provide additional guidance to our panelists in drafting the version 2.0 document that is being shared now.

Phase 3 (November -December 2020)

Goal: Ensure the **Commitment to the Focus and Coherence** of the standards is maintained or improved.

- Primary Task: Review the March 2020 draft (version 2.0) through the lens of focus (e.g. clarity, prioritization) and coherence (e.g. learning within established progressions).
- Create version 3.0 based on content panel feedback

The COVID-19 health crisis significantly disrupted all our lives in a number of ways included state-wide school closures starting mid-March just after we were able to have our in-person content panel meeting. This impacted our work on math standards which was also put on hold. Work will with math standards feedback will move to online only options starting in November 2020.

Phase 4 (January – February 2021)

Goal: Ensure the **Commitment to Expand the Conversation** beyond the content panel through a public review process.

- Primary Task: Public review the January 2021 (Version 3.0).
- Coordinate with Oregon Educational Service Districts (ESDs) to schedule virtual webinar options for feedback. Separate sessions for K-8 and High School standards will be provided and sign ups should be done with the hosting ESDs. Dates, times, and registration links can be found on the ODE math standards page.

Phase 5 (March – April 2021)

Goal: Ensure the **Commitment to Incorporate Feedback** collected to inform and improve the quality of the standards.

- Primary Task: Review feedback from the winter public feedback sessions and incorporate changes as needed.
- Connections will also be made to work happening nationally in other states to inform standards work in Oregon. Significant potential changes will be shared with educators and the State Board for possible inclusion in our state standards work (Version 4.0)

Phase 6 (Spring/Summer 2021)

Goal: Ensure the **Commitment to Alignment to the Vision** of mathematics education in Oregon and ensure standards support this vision.

- Primary Task: Present standards to the state board of education for adoption, or update timeline if additional time is needed to incorporate feedback and alignment to national work in math education.

1J: Acknowledgements

It is with tremendous gratitude that the staff at the Oregon Department of Education recognizes the work of the math content panel and the countless hours invested in the work invested in this draft document. This work will continue to be reviewed in Spring and Summer 2020 with the goal of a public review in Fall 2020. Please join us in thanking the content panelists and co-chairs (indicated in bold) for their efforts to support the review of our math standards.

High School Algebra Team

- **Maddy Ahearn, Lane ESD**
- **Tammy Anderson, Ashland School District**
- **Dev Sinha, University of Oregon**
- **Katie Weybright, Redmond School District**
- James Akers, La Grande School District
- Jacob Alburn, Woodburn School District
- Marla Baber, Portland Public Schools
- Barry Bowers, Imbler School District
- David Duncan, Wallowa School District
- Kristen Faust, North Clackamas School District
- Mark Gano, Woodburn School District
- Olivia Green, Gresham-Barlow School District
- Charlene Herron, Klamath Falls City Schools
- Jaclyn Herzog, Portland Public Schools
- Courtney Jentzsch, North Clackamas SD
- Laura Lethe, Salem-Keizer School District
- Shannon Mashinchi, Reynolds School District
- Joeal Mazurowski, Portland Christian HS
- Stephanie Partlow, Woodburn School District
- Erica Pifer, Springfield SD/Lane ESD
- Steve Rhine, Pacific University
- Sarah Savage, Reynolds School District
- Charles Slusher, Portland Public Schools
- Teresa Swake, Sherwood School District
- Clair Thiel, Reynolds School District
- Cynthia Townsend, Hermiston School District
- Lori Vandine, Molalla School District
- Jerry Young, Portland State University
- Kee Zublin, Bethel School District

High School Data Science & Statistics Team

- **Wendy Clark, Ontario School District**
- **Shaun Gross, Grants Pass School District**
- **Nancy Swarat, Umatilla School District**
- Chris Garrigues, Hermiston School District
- Susan Kaller, Portland Public Schools
- Andria Lindsey, Bend-LaPine School District
- Miguel Mejia, Portland Public Schools
- Brian Sica, Beaverton School District
- Brad Simmons, Gervais School District
- Lisa Wileman, Southern Oregon University

High School Geometry Team

- **Jeremy Knight, Grants Pass School District**
- **Larry Susuki, Harper School District**
- **Sarah Wall, Portland Public Schools**
- Linda Adams, Jefferson County School District
- Corrie Aljian, Springfield School District
- Steve Boyce, Portland State University
- Alison Bryant, Albany School District
- Malika Gillette, Woodburn School District
- Doug Mella, Portland Public Schools
- Kayla Stolte, Portland Public Schools
- Michael Whalen, Klamath County SD
- Patrice Woods, Portland Public Schools
- Deanna Yule, Culver School District

Oregon Department of Education Staff

- Colt Gill, Director of the Oregon Department of Education
- Jennifer Patterson, Assistant Superintendent of Teaching and Learning
- Alexa Pearson, Director of Standards and Instructional Supports
- Mark Freed, Math Education Specialist
- Andy Byerley, Math Assessment Specialist
- Tom Thompson, STEM Education Specialist
- Aujalee Moore, Administrative Specialist

1K: How to Read the Document

Priority content identified in this document was done by a panel of Oregon educators as part of the scheduled review and revision of the adopted high school standards during the 2019-20 school year. The next stage of the review process will include a public comment period with the aim of presenting to the State Board of Education for adoption.

As a system, we need all high school teachers to commit to ensuring that students are given the opportunity to learn the identified content by the end of a three credit sequence.

Content identified in this document should be thought of approximately as 1 credit of algebra content, ½ credit of geometry, and ½ credit of data science. This would open up the opportunity of a full credit to accelerate unfinished learning across a three credit sequence.

- Content not identified in this document could certainly be taught once teachers are confident students are proficient in the core content.
- Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout core content in this document as indicated by a star symbol (★).

Focus within high school courses could occur in two ways including:

(1) identification of a subset of our standards on which to focus, and

(2) narrowing the focus within each standard themselves.

Attention should be paid to not only the named standards in this document, but the specific content identified as a focus within each standard as well. Organization of specific units within a course will need to be determined at the local district and/or school level.

Part A – Draft Statements

The 2021 review and revision of high school math standards including efforts to improve readability and access to a wide range of potential readers. The standards document itself is one of the most downloaded document on the state website and provides an opportunity to share important information to not only educators, but additional audiences such as parents and community members as well.

Grade level overview

Following the critical areas for each grade will include an overview of the grade level domains and clusters. This content is presented unedited from the CCSS (2010), and is proposed to remain the same for the K-8 standards to assist in providing continuity to the Oregon draft 2021 math standards. In general, these terms refer to:

Standards define what students should understand and be able to do.

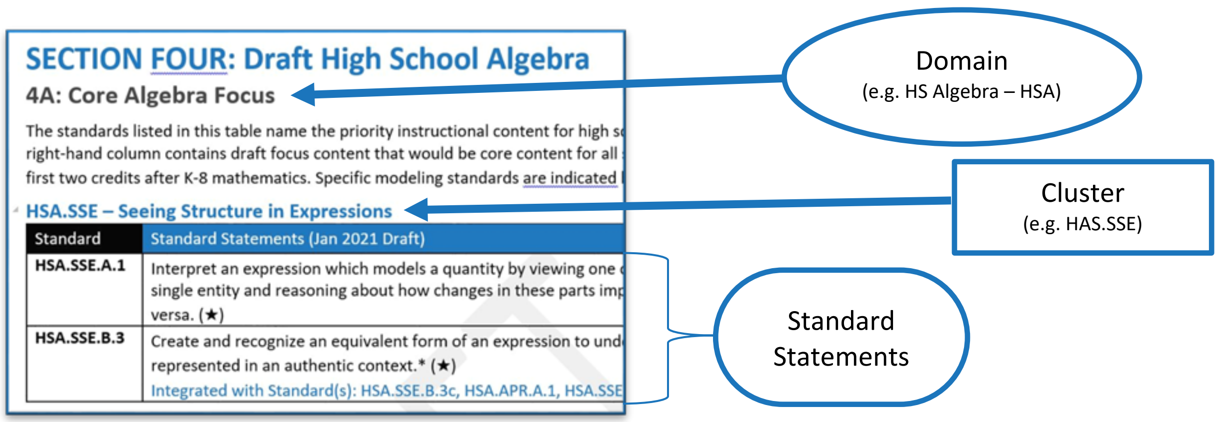
Clusters summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Note that the original CCSS (2010) included 147 high school standards, which the 2019 content panel was directed to identify a subset of that could reasonably be covered in a two-credit course sequence. The panel identified 55 standards, or approximately 37% of the original number of standards, as part of the core two credits. With this change, many of the original clusters did not make sense if they only included no standards, or just one. So for the purposes of this document, domains will refer to course level organization (e.g. Algebra – HSA), and clusters will be one level below this, such as "HAS.SSE – Seeing structures in equations."

