Oregon Mathematics Guidance Document

2021 Oregon Mathematics Standards

Guidance Version 5.2.7

Standards Adopted: October 2021

Guidance Version Update: May 2023

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Table of Contents

Table of Contents 2
SECTION ONE: Introduction 4
1A: Letter to Educators 4
1B: How to Read the Document 6
1C: High School Math Pathways 13
SECTION TWO: 2021 Adopted Math Practice Standards 17
2A: Standards for Mathematical Practices 17
2B: Mathematical Modeling 20
SECTION THREE: 2021 Adopted Math Content Standards 22
3A: Grade K Math Standards (2021) 22
3B: Grade 1 Math Standards (2021) 24
3C: Grade 2 Math Standards (2021) 26
3D: Grade 3 Math Standards (2021) 28
3E: Grade 4 Math Standards (2021) 30
3F: Grade 5 Math Standards (2021) 32
3G: Grade 6 Math Standards (2021) 34
3H: Grade 7 Math Standards (2021) 36
3I: Grade 8 Math Standards (2021) 38
3J: High School Algebra Standards (2021) 40
3K: High School Geometry Standards (2021) 42
3L: High School Data & Statisitcs Standards (2021) 43
SECTION FOUR – Grade/Course Level Guidance 44
4A: Grade K Math Standards and Guidance 44
4B: Grade 1 Math Standards and Guidance 68
4C: Grade 2 Math Standards and Guidance 93
4D: Grade 3 Math Standards and Guidance 121
4E: Grade 4 Math Standards and Guidance 148
4F: Grade 5 Math Standards and Guidance 180
4G: Grade 6 Math Standards and Guidance 208
4H: Grade 7 Math Standards and Guidance 238
4I: Grade 8 Math Standards and Guidance 263
4J: HS Algebra Standards and Guidance 293
4K: HS Geometry Standards and Guidance 322

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4L: HS Data & Statistics Standards and Guidance 338
SECTION FIVE: Oregon/CCSS Crosswalk 355
5A: Grade K Math Crosswalk (2021) 355
5B: Grade 1 Math Crosswalk (2021) 361
5C: Grade 2 Math Crosswalk (2021) 369
5D: Grade 3 Math Crosswalk (2021) 377
5E: Grade 4 Math Crosswalk (2021) 387
5F: Grade 5 Math Crosswalk (2021) 399
5G: Grade 6 Math Crosswalk (2021) 411
5H: Grade 7 Math Crosswalk (2021) 424
5I: Grade 8 Math Crosswalk (2021) 436
5J: High School Algebra Crosswalk (2021) 448
5K: High School Geometry Crosswalk (2021) 456
5L: High School Data Reasoning and Statisitcs Crosswalk (2021) 462

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

SECTION ONE: Introduction

1A: Letter to Educators

The revised Oregon math standards were created in collaboration with educators from across the state, who committed their time and expertise to drafting, reviewing, and finalizing these guidance documents. The revision process was a two-year project starting in the summer of 2019, led by a total of 118 educators divided into 12 grade level teams.

Public draft documents were published in January 2021. The Oregon Department of Education (ODE) hosted 18 virtual sessions in partnership with nine Oregon Educational Service Districts. Separate K-8 and High School sessions were scheduled that engaged over 400 educators to provide additional input. ODE staff is grateful for the contributions of so many educators throughout this process, who continued their work throughout the disruptions of a global pandemic.

In October of 2022, the Oregon State Board of Education adopted the final revised standards. This document is a testament to the shared vision of a next generation math education system that equitably meets the needs of each student.

Commitment to Engineering an Equitable Math Sytem for All

> The vision of mathematics education in Oregon is to provide all students with a mathematical foundation that supports them to make sense of the world around us, to communicate effectively, and to discover innovative solutions.

Mathematics is both a logical framework for understanding the world and a creative endeavor that opens up opportunities to explore relationships, recognize patterns, and find symmetry and beauty in the world. In its broadest sense, mathematics is about more than just a single track to a right answer and includes multiple pathways that engage relevancy, invite joy, and develop wonder to explore the world.

Engineering an equitable system of math education requires a collective commitment by policymakers, leaders, educators, students, and families. It also requires replacing the historical system that filters students with one that elevates and develops each student's ability to use mathematics to make sense of the world. Such a system supports all students bring their identities to use math to solve the problems that they are interested in, and to make the world a better place. ODE is dedicated to building a math education system that propels students towards their career and college goals, through the following commitments:

- **Commitment 1:** Provide a strong foundation of mathematical understanding and fluency for every student.
- **Commitment 2:** Apply mathematics through authentic problem solving opportunities for students.
- **Commitment 3:** Implement inclusive active learning strategies that engage students with content both inside and outside the classroom.
- **Commitment 4:** Dismantle systemic barriers based on perceptions of student's mathematical readiness rooted in racially biased or otherwise invalid assumptions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Oregon Math Project – Meaningful Math For All

The Oregon Math Project (OMP) advances mathematics education in our state by cultivating a network of educators that promotes equitable math experiences for all students through guidance and the support of policies, standards, curricula, assessments, and instructional best practices. Realizing the vision of math education in Oregon includes ensuring that all students attain mathematics proficiency by having access to high-quality instruction that includes challenging and coherent content in a learning environment where each student receives the support they need to succeed in mathematics.



Uri Treisman of the University of Texas at Austin shared a metaphor of math education being engineered as a filter, one that sorts and labels our students as "math" or "non-math" people.



In place of the filter, the challenge before us is the work of reimagining an equitable system, engineered like a pump that moves and lifts all students to the goals they want to achieve. This metaphor resonated deeply with the State Board of Education as we presented the 2021 math standards for adoption.

The work of engineering a more equitable math system is centered on four cornerstone principles of the Oregon Math Project: **Focus, Engagement, Pathways, and Belonging.**

Focus: Learning experiences in every grade and course are focused on core mathematical content and practices that progress purposefully across grade levels.



Engagement: Mathematical learning happens in environments that motivate all students to engage with relevant and meaningful issues in the world around them.



Pathways: All students are equipped with the mathematical knowledge and skills necessary to identify and productively pursue any postsecondary paths in their future. Students have agency to choose from a variety of courses, contexts, and applications they find relevant.



Belonging: Participation in mathematical learning builds students' identities as capable math learners and fosters a positive self-concept. Students' cultural and linguistic assets are valued in ways that contribute to a sense of belonging to a community of learners.



Any proposed instructional approach, curricular change, or system design element should be evaluated by the degree to which it builds on these four cornerstones. When new approaches are built within the framework of all four-cornerstone principles, we will be on our way to engineering a reimagined system.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

1B: How to Read the Document

The 2021 Oregon Mathematics Standards are organized into grade level content for K-8 and within the domains of algebra, geometry, and data science/statistics for high school and can be found in the ODE math standards webpage.

This document provides and overview for each grade level, including critical areas of focus, domains, and cluster headings. Each standard section is divided into sections: (1) standards statement; (2) content connections; and (3) standards guidance. Hyperlinks are provided to assist with navigation to jump to a particular grade, cluster, or standard of interest.

Navigating Section Two – Adopted 2021 Mathematical Practices

The Standards for Mathematical Practices (SMPs) were first introduced in the Common Core State Standards (CCSS, 2010) and appear in the revised Oregon Mathematics Standards (2021). These eight standards were adopted in the same form within the revised standards. At this time, section two includes this wording and additional information on math modeling (MP4).

Future versions of this document will include additional information on specific math practices, starting with an overview of math modeling in this version. Additional resources on the SMPs can be found on sites outside of Oregon, such as from Inside Mathematics, or the Math Assessment Project, that can provide additional details and sample tasks.