Additional re-organization and naming of clusters may be needed, but not included as part of the draft document at this time. Requested feedback would be on the appropriateness of the subset identified, and any wording to found in the proposed draft document. References to these levels within section A can be found in Figure 2 below.

Figure 1: High School Domain, Clusters, and Standard Statements within Part B



Many of the CCSS (2010) standards were long and technical that presented barriers to access for many readers interested in grade level expectations. For the 2021 review, the original standards were divided into two parts, which were:

1. Standards statements that will be adopted by the State Board of Education
2. Clarifying guidance that will be used in supporting documents for use by educators to understand the boundaries and examples of a given standard.

Table 2: Overview of Standards Statements

Audience	Everyone
Definition	A standard is a statement what a student should know, understand, or do.
Description	Standards statements need to be written as stand-alone statement(s) in the final document. Could include more than one sentence, but overall word count needs to remain below approximately 40-50 words.
Considerations	• Lead with clarity o Start with key ideas o First Sentence approximately 10-20 words o Minimize use of conjunctions (and/or) • Include information from CCSSM o Total word count approximately 40-50 words • No Parentheticals o Examples moved to clarifying statements • Technical Considerations o Size (word count, character count, number of paragraphs) o Complexity (words per sentence, characters per word) o Readability (Flesch Reading Ease, Flesch-Kincaid Grade Level)

Part B – Remaining content considerations

As part of the high school standards review, the 2019 panel was directed to identify a subset of that could reasonably be covered in a two-credit course sequence. A total of 92 standards were identified for removal which included: 37 advanced (+) standards; 9 standards to be merged; and 46 non-advanced standards for removal from the core two credit requirement. Totals for core standards identified in comparison to the original CCSS (2010) can be found below in Table 3.

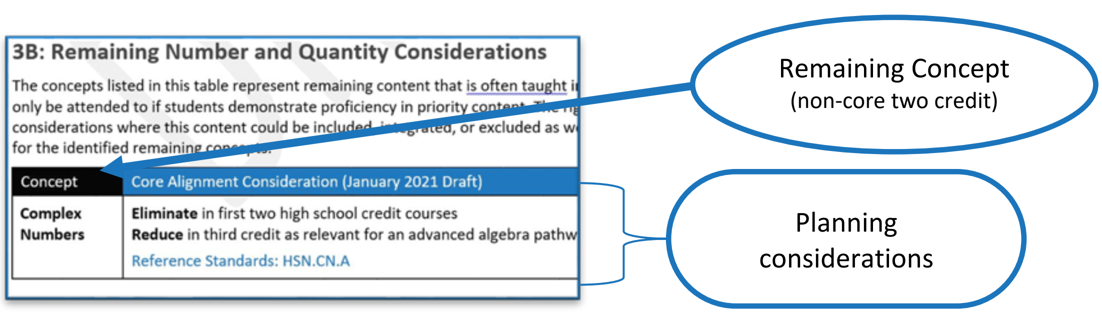
Table 3: Total count of core high school standards in January 2021 draft

High School Domain	Original number of standards [CCSS, 2010]	Number of core standards [Jan 2021 Draft]	Advanced (+) standards removed [Jan 2021 draft]	Non-advanced standards removed [Jan 2021 draft]	Proposed merged standards [Jan 2021 draft]
HS Algebra (HSA)	27	11	4	12	0
HS Functions (HSF)	28	10	7	11	0
HS Number (HSN)	27	4	18	5	0
HS Geometry	41	14	6	12	9
HS Data Science & Statistics (HSS)	24	16	2	6	0
TOTAL COUNT	147	55	37	46	9

As part of the high standards work then will be to understand what topics may be considered in the past as core content for all students, but is not consider core in the January 2021 draft. To support conversations during this review period, topics have been identified in part B for each domain with guidance in terms of how to plan. Many of the topics can still be included in specialized third or fourth credits of high school, such as in an algebra path to prepare for calculus, but not required for all students. An example of the tables found in part B can be found in Figure 2 below.

This guidance was originally provided in the high school 2020-21 planning guidance published in August 2020, and would still apply to future course planning. Feedback on the boundaries of core and non-core content would be welcome during this feedback phase.

Figure 2: Example of additional considerations found in part B



Part C – Crosswalk with Clarifying Guidance

A crosswalk between the CCSS (2010) text and the draft Oregon 2021 standards statements are provided in section C for each grade level. Additionally, clarifying guidance is provided that will be included in supporting documents, but not presented to the State Board for adoption. This will allow guidance to be revised and updated as needed without board adoption. Guidance is provided for educators to better understand the scope and boundaries of a given content standard.

Figure 3: Domain and cluster headings within Part C: Crosswalk with Clarifying Guidance

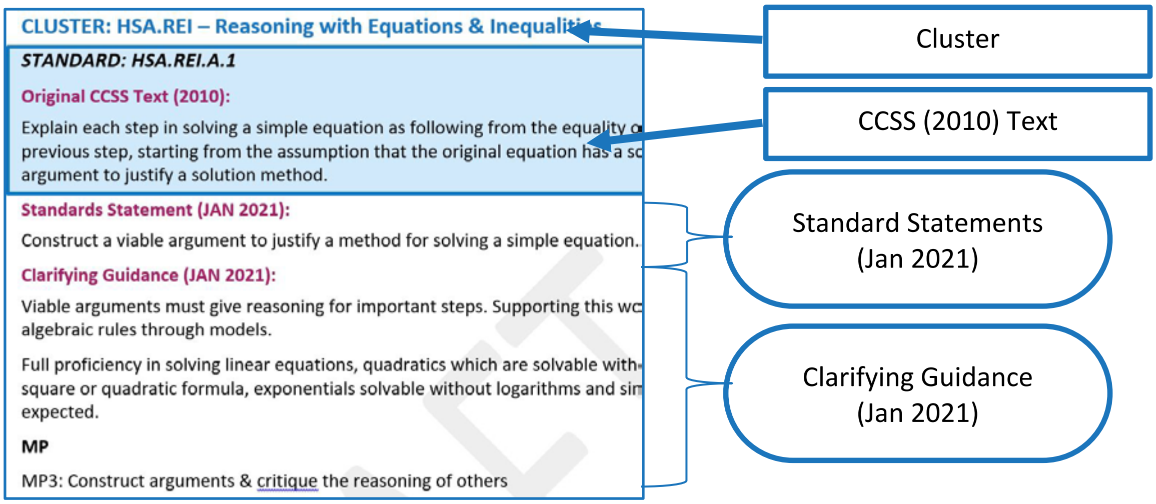


Table 3: Overview of Clarifying Guidance

Audience	Teachers, Administrators, Test & Curriculum Developers
Definition	Clarifying statements extend expectations within standards to decrease possible confusion or ambiguity.
Description	The intent of clarifying statements is to provide additional guidance for educators to communicate the intent of the standard as supporting resources are developed. Clarifying statements can be in the form of succinct sentences or paragraphs that attend to one of four types of clarifications: (1) Student Experiences; (2) Examples; (3) Boundaries; and (4) Connection to Math Practices.
Considerations	Clarifying statements are encouraged to draft, but optional so could potentially be left blank. The use of sentence frames is also encouraged as well as titles to indicate why type of clarifying statement this would be. Information could be used in the development of assessments and instructional materials, but not a requirement in that all students have the identical experience. They are guideposts that help reduce potential confusion and increase fidelity as educators implement the standards. • Examples found within the current CCSS document in the form of "i.e." or "e.g." statements should be moved to clarifying statements or removed. • Standards with and additional level, such as a "4a", "4b", or "4c" statement, should include relevant content in the standards statement, incorporate into the clarifying paragraphs, or removed.