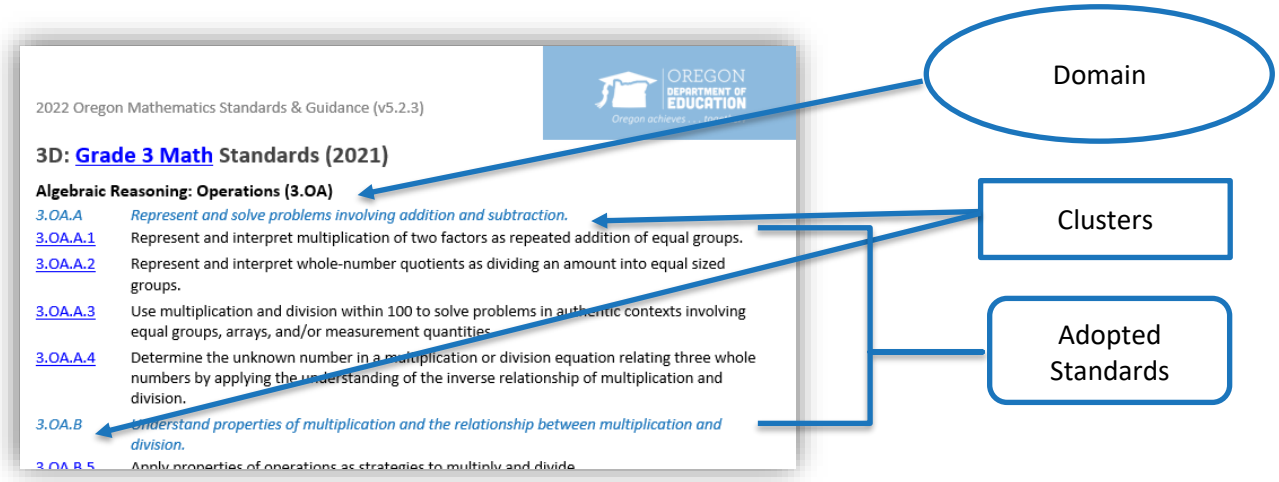
Navigating Section Three – Adopted 2021 Oregon Mathematics Standards

Overall Structure: Domains, clusters, and standards

The 2021 review and revision of K-12 math standards includes efforts to improve readability and access for a wide range of potential readers. The standards document itself provides an opportunity to share important information to not only educators, but additional audiences—such as parents and community members—as well. The standards within section three reflect efforts to focus the wording of the standards statements, and is the content adopted by the State Board.

Additional information in the 2010 standards can generally be found within the guidance sections, now within section four. A sample grade from section three is shown in Figure 1 below identifying domains, clusters, and standards used.

Figure 1: Domain, Clusters, and Standards Statements within Section Three

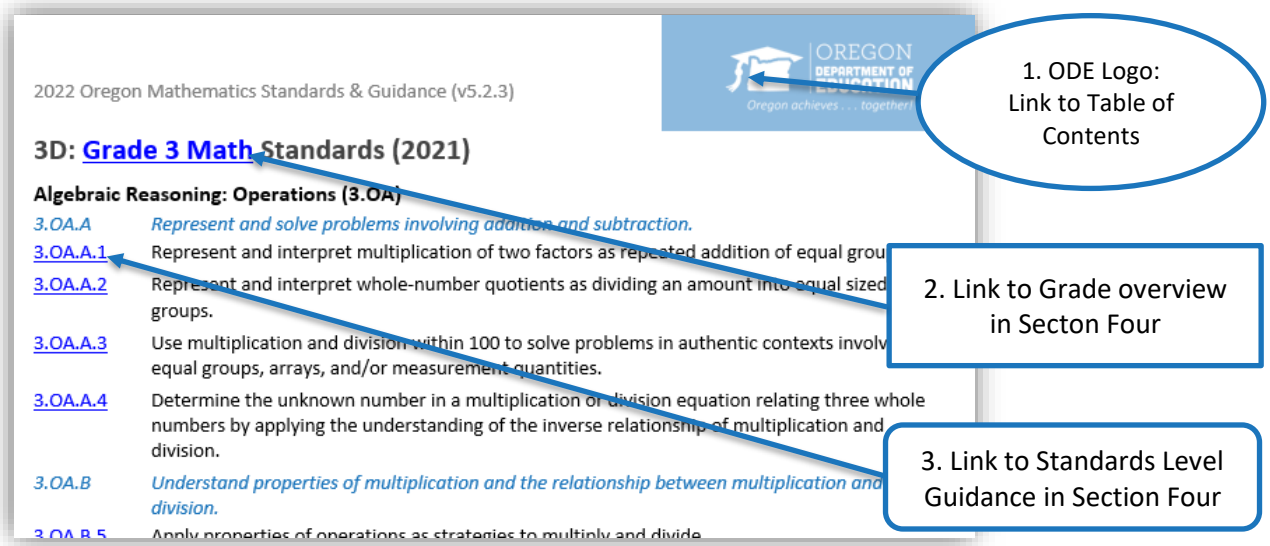


This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Navigation Links in Section Three

Section three includes hyperlinks to help navigate to specific content standards. These links include: (1) hyperlink to table of contents (ODE Logo), (2) hyperlink to navigate to the given grade overview in section four, and (3) direct hyperlinks to standards level guidance found in section four. Figure 2 below shows were to find these hyperlinks from a sample section of section three.

Figure 2: Navigation Hyperlinks in Section Three



Navigating Part Four – Grade and Standards Level Guidance

Critical Areas for Grade Level Mathematics

For each grade level from kindergarten through grade 8, the Critical Areas outline essential mathematical ideas. This included in high school for algebra, geometry, and data/statistics content. The critical areas are designed to bring focus to the standards by describing two to four big ideas that educators can use to build their curriculum and to guide instruction.

Grade Level Overview

Following the critical areas, information is provided on grade level domains and clusters. This content is presented unedited from the CCSS (2010) K-8 standards in order to provide continuity with the 2021 Oregon math standards. In general, these terms include:

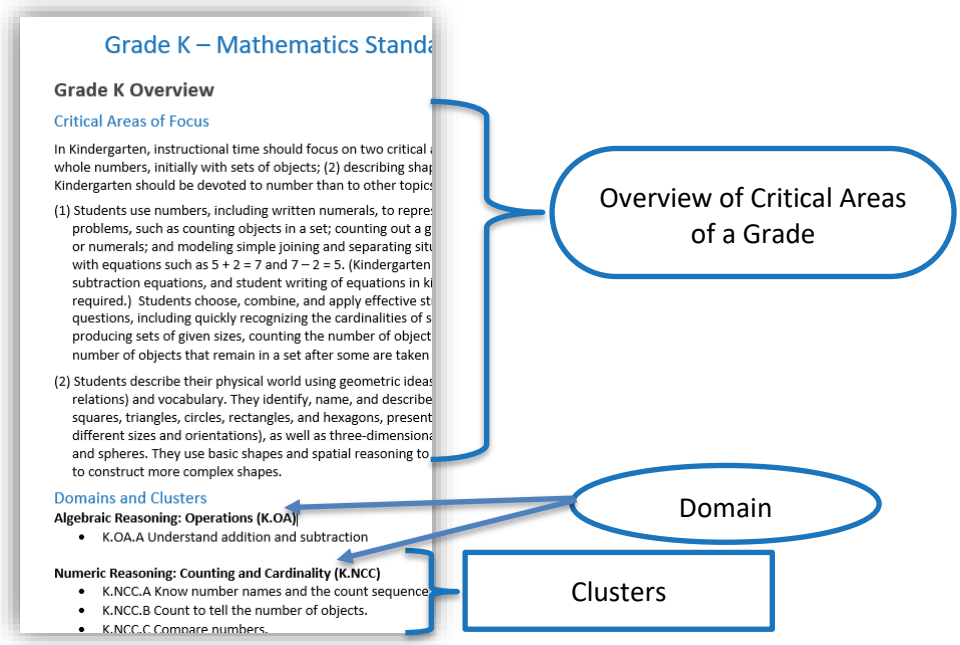
Standards define what students should understand and be able to do.

Clusters summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

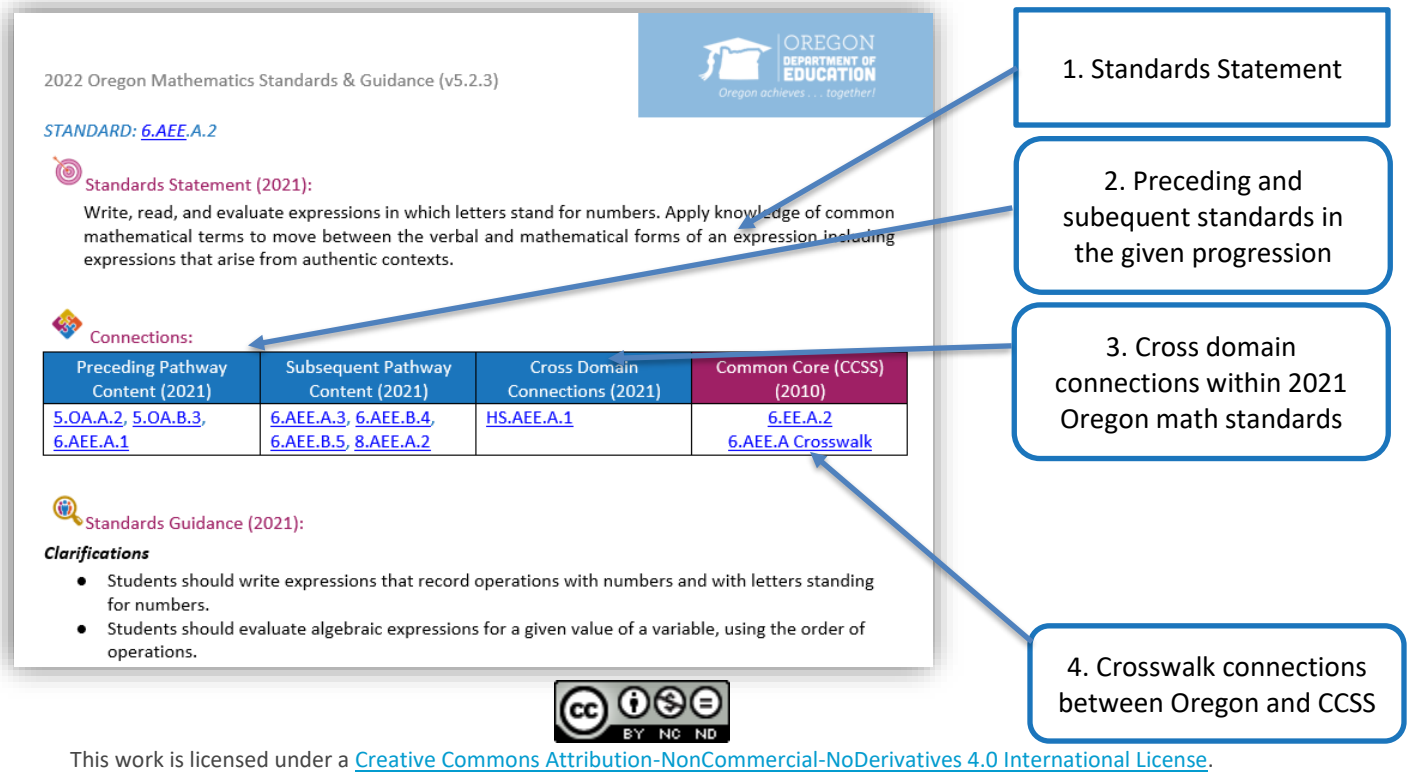
Figure 3: Domain and cluster headings within Section Three



Overall Sturcture in Section Four: Standards, Connections, and Guidance

Guidance within section four generally includes one page of guidance for each adopted standard. On this page, you will find (1) the adopted wording from the 2021 Oregon Math Standards; (2) content standards preceding and follow a given standards as well as cross domain connections; (3) Cross domain connections to the standard; and (4) links to the CCSS (2010) and the crosswalk in section five.

Figure 4: Navigation Links in Section Four

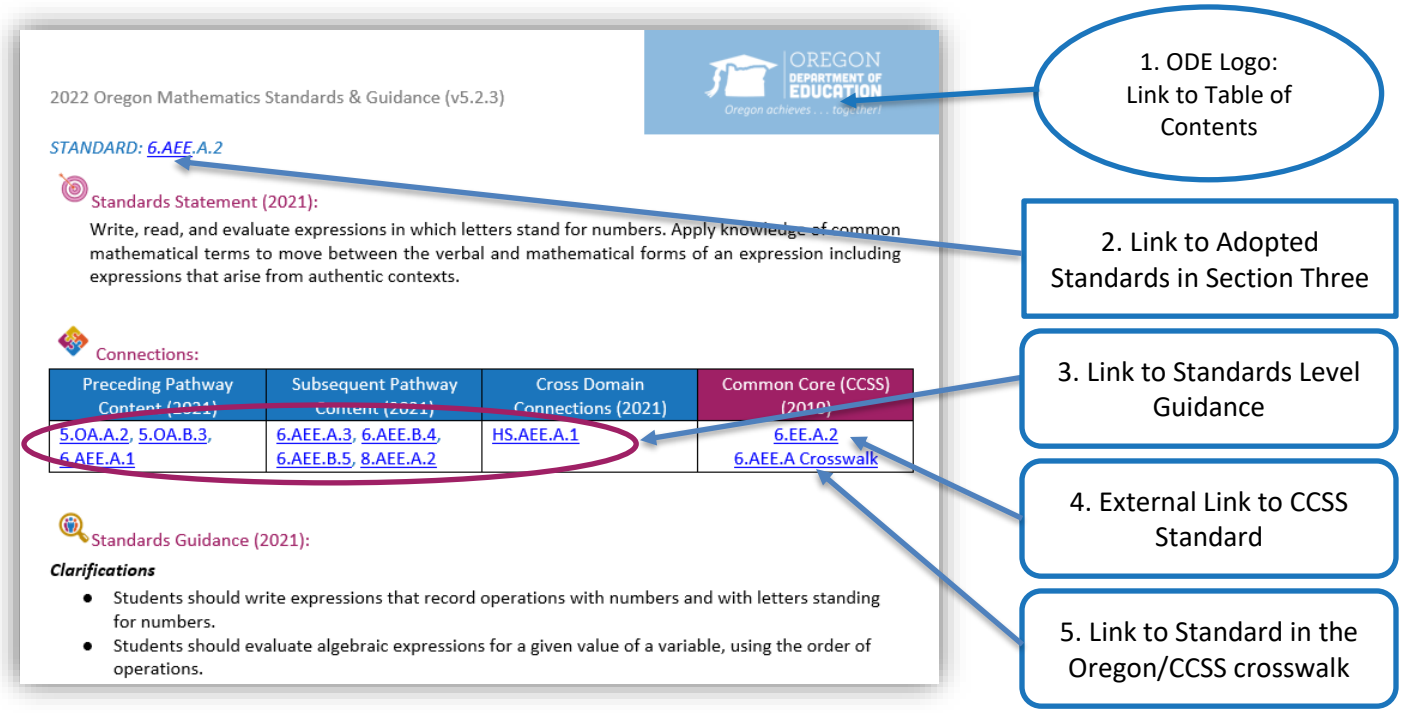


This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Navigation Links in Section Four

Section four includes hyperlinks to help navigate to specific content standards. These links include: (1) hyperlink to table of contents (ODE Logo), (2) hyperlink to the given grade overview in section three, and (3) direct hyperlinks to standards level guidance found within a learning a progression, (4) external link to wording directly from CCSS website related to the given Oregon standard; and (5) link to the relevant section of the ODE/CCSS crosswalk in section five. Figure 2 below shows were to find these hyperlinks from a sample section of section three.

Figure 5: Navigation Links in Section Four



Categories for Standards Guidance in Section Four

Many of the CCSS (2010) standards were long and technical, and presented barriers to access for many readers interested in grade level expectations. For the 2021 review, the original standards were divided into two parts:

1. Standards statements that will be adopted by the State Board of Education
2. Clarifying guidance that will be used in supporting documents for use by educators to understand the boundaries and examples of a given standard

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Table 1: Overview of Standards Statements

Audience	Everyone
Definition	A standard is a statement of what a student should know, understand, or do.
Description	Standards statements need to be written as stand-alone statement(s) in the final document. They could include more than one sentence, but overall word count needs to remain below approximately 40-50 words.

Considerations	• Lead with clarity
	  ○ Start with key ideas
	  ○ First sentence approximately 10-20 words
	  ○ Minimize use of conjunctions (and/or)
	• Include information from CCSSM
	  ○ Total word count approximately 40-50 words
	• No Parentheticals
	  ○ Examples moved to clarifying statements
	• Technical Considerations
	  ○ Size (word count, character count, number of paragraphs)
	  ○ Complexity (words per sentence, characters per word)
	  ○ Readability (Flesch Reading Ease, Flesh-Kincaid Grade Level)

Table 2: Overview of Clarifying Guidance

Audience	Teachers, Administrators, Test & Curriculum Developers
Definition	Clarifying statements extend expectations within standards to decrease possible confusion or ambiguity.
Description	The intent of clarifying statements is to provide additional guidance for educators to communicate the intent of the standard to support the future development of curricular resources and assessments aligned to the 2021 math standards. Clarifying statements can be in the form of succinct sentences or paragraphs that attend to one of four types of clarifications: (1) Student Experiences; (2) Examples; (3) Boundaries; and (4) Connection to Math Practices.
Considerations	Clarifying statements are encouraged but are optional. The use of sentence frames and titles is also encouraged to indicate the type of clarifying statement. Information could be used in the development of assessments and instructional materials. These guideposts help reduce potential confusion and increase fidelity as educators implement the standards. • Examples found within the current CCSS document in the form of "i.e." or "e.g." statements should be moved to clarifying statements or removed. • Standards with an additional level, such as a "4a", "4b", or "4c" statement, should include relevant content in the standards statement, incorporate into the clarifying paragraphs, or be removed.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Types of guidance is provided within the categories found in Table 3 below. Not all categories were necessarily used for each standard, but all guidance was categorized using these types and presented in the order described in the table.