Example sentence frames for clarifying statements could include, but are not limited to:

- Student Experiences
  - "Students should have the opportunity to ____"
  - "Build conceptual understanding by ____"
- Examples
  - "Some examples include ____"
- Boundary Statements
  - "Students are not expected to _____"
  - "Expectations of the standard include ____"
- Math Practices
  - "Opportunities to engage in math practices include ____"

Future work of the clarifying guidance could pull from additional sources outside of Oregon to create supporting documents to guide implementation of the adopted standards.

At this time, please provide feedback on the balance between adopted content standards and clarifying guidance using the provided forms on the Oregon Department of Education mathematics standards webpage, or contact Mark Freed, ODE Math Education Specialist, if you have additional questions or comments about the 2021 draft standards document.

SECTION TWO: Mathematical Practices and Modeling

2A: Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), (x - 1)(x2 + x + 1), and (x - 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

2B: Mathematical Modeling

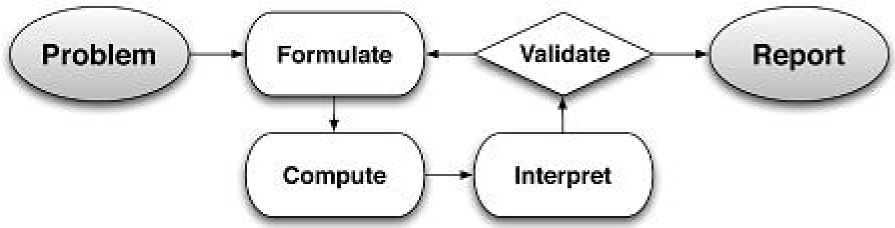
Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.



One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

SECTION THREE: Draft High School Number and Quantity

3A: Core Number and Quantity Focus

The standards listed in this table name the priority instructional content for high school Number and Quantity (HSN). The right-hand column contains draft focus content that would be essential for all students, and part of a core content in a student's first two credits after K-8 mathematics. Specific modeling standards appear throughout core content in this document as indicated by a star symbol (★).

HSN.RN - The Real Number System

Standard	Standard Statements (Jan 2021 Draft)
HSN.RN.A.1	Establish properties of positive integer exponents. Use these properties to extend the definition of exponentiation to negative and rational exponents. Integrated with Standard(s): HSN.RN.A.2

HSN.Q - Quantities

Standard	Standard Statements (Jan 2021 Draft)
HSN.Q.A.1	Choose and interpret units consistently in formulas, graphs, and data displays, as a way to understand problems and to guide the solution of multi-step problems. (★)
HSN.Q.A.2	Define appropriate quantities in real world situations for the purpose of modeling them and justify these choices. (★) Integrated with Standard(s): HSA.CED.A.1
HSN.Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in modeling situations. (★)

3B: Remaining Number and Quantity Considerations

The concepts listed in this table represent remaining content that is often taught in high school but should only be attended to if students demonstrate proficiency in priority content. The right-hand column contains considerations where this content could be included, integrated, or excluded as well as reference standards for the identified remaining concepts.

Concept	Core Alignment Consideration (January 2021 Draft)
Complex Numbers	**Eliminate** in first two high school credit courses **Reduce** in third credit as relevant for an advanced algebra pathway Reference Standards: HSN.CN.A

3C: High School Number Crosswalk with Clarifying Guidance

CLUSTER: HSN.RN - The Real Number System

STANDARD: HSN.RN.A.1

DRAFT Standards Statement (JAN 2021):

Establish properties of positive integer exponents. Use these properties to extend the definition of exponentiation to negative and rational exponents.

DRAFT Clarifying Guidance (JAN 2021):

MP

MP8: generalizing patterns

Original CCSS Text (2010):

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

CLUSTER: HSN.Q - Quantities

STANDARD: HSN.Q.A.1

DRAFT Standards Statement (JAN 2021):

Choose and interpret units consistently in formulas, graphs, and data displays, as a way to understand problems and to guide the solution of multi-step problems.*

DRAFT Clarifying Guidance (JAN 2021):

Note: This standard applies universally in modeling situations.
This includes real world problems that require changing units to understand a given context.
MP
MP2: quantitative and abstract reasoning
MP4: mathematical modeling
Original CCSS Text (2010):

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

STANDARD: HSN.Q.A.2

DRAFT Standards Statement (JAN 2021):

Define appropriate quantities in real world situations for the purpose of modeling them and justify these choices.

DRAFT Clarifying Guidance (JAN 2021):

Note: This standard applies universally in modeling situations.

Original CCSS Text (2010):

Define appropriate quantities for the purpose of descriptive modeling.

STANDARD: HSN.Q.A.3

DRAFT Standards Statement (JAN 2021):

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in modeling situations.

DRAFT Clarifying Guidance (JAN 2021):

Note: This standard applies universally in modeling situations.
MP
MP4: mathematical modeling

Original CCSS Text (2010):

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

SECTION FOUR: Draft High School Algebra

4A: Core Algebra Focus

The standards listed in this table name the priority instructional content for high school algebra (HSA). The right-hand column contains draft focus content that would be core content for all students in a student's first two credits after K-8 mathematics. Specific modeling standards are indicated by a star symbol (★).

HSA.SSE – Seeing Structure in Expressions

Standard	Standard Statements (Jan 2021 Draft)
HSA.SSE.A.1	Interpret an expression which models a quantity by viewing one or more of its parts as a single entity and reasoning about how changes in these parts impact the whole, and vice versa. (★)
HSA.SSE.B.3	Create and recognize an equivalent form of an expression to understand the quantity represented in an authentic context.* (★) Integrated with Standard(s): HSA.SSE.B.3c, HSA.APR.A.1, HSA.SSE.A.2

HSA.CED – Creating Equations

Standard	Standard Statements (Jan 2021 Draft)
HSA.CED.A.1	Define variables and create inequalities with one or more variables and use them to solve problems in real life contexts. (★) Integrated with Standard(s): HSA.CED.A.2
HSA.CED.A.2	Define variables and create equations with two or more variables to represent relationships between quantities in order to solve problems in real life contexts. (★) Integrated with Standard(s): HSA.CED.A.1, HSF.BF.A.1
HSA.CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities; interpret solutions as viable or nonviable options in a modeling context. (★)
HSA.CED.A.4	Rearrange formulas and equations to highlight a specific quantity. (★) Integrated with Standard(s): HSA.SSE.A.2

HSA.REI – Reasoning with Equations & Inequalities

Standard	Standard Statements (Jan 2021 Draft)
HSA.REI.A.1	Construct a viable argument to justify a method for solving a simple equation. Integrated with Standard(s): HSA.REI.B.4, HSA.REI.A.2
HSA.REI.C.6	Solve systems of linear equations through algebraic means for simple systems and strategically using technology when needed. Integrated with Standard(s): HSA.REI.C.7
HSA.REI.D.10	Understand the solutions to an equation in two variables is a set of points in the coordinate plane that form a curve, which could be a line.
HSA.REI.D.11	Recognize and explain why the point(s) of intersection of the graphs of f(x) and g(x) are solutions to the equation f(x)=g(x). Interpret the meaning of the coordinates of these points. (★) Integrated with Standard(s): HSA.REI.C.6
HSA.REI.D.12	Graph and explain why the points in a half plane are solutions to a linear inequality and the solutions to a system of inequalities are the points in the intersection of corresponding half planes. Interpret the meaning of the coordinates of these points in context.

4B: Remaining Algebra Considerations

The concepts listed in this table represent remaining content that is often taught in high school but should only be attended to if students demonstrate proficiency in priority content. The right-hand column contains considerations where this content could be included, integrated, or excluded as well as reference standards for the identified remaining concepts.