Table 3: Types of Standards Guidance

Order	Title	Description
1	Clarifications	A foundational or primary concept, rule, or principle that a student would have the opportunity to learn prior to the given standard (such as clarifying statements and elaborations).
2	Terminology	Definitions of mathematical terms or nomenclature associated with a given content standard (such as terms, vocabulary, and nomenclature).
3	Boundaries	Boundary statements for a given standard such as inclusions and exclusions for a given grade or course.
4	Teaching Strategies	Strategies and methods for consideration by an educator to better understand types of instructional activities that could be used to support teaching a given standard.
5	Progressions	The mathematics standardds were built on progressions: narrative documents describing the progression of a topic across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics. All the progression documents can be found on this website, with specific standards level connections to these documents will be made in this section.
6	Examples	Specific examples of tasks or resources that help illustrate the type of work associated with a given standard.

Sentence frames were often used within the guidance development to assist with clarity, and could be edited to reflect these types of structures if additional clarity is needed. Example sentence frames for clarifying statements could include, but are not limited to:

- Student Experiences
  - "Students should have the opportunity to_____"
  - "Build conceptual understanding by_____"
- Examples
  - "Some examples include_____"
- Boundary Statements
  - "Students are not expected to_____"
  - "Expectations of the standard include_____"
- Math Practices
  - "Opportunities to engage in math practices include_____"

Future work of the clarifying guidance could pull from additional sources from within and outside of Oregon. Resources, edits, or updates can be sent anytime to ODE.MathProject@ode.oregon.gov

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Navigation Section Five – Oregon/CCSS Crosswalk

A crosswalk between the 2021 Oregon math standards statements and the Common Core (2010) is provided in section five for each grade level. Access to specific standards can be through links provided in the "connections" table within the grade level guidance in section four.

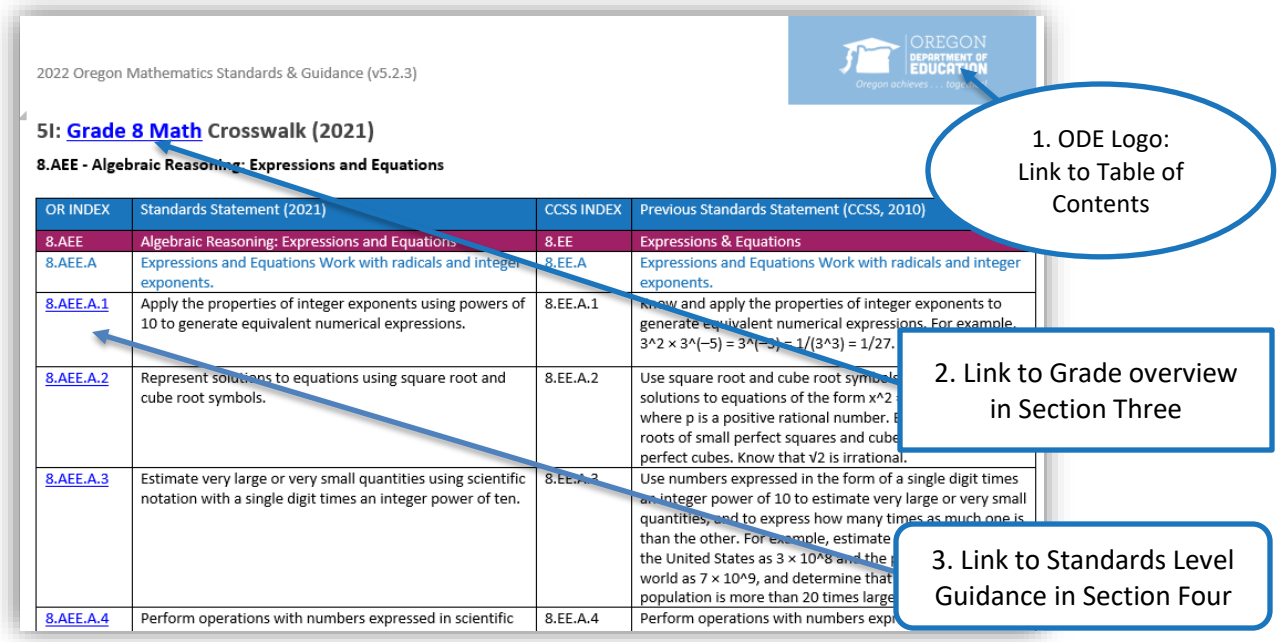
The tables in section five provide a side-by-side comparison of the domains, clusters, and standards within the 2021 Oregon math standards. An example of one grade can be seen below in Figure 6.

Navigation Links in Part Five

Links within the "connections" tables in section four combine with return links in the crosswalk tables in section five and are designed to ease back and forth viewing between the sections.

A return link can be found for (1) the table of contents (ODE Logo); (2) link to standards level guidance in section four; and (3) standards level guidance found in section four.

Figure 6: Navigation Links in Section Five



OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.AEE	Algebraic Reasoning: Expressions and Equations	8.EE	Expressions & Equations
8.AEE.A	Expressions and Equations Work with radicals and integer exponents.	8.EE.A	Expressions and Equations Work with radicals and integer exponents.
8.AEE.A.1	Apply the properties of integer exponents using powers of 10 to generate equivalent numerical expressions.	8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 3^2 × 3^(−5) = 3^(−3) = 1/(3^3) = 1/27.
8.AEE.A.2	Represent solutions to equations using square root and cube root symbols.	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form x^2 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.
8.AEE.A.3	Estimate very large or very small quantities using scientific notation with a single digit times an integer power of ten.	8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the United States as 3 × 10^8 and the world as 7 × 10^9, and determine that population is more than 20 times large
8.AEE.A.4	Perform operations with numbers expressed in scientific	8.EE.A.4	Perform operations with numbers expr

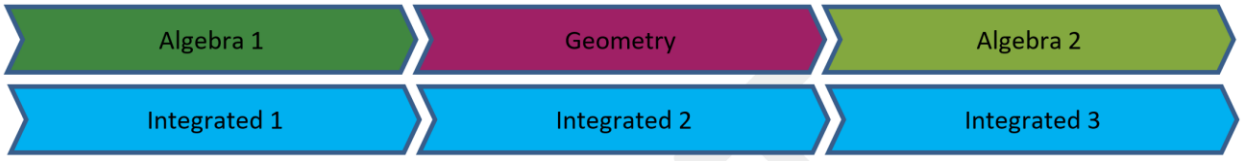
This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

1C: High School Math Pathways

Common Historical Course Sequences

In Oregon, districts and schools need to plan courses that provide the opportunity for students to have access to the adopted high school standards by the end of a three credit sequence or sooner. State law does not prescribe any specific high school math course sequence, rather only that students have access to the content identified in the adopted state standards. Traditionally, three credits of high school course content have been arranged into either an Algebra, Geometry, Algebra 2 sequence (AGA), or an integrated equivalent.

Figure 7: Common High School Course Options



In 2010, Oregon adopted high school math standards based on the Common Core State Standards (CCSS), which currently identify 111 non-advanced standards for all students. In addition, there are 36 optional advanced (+) standards that could either be included in the core three-credit sequence or as additional study. Content is divided into six domains of Number, Algebra, Functions, Modeling, Geometry, and Statistics & Probability.

Standards Revision and Future High School Pathways

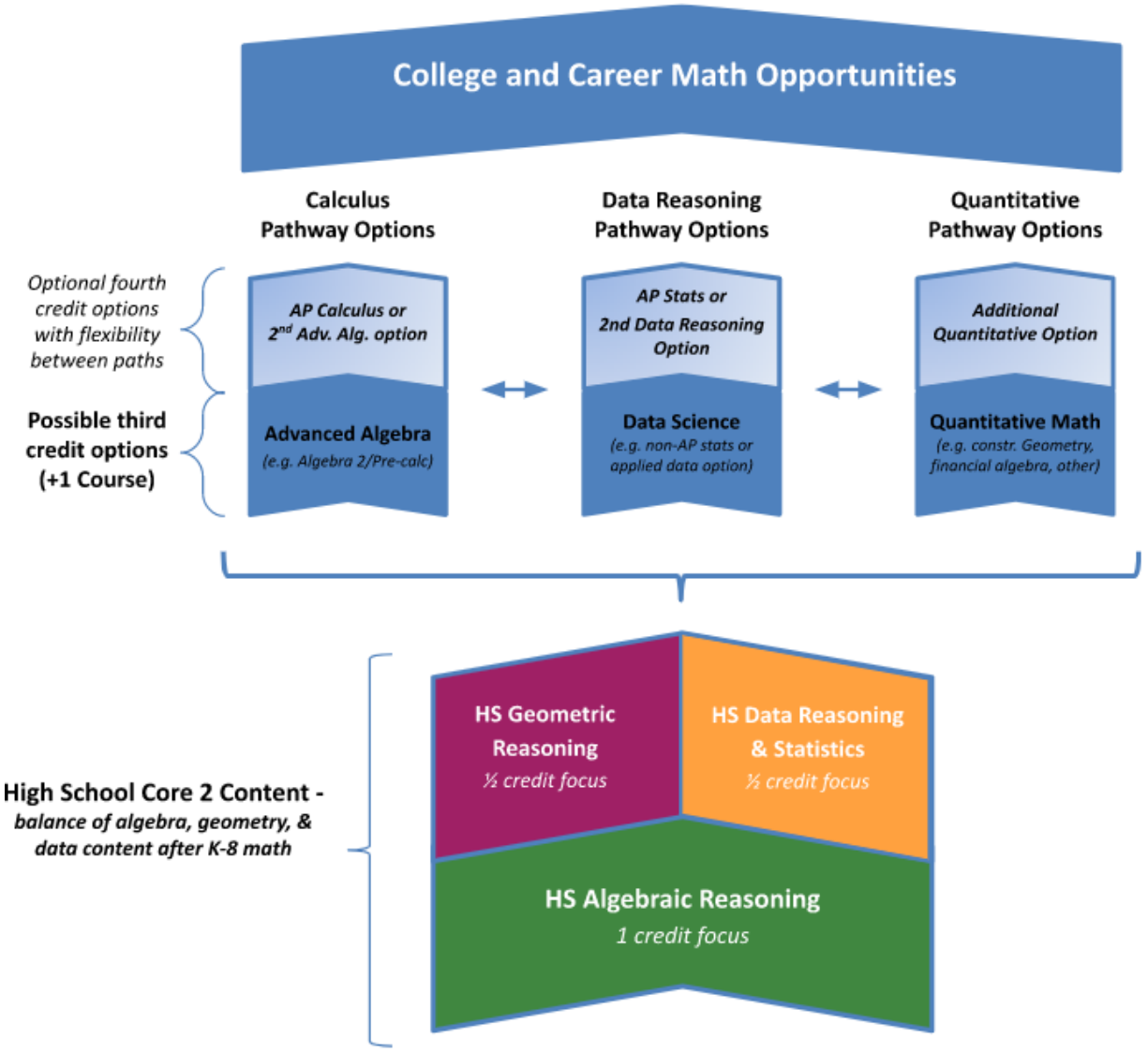
Oregon State Standards are reviewed and revised on a regular basis. In mathematics, the review and revsion by educators occurred between 2019 and 2021 and was adopted by the State Board of Education in October 2021. This timeline includes preparing for a state review of instructional materials in the summer of 2022 and district updating of materials in the 2022-23 school year. From the student perspective, statewide implementation in classrooms would occur by the fall of the 2023-24 school year. Districts do have and additional two years of flexilbity, which would give the option for updated materials used by students in Fall 2024 or Fall 2025.

With the State Board of Education adoption of the revised high school standards, the work will shift to implementation of the revised 2+1 course pathway model with two credits of core content for all students, and create third credit pathway options that align to student interests and goals.

Core content would be balanced between approximately one credit of algebra content, ½ credit of geometry, and ½ credit of data science and statistics. For third credit options, Oregon high school staff are invited to innovate by offering new specialized courses within three general paths: (1) a pathway to calculus; (2) a pathway to data science; (3) and a pathway to quantitative mathematics. Figure 8 helps visualize the long-term goal of what high school math pathways in the 2+1 model could look like as early as the 2023-24 school year.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Figure 8: Long Term (2023-24) Course Pathway Options



Centering on Equitable High School Mathematics

Equitable outcomes for all Oregon students will require considerations for health, saftety, and access to high quality instruction. Historically, students in mathematics have inequitable access to grade level content; unfinished learning can accumulate over time, creating a system where students find themselves trapped in tracks repeating K-8 mathematics within high school courses.

Ensuring students have access to high school content will take innovation and creativity to reimagine what course experiences could look like for Oregon students. The intent of this document is to provide additional guidance specific to planning high school math courses as we plan course options that set up each and every Oregon student for equitable access to mathematics. Additional information and example practices can be found in the Oregon Math Project Practice Brief: Promoting Equity

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Detracking Mathematics and Creating Pathways in High School

ODE supports national calls to consider detracking math experiences for our students and teachers. Detracking the first two credits of core math content is a long-term goal for Oregon, and can begin as soon as this next school year. At the same time, ODE supports efforts to create grade 11-14 math pathways that could include specialized third credit course options, such as construction geometry or financial algebra, that align to student goals and aspirations. For the 2022-23 school year, it is likely that existing courses such as Algebra 1, Geometry, and Algebra 2 would still be provided, with a focus on prioritized content.

For the purposes of ODE guidance, the term "tracking" will refer to the practice of creating different levels of the same course that group students by perceived abilities. Detracking high school courses would ensure that all students have access to the same content and experiences for any given course.

The term "pathway" refers to a specialized math course sequence that leads to career and college readiness. This could include the traditional advanced algebra pathway to prepare for calculus that all students were locked into, but could also include a statistics pathway or quantitative applied pathway that leads to specific career and college options for students after high school graduation.

Further resources to understand this work in Oregon and nationally in support of grade 11-14 math pathways include Branching Out: Designing High School Math Pathways for Equity, Dana Center Launch Years Report, and CBMS High School to College Mathematics Pathways forum, at which our state was invited to share course pathway work described in this document. The time is right for Oregon educators to think innovatively and lead the nation in reimagining high school math.

- Additional information and examples can be found in the Oregon Math Project Practice Brief: Tracking

Opportunity to Deepen Equity, Innovation, Care, and Connection

High school pathways described in the 2+1 course model are an innovation that high school faculty can use to create equitable opportunities that connect mathematics to student goals and interests as educators plan pathway options to create math pathways options for students.

This includes leaning into new and innovative ways to incorporate instructional best practices, such as NCTM's Principles to Action, to create student-centered instructional experiences that should be a focus as we look to implement the standards this next decade. Resources and courses created today can lay a strong foundation for high school experiences in the future.

- Additional information and examples can be found in the Oregon Math Project Practice Brief: Classroom Discourse

Continued Focus on Math Practices and Modeling

Reimagining math pathway options that meet the needs of more students will require a focus on content rather than courses that students need for success. It will also require ensuring the Standards of Mathematical Practice are attended to as we accelerate learning. This includes finding new ways to infuse applications through mathematical modeling that supports the natural interconnectedness of math to other disciplines and to community-based problems. Modeling is an opportunity to see mathematics as relevant to students' lives and the questions confronting our world. Now more than ever, we need to find ways to increase student interest and enthusiasm in math by providing more opportunities to engage in interactive, student-centered problems that are based in applied

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

mathematics. Examples of mathematical modeling lessons can be found below, with additional examples added over time to the Oregon Open Learning Mathematics Group.

- American Statistical Association - STatistics Education Web (STEW)
- Council for Economic Education - EconEdLink Lessons
- Modeling with Mathematics through Three-Act Tasks

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout core content in this document as indicated by a star symbol (★).

- Additional information and examples can be found in the Oregon Math Project Practice Brief: Mathematical Modeling

Connections to National Conversations

The ideas described in this document have been informed and aligned to the call for re-humanizing school mathematics. The National Council of Teachers of Mathematics (NCTM) recognizes the need to focus on high school mathematics in *Catalyzing Change in High School Mathematics: Initiating Critical Conversations,* which outlines the need for change as well as recommendations to focus math content that lead to specialized pathways in high school.

Additional national conversations are occurring within professional math organizations that recognize the need to create more options for students that are contextual and connected to a student's interest. *The Common Vision Project* is a joint effort of five national math organizations led by the Mathematical Association of America (MAA) calling for a shift in college math instruction to introduce contemporary topics and applications and employ a broad range of examples and applications to motivate students and illustrate how math is used.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

SECTION TWO: 2021 Adopted Math Practice Standards

2A: Standards for Mathematical Practices

MP.1: Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.2: Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents— and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies.

Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

MP.4: Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.5: Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.6: Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time, they reach high school they have learned to examine claims and make explicit use of definitions.