Concept	Core Alignment Consideration (January 2021 Draft)
Rational Expressions	**Eliminate** lessons on rational expressions in first two credits. **Possible integration** in advanced third credit courses if needed for modeling applications. Reference Standard(s): HSA.REI.A.2, HSA.APR.D.6, HSA.REI.A.2
Rewriting Expressions (Factoring)	**Limited** to factoring quadratics with technology in the first two credit courses. **Reduced** emphasis on paper and pencil methods in third credit courses. Reference Standard(s): HSA.SSE.A.2
Polynomial Arithmetic	**Integrated** use with lessons with simple equivalent expressions across all courses. **Limited** to use with technology for complex and/or multi-step arithmetic. Reference Standard(s): HSA.APR
Systems of Equations	**Limit** paper and pencil lessons to systems of linear functions, and combine using technology when possible. **Integrate** use of technology to solve systems that use nonlinear functions. Reference Standard(s): HSA.REI.C
Sequences & Series	**Limited** lessons with modeling applications (e.g. HSF.BF.A.2) across all courses. Reference Standard(s): HSA.SSE.B.4, HSF.IF.A.3

4C: High School Algebra Crosswalk with Clarifying Guidance

CLUSTER: HSA.SSE – Seeing Structure in Expressions

STANDARD: HSA.SSE.A.1

DRAFT Standards Statement (JAN 2021):

Interpret an expression which models a quantity by viewing one or more of its parts as a single entity and reasoning about how changes in these parts impact the whole, and vice versa.

DRAFT Clarifying Guidance (JAN 2021):

Note:

Parts include terms, factors, coefficients, exponents, numerators and denominators.

MP

MP4: mathematical modeling

MP7: using structure

Original CCSS Text (2010):

*Interpret expressions that represent a quantity in terms of its context.**

HSA.SSE.A.1.A *Interpret parts of an expression, such as terms, factors, and coefficients.*

HSA.SSE.A.1.B *Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

STANDARD: HSA.SSE.B.3

DRAFT Standards Statement (JAN 2021):

Create and recognize an equivalent form of an expression to understand the quantity represented in an authentic context.

DRAFT Clarifying Guidance (JAN 2021):

Notes:

Algebraic manipulation for its own sake should be avoided.

Equivalent forms are found through application of algebraic properties including properties of exponents, combining like terms, and distributive property.

MP:

MP2, 7 & 8: quantitative & abstract reasoning, using structure & generalizing --Equivalent forms are found through application of algebraic properties including properties of exponents, combining like terms, and distributive property.

Original CCSS Text (2010):

*Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.**

HSA.SSE.B.3.A	*Factor a quadratic expression to reveal the zeros of the function it defines.*
HSA.SSE.B.3.B	*Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*
HSA.SSE.B.3.C	*Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

CLUSTER: HSA.CED – Creating Equations

STANDARD: HSA.CED.A.1

DRAFT Standards Statement (JAN 2021):

Define variables and create inequalities with one or more variables and use them to solve problems in real life contexts.

DRAFT Clarifying Guidance (JAN 2021):

Full proficiency in creating inequalities arising from linear situations and developing proficiency for exponential situations is expected. Opportunities to explore simple quadratic and rational situations when called for by context are also included.

MP

MP4: mathematical modeling

Original CCSS Text (2010):

Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

STANDARD: HSA.CED.A.2

DRAFT Standards Statement (JAN 2021):

Define variables and create equations with two or more variables to represent relationships between quantities in order to solve problems in real life contexts.

DRAFT Clarifying Guidance (JAN 2021):

Full proficiency in creating equations arising from linear situations and developing proficiency for exponential situations is expected. Opportunities to explore simple quadratic and rational situations when called for by context are also included.

MP

MP4: mathematical modeling

Original CCSS Text (2010):

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

STANDARD: HSA.CED.A.3

DRAFT Standards Statement (JAN 2021):

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities; interpret solutions as viable or nonviable options in a modeling context.

DRAFT Clarifying Guidance (JAN 2021):

Full proficiency in creating and interpreting equations or inequalities arising from linear situations is expected. Opportunities to explore exponentials, simple quadratic and rational situations when called for by context are also included.

MP

MP4: Mathematical Modeling

Original CCSS Text (2010):

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

STANDARD: HSA.CED.A.4

DRAFT Standards Statement (JAN 2021):

Rearrange formulas and equations to highlight a specific quantity.

DRAFT Clarifying Guidance (JAN 2021):

Full proficiency in rearranging linear equations and developing proficiency with exponential (solved via roots, not logs) is expected. Opportunities with simple quadratic and rational situations when called for by context are also included. MP - MP2: quantitative & abstract reasoning

Original CCSS Text (2010):

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.

CLUSTER: HSA.REI – Reasoning with Equations & Inequalities

STANDARD: HSA.REI.A.1

DRAFT Standards Statement (JAN 2021):

Construct a viable argument to justify a method for solving a simple equation.

DRAFT Clarifying Guidance (JAN 2021):

Viable arguments must give reasoning for important steps. Supporting this work may also involve justifying algebraic rules through models.

Full proficiency in solving linear equations, quadratics which are solvable without factoring, completing the square or quadratic formula, exponentials solvable without logarithms and simple rational equations are expected.

MP

MP3: Construct arguments & critique the reasoning of others

Original CCSS Text (2010):

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

STANDARD: HSA.REI.C.6

DRAFT Standards Statement (JAN 2021):

Solve systems of linear equations through algebraic means for simple systems and strategically using technology when needed.

DRAFT Clarifying Guidance (JAN 2021):

Simple systems of equations are those that could easily be solved by hand, including whole number coefficients and/or rational number solutions. Full proficiency with pairs of linear equations in two variables is expected. Opportunities with non-linear systems when called for by context are also included.

MP

MP5: Using graphing technology

Original CCSS Text (2010):

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

STANDARD: HSA.REI.D.10

DRAFT Standards Statement (JAN 2021):

Understand the solutions to an equation in two variables is a set of points in the coordinate plane that form a curve, which could be a line.

DRAFT Clarifying Guidance (JAN 2021):

Note:

Common graphs include lines, parabolas, circles, and exponential curves

Original CCSS Text (2010):

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

STANDARD: HSA.REI.D.11

DRAFT Standards Statement (JAN 2021):

Recognize and explain why the point(s) of intersection of the graphs of f(x) and g(x) are solutions to the equation f(x)=g(x). Interpret the meaning of the coordinates of these points.*

DRAFT Clarifying Guidance (JAN 2021):

MP

MP4: mathematical modeling

MP5: using graphing technology

Original CCSS Text (2010):

*Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.**

STANDARD: HSA.REI.D.12

DRAFT Standards Statement (JAN 2021):

Graph and explain why the points in a half plane are solutions to a linear inequality and the solutions to a system of inequalities are the points in the intersection of corresponding half planes. Interpret the meaning of the coordinates of these points in context.

DRAFT Clarifying Guidance (JAN 2021):

Graphs can be created by hand in simple cases but in general with technology to allow the emphasis on the interpretations of solutions.

MP

MP4: mathematical modeling

MP5: using graphing technology

Original CCSS Text (2010):

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

SECTION FIVE: Draft High School Functions

5A: Core Function Focus

The standards listed in this table name the priority instructional content for high school functions (HSF). The right-hand column contains draft focus content that would be core content for all students in a student's first two credits after K-8 mathematics. Specific modeling standards are indicated by a star symbol (★).

HSF.IF – Interpreting Functions

Standard	Standard Statements (Jan 2021 Draft)
HSF.IF.A.1	Understand a function as a rule that assigns a unique output for every input and that functions model situations where one quantity determines another. Integrated with Standard(s): HSF.IF.A.2
HSF.IF.A.2	Use function notation and interpret statements that use function notation in terms of the context and the relationship it describes. Integrated with Standard(s): HSF.IF.A.1
HSF.IF.B.4	Interpret key features of functions, from multiple representations, and conversely predict features of functions from knowledge of context. (★) Integrated with Standard(s): HSF.IF.A.1, HSF.IF.A.2
HSF.IF.B.5	Relate the domain of a function to its graph and to its context. (★)
HSF.IF.B.6	Calculate and interpret the average rate of change of a function over a specified interval. (★)
HSF.IF.C.7	Graph functions to show key features. (★)
HSF.IF.C.9	Compare properties of two functions using multiple representations. (★)

HSF.BF – Building Functions

Standard	Standard Statements (Jan 2021 Draft)
HSF.BF.A.2	Model situations involving arithmetic and geometric sequences. Use a variety of representations including an explicit formula for the sequence, and translate between the forms. (★)
HSF.BF.B.3	Identify and interpret the effect on the graph of a function when the equation has been transformed.