MP.7: Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP.8: Look for and express regularity in repeated reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x2 + x + 1)$, and $(x - 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

2B: Mathematical Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

SECTION THREE: 2021 Adopted Math Content Standards

3A: Grade K Math Standards (2021)

Algebraic Reasoning: Operations (K.OA)

K.OA.A *Understand addition and subtraction.*

K.OA.A.1 Represent addition as putting together and adding to and subtraction as taking apart and taking from using objects, drawings, physical expressions, numbers or equations.

K.OA.A.2 Add and subtract within 10. Model authentic contexts and solve problems that use addition and subtraction within 10.

K.OA.A.3 Using objects or drawings, and equations, decompose numbers less than or equal to 10 into pairs in more than one way.

K.OA.A.4 By using objects, drawings, or equations, find the unknown number that makes 10 when added to a given number from 1 - 9.

K.OA.A.5 Fluently add and subtract within 5 with accurate, efficient, and flexible strategies.

Numeric Reasoning: Counting and Cardinality (K.NCC)

K.NCC.A *Know number names and the count sequence.*

K.NCC.A.1 Orally count to 100 by ones and by tens in sequential order.

K.NCC.A.2 Count forward beginning from a given number within 100 of a known sequence.

K.NCC.A.3 Identify number names, write numbers, and the count sequence from 0-20. Represent a number of objects with a written number 0-20.

K.NCC.B *Count to tell the number of objects.*

K.NCC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

K.NCC.B.5 Count to answer “how many?” questions using up to 20 objects arranged in a variety of configurations or as 10 objects in a scattered configuration. Given a number from 1-20, count out that many objects.

K.NCC.C *Compare numbers.*

K.NCC.C.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.

K.NCC.C.7 Compare two numbers between 1 and 10 presented as written numerals.

Numeric Reasoning: Base Ten Arithmetic (K.NBT)

K.NBT.A *Work with numbers 11-19 to gain foundations for place value.*

K.NBT.A.1 Compose and decompose from 11 to 19 into groups of ten ones and some further ones using objects, drawings, or equations.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Geometric Reasoning and Measurement (K.GM)

K.GM.A *Identify and describe shapes.*

K.GM.A.1 Describe objects in the environment using names of shapes and describe the relative positions of these objects in their environment.

K.GM.A.2 Correctly name common two-dimensional and three-dimensional geometric shapes regardless of their orientations or overall size.

K.GM.A.3 Identify shapes as two-dimensional or three-dimensional.

K.GM.B *Analyze, compare, create, and compose shapes.*

K.GM.B.4 Analyze and compare two and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts and attributes.

K.GM.B.5 Represent shapes in the world by building shapes from components and drawing shapes.

K.GM.B.6 Compose common shapes to form larger shapes.

K.GM.C *Describe and compare measurable attributes.*

K.GM.C.7 Describe several measurable attributes of a single object using measurable terms, such as length or weight.

K.GM.C.8 Directly compare two objects with a measurable attribute in common, and describe which object has “more” or “less” of the attribute.

Data Reasoning (K.DR)

K.DR.A *Pose investigative questions and collect/consider data.*

K.DR.A.1 Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by sorting and counting.

K.DR.B *Analyze, represent, and interpret data.*

K.DR.B.2 Analyze data sets by counting the number of objects in each category and interpret results by classifying and sorting objects by count.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3B: Grade 1 Math Standards (2021)

Algebraic Reasoning: Operations (1.OA)

1.OA.A *Represent and solve problems involving addition and subtraction.*

1.OA.A.1 Use addition and subtraction within 20 to solve and represent problems in authentic contexts involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.

1.OA.A.2 Solve problems that call for addition of three whole numbers whose sum is less than or equal to 20 using objects, drawings or equations.

1.OA.B *Understand and apply properties of operations and the relationship between addition and subtraction.*

1.OA.B.3 Apply properties of operations as strategies to add and subtract.

1.OA.B.4 Understand subtraction as an unknown-addend problem.

1.OA.C *Add and subtract within 20.*

1.OA.C.5 Relate counting to addition and subtraction.

1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10 with accurate, efficient, and flexible strategies.

1.OA.D *Work with addition and subtraction equations.*

1.OA.D.7 Use the meaning of the equal sign to determine whether equations involving addition and subtraction are true or false.

1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

Numeric Reasoning: Base Ten Arithmetic (1.NBT)

1.NBT.A *Extend the counting sequence.*

1.NBT.A.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

1.NBT.B *Understand place value.*

1.NBT.B.2 Understand 10 as a bundle of ten ones and that the two digits of a two-digit number represent amounts of tens and ones.

1.NBT.B.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

1.NBT.C *Use place value understanding and properties of operations to add and subtract.*

1.NBT.C.4 Add within 100 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain why sometimes it is necessary to compose a ten.

1.NBT.C.5 Without having to count, mentally find 10 more or 10 less than a given two-digit number and explain the reasoning used.

1.NBT.C.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy and model used to a written method and explain the reasoning used.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Geometric Reasoning and Measurement (1.GM)

1.GM.A *Reason with shapes and their attributes.*

1.GM.A.1 Distinguish between defining attributes versus non-defining attributes for a wide variety of shapes. Build and draw shapes to possess defining attributes.

1.GM.A.2 Compose common two-dimensional shapes or three-dimensional shapes to create a composite shape, and create additional new shapes from composite shapes.

1.GM.A.3 Partition circles and rectangles into two and four equal shares. Describe the equal shares and understand that partitioning into more equal shares creates smaller shares.

1.GM.B *Describe and compare measurable attributes.*

1.GM.B.4 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

1.GM.B.5 Express the length of an object as a whole number of non-standard length units, by laying multiple copies of a shorter object (the length unit) end to end.

1.GM.C *Tell and write time.*

1.GM.C.6 Tell and write time in hours and half-hours using analog and digital clocks.

Data Reasoning (1.DR)

1.DR.A *Pose investigative questions and collect/consider data.*

1.DR.A.1 Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by representing data visually.

1.DR.B *Analyze, represent, and interpret data.*

1.DR.B.2 Analyze data sets with up to three categories by representing data visually, such as with graphs and charts, and interpret information presented to answer investigative questions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3C: Grade 2 Math Standards (2021)

Algebraic Reasoning: Operations (2.OA)

2.OA.A *Represent and solve problems involving addition and subtraction.*

2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step problems in authentic contexts by using drawings and equations with a symbol for the unknown.

2.OA.B *Add and subtract within 20.*

2.OA.B.2 Fluently add and subtract within 20 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

2.OA.C *Work with equal groups of objects to gain foundations for multiplication.*

2.OA.C.3 Determine whether a group up to 20 objects has an odd or even number by pairing objects or counting them by 2s; record using drawings and equations including expressing an even number as a sum of two equal addends.

2.OA.C.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Numeric Reasoning: Base Ten Arithmetic (2.NBT)

2.NBT.A *Understand place value.*

2.NBT.A.1 Understand 100 as a bundle of ten tens and that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

2.NBT.A.2 Count within 1000; skip-count by 5's, 10's, and 100's.

2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.A.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

2.NBT.B *Use place value understanding and properties of operations to add and subtract.*

2.NBT.B.5 Fluently add & subtract within 100 using accurate, efficient, & flexible strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.B.6 Add up to four two-digit numbers using strategies based on place value and properties of operations and describe how two different strategies result in the same sum.

2.NBT.B.7 Add and subtract within 1000 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain why sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.B.8 Without having to count, mentally find 10 more or 10 less and 100 more or 100 less than a given three-digit number.

2.NBT.B.9 Explain why strategies to add and subtract work using properties of operations and the relationship between addition and subtraction.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Geometric Reasoning and Measurement (2.GM)

2.GM.A *Reason with shapes and their attributes.*

2.GM.A.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.

2.GM.A.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

2.GM.A.3 Partition circles and rectangles into two, three, or four equal parts. Recognize that equal parts of identical wholes need not have the same shape.

2.GM.B *Measure and estimate lengths in standard units.*

2.GM.B.4 Measure the length of an object by selecting and using appropriate measurement tools.

2.GM.B.5 Measure the length of an object using two different length units and describe how the measurements relate to the size of the unit chosen.

2.GM.B.6 Estimate lengths using units of inches, feet, yards, centimeters, and meters.

2.GM.B.7 Measure two objects and determine the difference in their lengths in terms of a standard length unit.

2.GM.C *Relate addition and subtraction to length.*

2.GM.C.8 Use addition and subtraction within 100 to solve problems in authentic contexts involving lengths that are given in the same units.

2.GM.C.9 Represent whole number lengths on a number line diagram; use number lines to find sums and differences within 100.

2.GM.D *Work with time and money.*

2.GM.D.10 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

2.GM.D.11 Solve problems in authentic contexts involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and c (cents) symbols appropriately.

Data Reasoning (2.DR)

2.DR.A *Pose investigative questions and collect/consider data.*

2.DR.A.1 Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by using measurements with whole-number units.

2.DR.B *Analyze, represent, and interpret data.*

2.DR.B.2 Analyze data with a single-unit scale and interpret information presented to answer investigative questions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3D: Grade 3 Math Standards (2021)

Algebraic Reasoning: Operations (3.OA)

3.OA.A *Represent and solve problems involving multiplication and division.*

3.OA.A.1 Represent and interpret multiplication of two factors as repeated addition of equal groups.

3.OA.A.2 Represent and interpret whole-number quotients as dividing an amount into equal sized groups.

3.OA.A.3 Use multiplication and division within 100 to solve problems in authentic contexts involving equal groups, arrays, and/or measurement quantities.

3.OA.A.4 Determine the unknown number in a multiplication or division equation relating three whole numbers by applying the understanding of the inverse relationship of multiplication and division.

3.OA.B *Understand properties of multiplication and the relationship between multiplication and division.*

3.OA.B.5 Apply properties of operations as strategies to multiply and divide.

3.OA.B.6 Understand division as an unknown-factor in a multiplication problem.

3.OA.C *Multiply and divide within 100.*

3.OA.C.7 Fluently multiply and divide within 100 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

3.OA.D *Solve problems involving the four operations, and identify and explain patterns in arithmetic.*

3.OA.D.8 Solve two-step problems in authentic contexts that use addition, subtraction, multiplication, and division in equations with a letter standing for the unknown quantity.

3.OA.D.9 Identify and explain arithmetic patterns using properties of operations, including patterns in the addition table or multiplication table.

Numeric Reasoning: Base Ten Arithmetic (3.NBT)

3.NBT.A *Use place value understanding and properties of operations to perform multi-digit arithmetic.*

3.NBT.A.1 Use place value understanding to round whole numbers within 1000 to the nearest 10 or 100.

3.NBT.A.2 Fluently add and subtract within 1000 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

3.NBT.A.3 Find the product of one-digit whole numbers by multiples of 10 in the range 10-90, such as 9 x 80. Students use a range of strategies and algorithms based on place value and properties of operations.

Numeric Reasoning: Fractions (3.NF)

3.NF.A *Develop understanding of fractions as numbers.*

3.NF.A.1 Understand the concept of a unit fraction and explain how multiple copies of a unit fraction form a non-unit fraction.

3.NF.A.2 Understand a fraction as a number on the number line; Represent fractions on a number line diagram.

3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Geometric Reasoning and Measurement (3.GM)

3.GM.A *Reason with shapes and their attributes.*

3.GM.A.1 Understand that shapes in different categories may share attributes and that shared attributes can define a larger category.

3.GM.A.2 Partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.

3.GM.B *Solve problems involving measurement and estimation.*

3.GM.B.3 Tell, write, and measure time to the nearest minute. Solve problems in authentic contexts that involve addition and subtraction of time intervals in minutes.

3.GM.B.4 Measure, estimate and solve problems in authentic contexts that involve liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).

3.GM.C *Geometric measurement: understand concepts of area and relate area to multiplication and to addition.*

3.GM.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement presented in authentic contexts by tiling and counting unit squares.

3.GM.C.6 Measure areas by counting standard and non-standard unit squares.

3.GM.C.7 Relate area to multiplication and addition. Use relevant representations to solve problems in authentic contexts.

3.GM.D *Geometric measurement: recognize perimeter.*

3.GM.D.8 Solve problems involving authentic contexts for perimeters of polygons.

Data Reasoning (3.DR)

3.DR.A *Pose investigative questions and collect/consider data.*

3.DR.A.1 Generate questions to investigate situations within the classroom, school or community. Collect or consider measurement data that can naturally answer questions by using information presented in a scaled picture and/or bar graph.

3.DR.B *Analyze, represent, and interpret data.*

3.DR.B.2 Analyze measurement data with a scaled picture graph or a scaled bar graph to represent a data set with several categories. Interpret information presented to answer investigative questions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3E: Grade 4 Math Standards (2021)

Algebraic Reasoning: Operations (4.OA)

4.OA.A *Use the four operations with whole numbers to solve problems.*

4.OA.A.1 Interpret a multiplication equation as comparing quantities. Represent verbal statements of multiplicative comparisons as equations.

4.OA.A.2 Multiply or divide to solve problems in authentic contexts involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison.

4.OA.A.3 Solve multistep problems in authentic contexts using whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.

4.OA.B *Gain familiarity with factors and multiples.*

4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Determine whether a given whole number in the range of 1-100 is a multiple of a given one-digit number, and whether it is prime or composite.

4.OA.C *Generate and analyze patterns.*

4.OA.C.5 Analyze a number, visual, or contextual pattern that follows a given rule.

Numeric Reasoning: Base Ten Arithmetic (4.NBT)

4.NBT.A *Generalize place value understanding for multi-digit whole numbers.*

4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Use understandings of place value within these forms to compare two multi-digit numbers using >, =, and < symbols.

4.NBT.A.3 Use place value understanding to round multi-digit whole numbers to any place.

4.NBT.B *Use place value understanding and properties of operations to perform multi-digit arithmetic.*

4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

4.NBT.B.5 Use representations and strategies to multiply a whole number of up to four digits by a one-digit number, and a two-digit number by a two-digit number using strategies based on place value and the properties of operations.

4.NBT.B.6 Use representations and strategies to find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.

Numeric Reasoning: Fractions (4.NF)

4.NF.A *Extend understanding of fraction equivalence and ordering.*

4.NF.A.1 Use visual fraction representations to recognize, generate, and explain relationships between equivalent fractions.

4.NF.A.2 Compare two fractions with different numerators and/or different denominators, record the results with the symbols >, =, or <, and justify the conclusions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4.NF.B *Build fractions from unit fractions.*

4.NF.B.3 Understand a fraction (a/b) as the sum (a) of fractions of the same denominator (1/b). Solve problems in authentic contexts involving addition and subtraction of fractions referring to the same whole and having like denominators.

4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. Represent and solve problems in authentic contexts involving multiplication of a fraction by a whole number.

4.NF.C *Understand decimal notation for fractions, and compare decimal fractions.*

4.NF.C.5 Demonstrate and explain the concept of equivalent fractions with denominators of 10 and 100, using concrete materials and visual models. Add two fractions with denominators of 10 and 100.

4.NF.C.6 Use and interpret decimal notation for fractions with denominators 10 or 100.

4.NF.C.7 Use decimal notation for fractions with denominators 10 or 100. Compare two decimals to hundredths place by reasoning about their size, and record the comparison using the symbols >, =, or <.

Geometric Reasoning and Measurement (4.GM)

4.GM.A *Draw and identify lines and angles, and classify shapes by properties of their lines and angles.*

4.GM.A.1 Explore, investigate, and draw points, lines, line segments, rays, angles, and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.GM.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size.

4.GM.A.3 Recognize and draw a line of symmetry for a two dimensional figure.

4.GM.B *Solve problems involving measurement and conversion of measurements.*

4.GM.B.4 Know relative sizes of measurement units and express measurements in a larger unit in terms of a smaller unit.

4.GM.B.5 Apply knowledge of the four operations and relative size of measurement units to solve problems in authentic contexts that include familiar fractions or decimals.

4.GM.B.6 Apply the area and perimeter formulas for rectangles in authentic contexts and mathematical problems.

4.GM.C *Geometric measurement: understand concepts of angle and measure angles.*

4.GM.C.7 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. Understand and apply concepts of angle measurement.

4.GM.C.8 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.GM.C.9 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.

Data Reasoning (4.DR)

4.DR.A *Pose investigative questions and collect/consider data.*

4.DR.A.1 Generate questions to investigate situations within the classroom, school or community. Determine strategies for collecting or considering data involving addition and subtraction of fractions that can naturally answer questions by using information presented in line plots.

4.DR.B *Analyze, represent, and interpret data.*

4.DR.B.2 Analyze line plots to display a distribution of numerical measurement data, which include displays of data sets of fractional measurements with the same denominator. Interpret information presented to answer investigative questions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3F: Grade 5 Math Standards (2021)

Algebraic Reasoning: Operations (5.OA)

5.OA.A *Write and interpret numerical expressions.*

5.OA.A.1 Write and evaluate numerical expressions that include parentheses.