HSF.LE – Linear, Quadratic, & Exponential Models

Standard	Standard Statements (Jan 2021 Draft)
HSF.LE.A.1	Explain why a situation can be modeled with a linear function, an exponential function, or neither. (★) Explanations should connect to the reasoning required in HSF.LE.A.1a

5B: Remaining Function Considerations

The concepts listed in this table represent remaining content that is often taught in high school but should only be attended to if students demonstrate proficiency in priority content. The right-hand column contains considerations where this content could be included, integrated, or excluded as well as reference standards for the identified remaining concepts.

Concept	Core Alignment Consideration (January 2021 Draft)
Quadratic Functions	**Combine** lessons on quadratic functions with the study of expressions, equations, and functions in support of math modeling applications. **Eliminate** use of paper and pencil methods (e.g. quadratic formula, factoring trinomials, completing the square) in the first two credit courses, and possible **reduced** use in third credit courses as applicable for an advanced algebra course. Reference Standard(s): HSA.REI.B.4
Inverse Functions	**Eliminate** lessons on inverse functions in the first three credit courses with possible inclusion in fourth credit courses such as pre-calculus. Reference Standard(s): HSF.BF.B.4
Interpret expressions for functions	**Integrate** lessons on interpreting the parameters for functions in context of modeling applications only. Reference Standard(s): HSF.LE.B.5
Trigonometric Functions	**Limit** lessons to applications using right triangle trigonometry using appropriate technology in first two credits. Possible reduced emphasis of additional trigonometric applications in third and fourth credit courses as applicable for advanced algebra options. Reference Standard(s): HSF.TF
Unit Circle, Periodic Functions	**Eliminate** lessons in the first two credit courses. **Limit** emphasis of periodic behavior within third or fourth credit courses as applicable for advanced algebra options. Reference Standard(s): HSF.TF.B.5, HSF.TF.A.1, HSF.TF.A.2
Trigonometric Identities	**Eliminate** lessons in the first two credits and **limited** emphasis in a third or fourth credit option. Reference Standard(s): HSF.TF.C.8

5C: High School Functions Crosswalk with Clarifying Guidance

CLUSTER: HSF.IF – Interpreting Functions

STANDARD: HSF.IF.A.1

DRAFT Standards Statement (JAN 2021):

Understand a function as a rule that assigns a unique output for every input and that functions model situations where one quantity determines another.

DRAFT Clarifying Guidance (JAN 2021):

Functions are often represented by tables, expressions or graphs. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.

Modeling examples should include both contexts where only one quantity can be considered the independent variable as well as contexts where both quantities could.

MP

MP4: Mathematical Modeling

Original CCSS Text (2010):

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

STANDARD: HSF.IF.A.2

Original CCSS Text (2010):

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

DRAFT Standards Statement (JAN 2021):

Use function notation and interpret statements that use function notation in terms of the context and the relationship it describes.

DRAFT Clarifying Guidance (JAN 2021):

MP

MP4: mathematical modeling

STANDARD: HSF.IF.B.4

DRAFT Standards Statement (JAN 2021):

Interpret key features of functions, from multiple representations, and conversely predict features of functions from knowledge of context.

DRAFT Clarifying Guidance (JAN 2021):

Key features include: domain, range, discrete, continuous, intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Representations include: graphs, tables, spreadsheet representations, as well as symbolic.

MP

MP4: mathematical modeling

MP7: using structure

Original CCSS Text (2010):

*For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

STANDARD: HSF.IF.B.5

DRAFT Standards Statement (JAN 2021):

Relate the domain of a function to its graph and to its context.

DRAFT Clarifying Guidance (JAN 2021):

Contexts can demand discrete vs. continuous and domain restrictions.

MP

MP4: mathematical model

MP6: precision

Original CCSS Text (2010):

*Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

STANDARD: HSF.IF.B.6

DRAFT Standards Statement (JAN 2021):

Calculate and interpret the average rate of change of a function over a specified interval.

DRAFT Clarifying Guidance (JAN 2021):

Work with functions presented as graphs, tables or symbolically.

Students should choose intervals for analysis of functions with substantially varying rates of change.

MP

MP6: precision

MP7: structural thinking

Original CCSS Text (2010):

*Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.**

STANDARD: HSF.IF.C.7

DRAFT Standards Statement (JAN 2021):

Graph functions to show key features.

DRAFT Clarifying Guidance (JAN 2021):

Use technology to graph functions expressed symbolically or in tables, with intentional choices of window and scale. Graph functions by hand in simple cases or for approximations.

Key features include: specific values when context demands; domain and range; discrete or continuous; intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima.

MP

MP4: mathematical modeling

MP5: using graphing technology

Original CCSS Text (2010):

*Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.**

HSF.IF.C.7.A	*Graph linear and quadratic functions and show intercepts, maxima, and minima.*
HSF.IF.C.7.B	*Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
HSF.IF.C.7.C	*Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
HSF.IF.C.7.D	*(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.*
HSF.IF.C.7.E	*Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*

STANDARD: HSF.IF.C.9

DRAFT Standards Statement (JAN 2021):

Compare properties of two functions using multiple representations.

DRAFT Clarifying Guidance (JAN 2021):

Functions can be represented algebraically, graphically, numerically in tables, or by verbal descriptions.

Original CCSS Text (2010):

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

CLUSTER: HSF.BF – Building Functions

STANDARD: HSF.BF.A.2

DRAFT Standards Statement (JAN 2021):

Model situations involving arithmetic and geometric sequences. Use a variety of representations including an explicit formula for the sequence, and translate between the forms.*

DRAFT Clarifying Guidance (JAN 2021):

MP

MP2: quantitative and abstract reasoning

MP4: mathematical modeling

Original CCSS Text (2010):

*Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.**

STANDARD: HSF.BF.B.3

DRAFT Standards Statement (JAN 2021):

Identify and interpret the effect on the graph of a function when the equation has been transformed.

DRAFT Clarifying Guidance (JAN 2021):

Transformations include translations (f(x)+k, and f(x-h)), reflections (e.g. -f(x) and f(-x), and dilations (e.g. a*f(x)). Interpretations include accounting for different choices of variables, such as initial values or units.

Full proficiency with linear functions and developing proficiency with exponential functions is expected. Technology provides opportunities for exploration with other functions.

MP4: mathematical modeling

MP5: using graphing technology

Original CCSS Text (2010):

Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

CLUSTER: HSF.LE – Linear, Quadratic, & Exponential Models

STANDARD: HSF.LE.A.1

DRAFT Standards Statement (JAN 2021):

Explain why a situation can be modeled with a linear function, an exponential function, or neither.

DRAFT Clarifying Guidance (JAN 2021):

MP

MP4: Mathematical Modeling

Original CCSS Text (2010):

Distinguish between situations that can be modeled with linear functions and with exponential functions.

HSF.LE.A.1.A *Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.*

HSF.LE.A.1.B *Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*

HSF.LE.A.1.C *Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*

SECTION SIX: Draft High School Geometry

6A: Core Geometry Focus

The standards listed in this table name the priority instructional content for high school geometry (HSG). The right-hand column contains draft focus content that would be core content for all students in a student's first two credits after K-8 mathematics. Specific modeling standards are indicated by a star symbol (★).

HSG.CO – Congruence

Standard	Standard Statements (Jan 2021 Draft)
HSG.CO.A.1	Use definitions of geometric figures and geometric relationships to justify the solutions of problems.
HSG.CO.A.5	Develop definitions of rotations, reflections, and translations in authentic contexts. Apply these definitions to transform a shape or map between two given shapes. Integrated with Standard(s): HSG.CO.A.2; HSG.CO.A.4
HSG.CO.B.7	Apply and justify triangle congruence theorems in authentic contexts. Integrated with Standard(s): HSG.CO.B.8
HSG.CO.C.9	Justify theorems of line relationships, angles, triangles, and parallelograms; and use them to solve problems in authentic contexts. Integrated with Standard(s): HSG.CO.C.10
HSG.CO.D.12	Perform geometric constructions with a variety of tools and methods.

HSG.SRT – Similarity, Right Triangles, & Trigonometry

Standard	Standard Statements (Jan 2021 Draft)
HSG.SRT.A.5	Use similarity theorems to determine whether two triangles are similar. Verify experimentally the properties of dilations given by a center and a scale factor. Solve problems in authentic contexts involving similar triangles or dilations. Integrated with Standard(s): HSG.SRT.A.1, HSG.SRT.A.2, HSG.SRT.A.3
HSG.SRT.C.8	Apply sine, cosine, and tangent ratios, and the Pythagorean Theorem, to solve problems in authentic contexts. Integrated with Standard(s): HSG.SRT.C.6, HSG.SRT.C.7

HSG.GPE – Expressing Geometric Properties with Equations

Standard	Standard Statements (Jan 2021 Draft)
HSG.GPE.A.1	Apply the Pythagorean Theorem in authentic contexts, and develop the standard form for the equation of a circle.
HSG.GPE.B.4	Use Cartesian coordinates to determine parallel and perpendicular relationships, and distance in the coordinate plane.
HSG.GPE.B.5	Use the slopes of segments and the coordinates of the vertices of triangles, parallelograms, and trapezoids to solve problems in authentic contexts.

HSG.GMD – Geometric Measurement & Dimension

Standard	Standard Statements (Jan 2021 Draft)
HSG.GMD.A.1	Solve authentic modeling problems using area formulas for triangles, parallelograms, trapezoids, regular polygons, and circles. (★)
HSG.GMD.A.3	Use geometric shapes, their measures, and their properties to describe real world objects, and solve related authentic modeling and design problems. (★)

HSG.MG – Modeling with Geometry

Standard	Standard Statements (Jan 2021 Draft)
HSG.MG.A.1	Use geometric shapes, their measures, and their properties to describe real world objects, and solve related modeling and design problems. (★) Integrated with Standard(s): HSG.MG.A.3
HSG.MG.A.2	Apply concepts of density based on area and volume in authentic modeling situations. (★)

6B: Remaining Geometry Considerations

The concepts listed in this table represent remaining content that is often taught in high school but should only be attended to if students demonstrate proficiency in priority content. The right-hand column contains considerations where this content could be included, integrated, or excluded as well as reference standards for the identified remaining concepts.

Concept	Core Alignment Consideration (January 2021 Draft)
Congruence Proofs	**Limit** work to applications of triangle congruence in modeling contexts. Reference Standard(s): HSG.CO.C.9
Additional Proofs	**Integrate** lessons of logical reasoning with applications of priority geometry content as needed to construct viable arguments (MP.3). **Reduce** emphasis on the two-column proof procedure, instead emphasizing using deductive reasoning to support conjectures.
Similarity transformations	**Combine** lessons using dilations and justification of similarity transformations to contrast and complement the focus on congruence and rigid motions. Reference Standard(s): HSG.SRT.A
Polynomial Theorem Proofs	**Limit** to justification of theorems of line relationships, angles, triangles, and parallelograms in modeling contexts. Reference Standard(s): HSG.CO.C.10, HSG.CO.C.11
Pythagorean Theorem	**Integrate** use of the Pythagorean Theorem in context with right triangle applications. **Eliminate** proofs of Pythagorean identities in the first three credits. Reference Standard(s): HSA.APR.C.4, HSF.TF.C.8, HSG.SRT.B.4, HSG.SRT.C.6
Law of Sines and Cosines	**Eliminate** lessons in the first two credit courses. **Reduced** emphasis fourth credit courses as applicable for advanced algebra options. Reference Standard(s): HSG.SRT.D.11
Visualize 2-D and 3-D relationships	**Eliminate** lessons on cross-sections and rotations of two-dimensional objects; Limit applications use of two-dimensional nets of three-dimensional polyhedra. Reference Standard(s): HSG.GMD.B.4
Conic Sections	**Limit** use of the Pythagorean theorem to develop and apply the distance formula and the equation of a circle. **Eliminate** lessons deriving formulas for equations of additional conic sections. Reference Standard(s): HSG.GPE.A.1, HSG.GPE.A.2

6C: High School Geometry Crosswalk with Clarifying Guidance

CLUSTER: HSG.CO – Congruence

STANDARD: HSG.CO.A.1

DRAFT Standards Statement (JAN 2021):

Definitions should include angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

DRAFT Clarifying Guidance (JAN 2021):

Definitions should include angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

M.P. Students need to attend to precision as they use definitions to discuss their reasoning with others.

Original CCSS Text (2010):

Use definitions of geometric figures and geometric relationships to justify the solutions of problems.

STANDARD: HSG.CO.A.5, HSG.CO.A.2, HSG.CO.A.4

DRAFT Standards Statement (JAN 2021):

Develop definitions of rotations, reflections, and translations in authentic contexts. Apply these definitions to transform a shape or map between two given shapes.

DRAFT Clarifying Guidance (JAN 2021):

The focus here is on rigid transformations (rotation, reflection, and translations) that create congruent figures. This includes the use of transformation rules and functions.

HS expectation would be for any rigid transformation. Use for a purpose with technology.

Original CCSS Text (2010):

(A.2) Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

(A.4) Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

(A.5) Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

STANDARD: HSG.CO.B.7, HSG.CO.B.8

DRAFT Standards Statement (JAN 2021):

Apply and justify triangle congruence theorems in authentic contexts.

DRAFT Clarifying Guidance (JAN 2021):

Note: *B.8 will be left for a "+1" course. Rigid transformations are removed from this standard, but included as part of proposed HSG. CO.A.2, A.4, A.5 standard. Use of triangle congruence theorems (SSS, SAS, ASA, AAS, or HL) should be used to solve problems in authentic contexts.

The focus here is to develop an understanding of techniques for proving that two triangles are congruent. Opportunities should also be available for students to understand when the conditions do not result in congruence.

M.P. Construct viable arguments and critique the reasoning of others when showing that two triangular roof trusses must be congruent.

Original CCSS Text (2010):

(B.7) Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

(B.8) Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

STANDARD: HSG.CO.C.9, HSG.CO.C.10

DRAFT Standards Statement (JAN 2021):

Justify theorems of line relationships, angles, triangles, and parallelograms; and use them to solve problems in authentic contexts.

DRAFT Clarifying Guidance (JAN 2021):

Theorems should include angles formed by parallel lines, angles formed by polygons, properties of special quadrilaterals (sides, angles, and diagonals), and properties of special triangles (isosceles, equilateral, and right).

Justification should require a precise chain of reasoning that verifies the validity of a mathematical theorem.

M.P. Construct viable arguments and critique the reasoning of others when justifying the congruence of diagonals in a rectangle that is built by a contractor installing a rectangular window.

Original CCSS Text (2010):

(C.9) Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

(C.10) Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

STANDARD: HSG.CO.D.12

DRAFT Standards Statement (JAN 2021):

Perform geometric constructions with a variety of tools and methods.

DRAFT Clarifying Guidance (JAN 2021):

Tools to include compass and straightedge, string, reflective devices, paper folding, and/or dynamic geometric software. Constructions to include copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

M.P. Use appropriate tools strategically when choosing the physical method and appropriate procedures for performing a construction.

Original CCSS Text (2010):

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

CLUSTER: HSG.SRT – Similarity, Right Triangles, & Trigonometry

STANDARD: HSG.SRT.B.5, HSG.SRT.A.1, HSG.SRT.A.2, HSG.SRT.A.3

DRAFT Standards Statement (JAN 2021):

Use similarity theorems to determine whether two triangles are similar. Verify experimentally the properties of dilations given by a center and a scale factor. Solve problems in authentic contexts involving similar triangles or dilations.

DRAFT Clarifying Guidance (JAN 2021):

Triangles can be shown to be similar using transformations and triangle similarity theorems. Apply theorems of AA similarity, SSS similarity, and SAS similarity to prove that two given triangles are similar.

M.P. Model with Mathematics to use similarity to solve real world problems to measure lengths and distances indirectly.

Original CCSS Text (2010):

(B.5) Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

(A.1) Verify experimentally the properties of dilations given by a center and a scale factor:

HSG.SRT.A.1a A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

HSG.SRT.A.1b The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

(A.2) Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

(A.3) Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

STANDARD: HSG.SRT.C.8, HSG.SRT.C.6, HSG.SRT.C.7

DRAFT Standards Statement (JAN 2021):

Apply sine, cosine, and tangent ratios, and the Pythagorean Theorem, to solve problems in authentic contexts.

DRAFT Clarifying Guidance (JAN 2021):

Applications should involve finding angle and side measures of right triangles.

Understand the relationship between the sine and cosine of complementary angles.

Original CCSS Text (2010):

*(C.8) Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.**

(C.6) Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

(C.7) Explain and use the relationship between the sine and cosine of complementary angles.

CLUSTER: HSG.GPE – Expressing Geometric Properties with Equations

STANDARD: HSG.GPE.A.1

DRAFT Standards Statement (JAN 2021):

Apply the Pythagorean Theorem in authentic contexts, and develop the standard form for the equation of a circle.

DRAFT Clarifying Guidance (JAN 2021):

Given the coordinates of the center and length of the radius, write the equation of the circle in standard form. Given the equation of a circle in standard form, determine the coordinates of its center and the length of its radius.

Use the Pythagorean Theorem to develop and apply the distance formula.

M.P. Look for and make use of structure to make connections to the Pythagorean Theorem and distance formula.

Original CCSS Text (2010):

Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

STANDARD: HSG.GPE.B.4

DRAFT Standards Statement (JAN 2021):

Use Cartesian coordinates to determine parallel and perpendicular relationships, and distance in the coordinate plane.

DRAFT Clarifying Guidance (JAN 2021):

Applications include the use of coordinates to compute perimeters of polygons and areas of triangles and rectangles. The distance formula will play an important role in these applications.

M.P. Use appropriate tools strategically to choose between tools such as the slope formula, distance formula, midpoint formula, or Pythagorean Theorem.

Original CCSS Text (2010):

Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).

STANDARD: HSG.GPE.B.5

DRAFT Standards Statement (JAN 2021):

Use the slopes of segments and the coordinates of the vertices of triangles, parallelograms, and trapezoids to solve problems in authentic contexts.

DRAFT Clarifying Guidance (JAN 2021):

Possible applications include using slopes to determine parallel sides in parallelograms and trapezoids, perpendicular diagonals in rhombuses, perpendicular sides in a rectangle, as well as verifying mid-segment properties in triangles and trapezoids. Use coordinates of vertices for lengths of sides and diagonals to classify quadrilaterals and triangles.

Original CCSS Text (2010):

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

CLUSTER: HSG.GMD – Geometric Measurement & Dimension

STANDARD: HSG.GMD.A.1

DRAFT Standards Statement (JAN 2021):

Solve authentic modeling problems using area formulas for triangles, parallelograms, trapezoids, regular polygons, and circles.*

DRAFT Clarifying Guidance (JAN 2021):

Students should give informal arguments for area formulas, and combine them to solve problems with composite figures.

M.P. Model with Mathematics can be used here to solve a variety of problems involving area.

Original CCSS Text (2010):

Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

STANDARD: HSG.GMD.A.3

DRAFT Standards Statement (JAN 2021):

Use volume and surface area formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems and apply to authentic contexts.

DRAFT Clarifying Guidance (JAN 2021):

Students should give informal arguments for area and volume formulas, and combine them to solve problems with composite figures. This standard is limited to right solids.

M.P. Make sense of problems and persevere in solving them when finding the volume of prisms and pyramids with regular polygon bases (possibly using trigonometry).

Original CCSS Text (2010):

*Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.**

CLUSTER: HSG.MG – Modeling with Geometry

STANDARD: HSG.MG.A.1, HSG.MG.A.3

DRAFT Standards Statement (JAN 2021):

Use geometric shapes, their measures, and their properties to describe real world objects, and solve related authentic modeling and design problems.

DRAFT Clarifying Guidance (JAN 2021):

This includes the use of volume formulas for prisms, cylinders, pyramids, cones, and spheres.

M.P. Model with Mathematics can be used here to solve a variety of problems such as designing a real world object with CAD design tools for 3D printing or CNC machining.

Original CCSS Text (2010):

*(A.1) Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).**

*(A.3) Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).**

STANDARD: HSG.MG.A.2

DRAFT Standards Statement (JAN 2021):

Apply concepts of density based on area and volume in authentic modeling situations.

DRAFT Clarifying Guidance (JAN 2021):

The focus is on geometric probability and proportional reasoning.

This should include an understanding of the ratios of areas (area ratio = (scale factor)^2) and volumes (volume ratio = (scale factor)^3) of similar figures.

M.P. Model with Mathematics to compute persons per square miles, BTUs per cubic foot, or specimens per acre.

Original CCSS Text (2010):

*Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).**

SECTION SEVEN: Draft High School Data Science and Statistics

7A: Core Data Science and Statistics Focus

The standards listed in this table name the priority instructional content for high school functions (HSF). The right-hand column contains draft focus content that would be core content for all students in a student's first two credits after K-8 mathematics. Specific modeling standards are indicated by a star symbol (★).

HSS.ID – Interpreting Categorical & Quantitative Data

Standard	Standard Statements (Jan 2021 Draft)
HSS.ID.A.1	Represent the distribution of data multiple ways with plots on the real number line.
HSS.ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets.
HSS.ID.A.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
HSS.ID.A.4	Use the mean and standard deviation of an approximately normally distributed data set to estimate population percentages.
HSS.ID.B.5	Analyze the association between two categorical variables by using two-way tables and comparative bar graphs.
HSS.ID.B.6	Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
HSS.ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
HSS.ID.C.8	Compute, using technology, and interpret the correlation coefficient of a linear fit.
HSS.ID.C.9	Distinguish between correlation and causation.

HSS.IC – Making Inferences & Justifying Conclusions

Standard	Standard Statements (Jan 2021 Draft)
HSS.IC.A.1	Understand the process of statistical reasoning, formulate questions, collect, analyze, and interpret data to answer statistical investigative questions.
HSS.IC.B.3	Recognize the difference between sample surveys, experiments and observational studies and understand the role of randomization in each.
HSS.IC.B.4	Use data from a sample survey to estimate a population parameter.
HSS.IC.B.5	Use data from a randomized experiment to compare two treatments to decide if differences between parameters are significant based on the statistics.
HSS.IC.B.6	Evaluate reports based on data.

HSS.CP – Conditional Probability & the Rules of Probability

Standard	Standard Statements (Jan 2021 Draft)
HSS.CP.A.1	Describe the possible outcomes for a situation as subsets of a sample space.
HSS.CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. Integrated with Standard(s): HSS.CP.A.4

7B: Remaining Data Science and Statistics Considerations

The concepts listed in this table represent remaining content that is often taught in high school but should only be attended to if students demonstrate proficiency in priority content. The right-hand column contains considerations where this content could be included, integrated, or excluded as well as reference standards for the identified remaining concepts.

Concept	Core Alignment Consideration (January 2021 Draft)
Simulations	**Eliminate** lessons using simulations to develop a margin of error or decide if differences between parameters are significant. Reference Standard(s): HSS.IC.B.4, HSS.IC.B.5
Independent Events	**Limit** lessons to conceptual understanding; **Eliminate** product of probabilities. Reference Standard(s): HSS.CP.A.2, HSS.CP.A.3
Conditional Probability	**Limit** lessons to conceptual understanding; **Eliminate** lessons on computation of conditional probabilities. Reference Standard(s): HSS.CP.A.3; HSS.CP.B.6
Addition Rule	**Eliminate** lessons on applying the addition rule. Reference Standard(s): HSS.CP.B.7
Multiplication Rule	**Eliminate** lessons on applying the multiplication rule. Reference Standard(s): HSS.CP.B.8
Permutations and Combinations	**Limit** lessons to conceptual understanding; **Eliminate** lessons on computation of permutations and combinations. Reference Standard(s): HSS.CP.B.9

7C: High School Data Science and Statistics Crosswalk with Clarifying Guidance

CLUSTER: HSS.ID – Interpreting Categorical & Quantitative Data

STANDARD: HSS.ID.A.1

DRAFT Standards Statement (JAN 2021):

Represent the distribution of data multiple ways with plots on the real number line.

DRAFT Clarifying Guidance (JAN 2021):

Graph numerical data on a real number line using dot plots, histograms, and box plots. Data are displayed visually to discover patterns and deviations from patterns. Analyze the strengths and weakness inherent in each type of plot by comparing different plots of the same data. Describe and give simple conclusions and interpretations of a graphical representation of data.

Original CCSS Text (2010):

Represent data with plots on the real number line (dot plots, histograms, and box plots).

STANDARD: HSS.ID.A.2

DRAFT Standards Statement (JAN 2021):

Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets.

DRAFT Clarifying Guidance (JAN 2021):

Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, uniform, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Students should have the opportunity to gain an understanding of this concept through the use of technology tools.

Original CCSS Text (2010):

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

STANDARD: HSS.ID.A.3

DRAFT Standards Statement (JAN 2021):

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

DRAFT Clarifying Guidance (JAN 2021):

Use data from multiple sources to interpret differences in shape, center and spread. Discuss the effect of outliers on measures of center and spread. Students should use spreadsheets, graphing utilities and statistical software to identify outliers and analyze data sets with and without outliers as appropriate.

Original CCSS Text (2010):

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

STANDARD: HSS.ID.A.4

DRAFT Standards Statement (JAN 2021):

Use the mean and standard deviation of an approximately normally distributed data set to estimate population percentages.

DRAFT Clarifying Guidance (JAN 2021):

Data may be displayed using histograms, dot plots, or smooth normal curves. Recognize that there are data sets for which the empirical rule is not appropriate. The use of calculators, spreadsheets, z-score tables, to estimate the area under the curve is not appropriate for the first two years of study.

Original CCSS Text (2010):

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

STANDARD: HSS.ID.B.5

DRAFT Standards Statement (JAN 2021):

Analyze the association between two categorical variables by using two-way tables and comparative bar graphs.

DRAFT Clarifying Guidance (JAN 2021):

Read, interpret and write clear summaries of data displayed in a two-way frequency table. Calculate joint, marginal, and conditional relative frequencies. Make appropriate displays of joint, marginal, and conditional distributions. Describe patterns observed in the data. Recognize the association between two variables by comparing conditional and marginal percentages. Students may use spreadsheets, graphing calculators, and statistical software to create frequency tables and determine associations or trends in the data.

Original CCSS Text (2010):

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

STANDARD: HSS.ID.B.6

DRAFT Standards Statement (JAN 2021):

Represent data on two quantitative variables on a scatter plot and describe how the variables are related.

DRAFT Clarifying Guidance (JAN 2021):

This is a good opportunity for students to collect and graph their own data and use modeling to fit a function to the data; use a function fitted to data to solve problems in the context of the data (Emphasize linear models).

Fit a linear function for a scatter plot that suggests a linear association.

Students should use spreadsheets, graphing calculators, and statistical software to analyze the bivariate data.

Original CCSS Text (2010):

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

HSS.ID.B.6a *Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

HSS.ID.B.6b *Informally assess the fit of a function by plotting and analyzing residuals.*

HSS.ID.B.6c *Fit a linear function for a scatter plot that suggests a linear association.*

STANDARD: HSS.ID.C.7

DRAFT Standards Statement (JAN 2021):

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

DRAFT Clarifying Guidance (JAN 2021):

Students demonstrate interpreting slope in the context of a given situation when examining two variable statistics as being “for each additional known unit increase in an explanatory variable, we expect or predict a known unit increase (or decrease) in the response variable.”

Students demonstrate interpreting intercept in the context of a given situation when examining two variable statistics as being “the predicted known unit of a response variable when the explanatory variable is zero known units.”

Students would use technology to develop an awareness of how outliers might affect the rate of change and the intercept of a given model.

Students should be able to explain when intercepts might be outside the scope of the model.

Original CCSS Text (2010):

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

STANDARD: HSS.ID.C.8

DRAFT Standards Statement (JAN 2021):

Compute, using technology, and interpret the correlation coefficient of a linear fit.

DRAFT Clarifying Guidance (JAN 2021):

Explain that the correlation coefficient must be between −1 and 1 inclusive and explain what each of these values means. Determine whether the correlation coefficient shows a weak positive, strong positive, weak negative, strong negative, or no linear correlation. Interpret what the correlation coefficient is telling about the data. Students should use spreadsheets, graphing calculators and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals and correlation coefficients.

Original CCSS Text (2010):

Compute (using technology) and interpret the correlation coefficient of a linear fit.

STANDARD: HSS.ID.C.9

DRAFT Standards Statement (JAN 2021):

Distinguish between correlation and causation.

DRAFT Clarifying Guidance (JAN 2021):

Understand and explain the difference between correlation and causation. Understand and explain that a strong correlation does not mean causation. Determine if statements of causation seem reasonable or unreasonable and justify reasoning.

Original CCSS Text (2010):

Distinguish between correlation and causation.

CLUSTER: HSS.IC – Making Inferences & Justifying Conclusions

STANDARD: HSS.IC.A.1

DRAFT Standards Statement (JAN 2021):

Understand the process of statistical reasoning, formulate questions, collect, analyze, and interpret data to answer statistical investigative questions.

DRAFT Clarifying Guidance (JAN 2021):

This is an opportunity for students to create a survey, collect data, and use graphical displays, sample statistics or two way tables to help estimate population parameters which are unknown values. It is important to understand samples used on social media or in the news.

Original CCSS Text (2010):

Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

STANDARD: HSS.IC.B.3

DRAFT Standards Statement (JAN 2021):

Recognize the difference between sample surveys, experiments and observational studies and understand the role of randomization in each.

DRAFT Clarifying Guidance (JAN 2021):

[no additional clarifying guidance at this time]

Original CCSS Text (2010):

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

STANDARD: HSS.IC.B.4

DRAFT Standards Statement (JAN 2021):

Use data from a sample survey to estimate a population parameter.

DRAFT Clarifying Guidance (JAN 2021):

This is an opportunity for students to look at real data, margin of error and discuss what it means to estimate a population parameter.

Original CCSS Text (2010):

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

STANDARD: HSS.IC.B.5

DRAFT Standards Statement (JAN 2021):

Use data from a randomized experiment to compare two treatments to decide if differences between parameters are significant based on the statistics.

DRAFT Clarifying Guidance (JAN 2021):

Limit to population proportion, graphical representations, and visual overlap.

Original CCSS Text (2010):

Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

STANDARD: HSS.IC.B.6

DRAFT Standards Statement (JAN 2021):

Evaluate reports based on data.

DRAFT Clarifying Guidance (JAN 2021):

[no additional clarifying guidance at this time]

Original CCSS Text (2010):

Evaluate reports based on data.

CLUSTER: HSS.CP – Conditional Probability & the Rules of Probability

STANDARD: HSS.CP.A.1

DRAFT Standards Statement (JAN 2021):

Describe the possible outcomes for a situation as subsets of a sample space.

DRAFT Clarifying Guidance (JAN 2021):

This provides an opportunity for students to engage with finding the outcomes of situations which include words such as **and**, **or**, **not**, **if**, and **all**, and to grammatical constructions that reflect logical connections.

Original CCSS Text (2010):

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

STANDARD: HSS.CP.A.5

DRAFT Standards Statement (JAN 2021):

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

DRAFT Clarifying Guidance (JAN 2021):

[no additional clarifying guidance at this time]

Original CCSS Text (2010):

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.