5.OA.A.2 Write expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

5.OA.B *Analyze patterns and relationships.*

5.OA.B.3 Generate two numerical patterns using two given rules. Identify and analyze relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph them on a coordinate plane.

Numeric Reasoning: Base Ten Arithmetic (5.NBT)

5.NBT.A *Understand the place value system.*

5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.A.2 Use whole number exponents to denote powers of 10 and explain the patterns in placement of digits that occur when multiplying and/or dividing whole numbers and decimals by powers of 10.

5.NBT.A.3 Read, write, and compare decimals to thousandths.

5.NBT.A.4 Use place value understanding to round decimals to any place.

5.NBT.B *Perform operations with multi-digit whole numbers and with decimals to hundredths.*

5.NBT.B.5 Fluently multiply multi-digit whole numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

5.NBT.B.6 Use a variety of representations and strategies to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.

5.NBT.B.7 Use a variety of representations and strategies to add, subtract, multiply, and divide decimals to hundredths. Relate the strategy to a written method and explain the reasoning used.

Numeric Reasoning: Fractions (5.NF)

5.NF.A *Use equivalent fractions as a strategy to add and subtract fractions.*

5.NF.A.1 Add and subtract fractions with unlike denominators, including common fractions larger than one and mixed numbers.

5.NF.A.2 Solve problems in authentic contexts involving addition and subtraction of fractions with unlike denominators, including common fractions larger than one and mixed numbers.

5.NF.B *Apply and extend previous understandings of multiplication and division.*

5.NF.B.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve problems in authentic contexts involving division of whole numbers that result in answers that are common fractions or mixed numbers.

5.NF.B.4 Apply and extend previous understanding and strategies of multiplication to multiply a fraction or whole number by a fraction. Multiply fractional side lengths to find areas of rectangles, and represent fractional products as rectangular areas.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5.NF.B.5 Apply and extend previous understandings of multiplication and division to represent and calculate multiplication and division of fractions. Interpret multiplication as scaling (resizing) by comparing the size of products of two factors.

5.NF.B.6 Solve problems in authentic contexts involving multiplication of common fractions and mixed numbers.

5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions, including solving problems in authentic contexts.

Geometric Reasoning and Measurement (5.GM)

5.GM.A *Graph points on the coordinate plane to solve real-world and mathematical problems.*

5.GM.A.1 Graph and name coordinate points in the first quadrant using the standard (x, y) notation. Understand the coordinate points values represent the distance traveled along the horizontal x-axis and vertical y-axis.

5.GM.A.2 Represent authentic contexts and mathematical problems by graphing points in the first quadrant of the coordinate plane. Interpret the meaning of the coordinate values based on the context of a given situation.

5.GM.B *Classify two-dimensional figures into categories based on their properties.*

5.GM.B.3 Classify two-dimensional figures within a hierarchy based on their geometrical properties, and explain the relationship across and within different categories of these figures.

5.GM.C *Convert like measurement units within a given measurement system.*

5.GM.C.4 Convert between different-sized standard measurement units within a given measurement system. Use these conversions in solving multi-step problems in authentic contexts.

5.GM.D *Geometric measurement: understand concepts of volume.*

5.GM.D.5 Recognize that volume is a measurable attribute of solid figures.

5.GM.D.6 Measure the volume of a rectangular prism by counting unit cubes using standard and nonstandard units.

5.GM.D.7 Relate volume of rectangular prisms to the operations of multiplication and addition. Solve problems in authentic contexts involving volume using a variety of strategies.

Data Reasoning (5.DR)

5.DR.A *Pose investigative questions and collect/consider data.*

5.DR.A.1 Generate questions to investigate situations within the classroom, school or community. Determine strategies for collecting or considering data involving operations with fractions for this grade that can naturally answer questions by using information presented in line plots.

5.DR.B *Analyze, represent, and interpret data.*

5.DR.B.2 Analyze graphical representations and describe the distribution of the numerical data through line plots or categorical data through bar graphs. Interpret information presented to answer investigative questions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3G: Grade 6 Math Standards (2021)

Algebraic Reasoning: Expressions and Equations (6.AEE)

6.AEE.A *Apply and extend previous understandings of arithmetic to algebraic expressions.*

6.AEE.A.1 Write and evaluate numerical expressions involving whole-number bases and exponents.

6.AEE.A.2 Write, read, and evaluate expressions in which letters stand for numbers. Apply knowledge of common mathematical terms to move between the verbal and mathematical forms of an expression including expressions that arise from authentic contexts.

6.AEE.A.3 Apply the properties of operations to generate equivalent expressions and to determine when two expressions are equivalent.

6.AEE.B *Reason about and solve one-variable equations and inequalities.*

6.AEE.B.4 Understand solving an equation or inequality as a process of answering which values from a specified set, if any, make the equation or inequality true. Use substitution to determine which number(s) in a given set make an equation or inequality true.

6.AEE.B.5 Use variables to represent numbers and write expressions when solving problems in authentic contexts.

6.AEE.B.6 Write and solve equations of the form $x + p = q$ and $px = q$ in problems that arise from authentic contexts for cases in which p, q and x are all nonnegative rational numbers.

6.AEE.B.7 Write inequalities of the form $x > c$ and $x < c$ to represent constraints or conditions to solve problems in authentic contexts. Describe and graph on a number line solutions of inequalities of the form $x > c$ and $x < c$.

6.AEE.C *Represent and analyze quantitative relationships between dependent and independent variables.*

6.AEE.C.8 Use variables to represent and analyze two quantities to solve problems in authentic contexts. Including those that change in relationship to one another; write an equation to express one quantity in terms of the other quantity.

Proportional Reasoning: Ratios and Proportions (6.RP)

6.RP.A *Understand ratio concepts and use ratio reasoning to solve problems.*

6.RP.A.1 Understand the concept of a ratio in authentic contexts, and use ratio language to describe a ratio relationship between two quantities.

6.RP.A.2 Understand the concept of a unit rate in authentic contexts and use rate language in the context of a ratio relationship.

6.RP.A.3 Use ratio and rate reasoning to solve problems in authentic contexts that use equivalent ratios, unit rates, percents, and/or measurement units.

Numeric Reasoning: Number Systems (6.NS)

6.NS.A *Apply and extend previous understandings of multiplication and division to divide fractions by fractions.*

6.NS.A.1 Represent, interpret, and compute quotients of fractions to solve problems in authentic contexts involving division of fractions by fractions.

6.NS.B *Compute fluently with multi-digit numbers and find common factors and multiples.*

6.NS.B.2 Fluently divide multi-digit numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

6.NS.B.3 Fluently add, subtract, multiply, and divide positive rational numbers using accurate, efficient, and flexible strategies and algorithms.

6.NS.B.4 Determine greatest common factors and least common multiples using a variety of strategies. Apply the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in authentic contexts, explaining the meaning of zero in each situation.

6.NS.C.6 Represent a rational number as a point on the number line. Extend number line diagrams and coordinate axes to represent points on the line and in the coordinate plane with negative number coordinates.

6.NS.C.7 Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. Write, interpret, and explain statements of order for rational numbers and absolute value in authentic applications.

6.NS.C.8 Graph points in all four quadrants of the coordinate plane to solve problems in authentic contexts. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Geometric Reasoning and Measurement (6.GM)

6.GM.A Solve real-world and mathematical problems involving area, surface area, and volume.

6.GM.A.1 Find the area of triangles, quadrilaterals, and other polygons by composing into rectangles or decomposing into triangles and other shapes. Apply these techniques to solve problems in authentic contexts.

6.GM.A.2 Find the volume of a right rectangular prism with fractional edge lengths by filling it with unit cubes of appropriate unit fraction edge lengths. Connect and apply to the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths to solve problems in authentic contexts.

6.GM.A.3 Draw polygons in the four quadrant coordinate plane given coordinates for the vertices and find the length of a side. Apply these techniques to solve problems in authentic contexts.

6.GM.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures, including those from authentic contexts.

Data Reasoning (6.DR)

6.DR.A Formulate Statistical Investigative Questions.

6.DR.A.1 Formulate and recognize statistical investigative questions as those that anticipate changes in descriptive data related to the question and account for it in the answers.

6.DR.B Collect and Consider Data.

6.DR.B.2 Collect and record data with technology to identify and describe the characteristics of numerical data sets using quantitative measures of center and variability.

6.DR.C Analyze, summarize, and describe data.

6.DR.C.3 Analyze data representations and describe measures of center and variability of quantitative data using appropriate displays.

6.DR.D Interpret data and answer investigative questions.

6.DR.D.4 Interpret quantitative measures of center to describe differences between groups from data collected to answer investigative questions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3H: Grade 7 Math Standards (2021)

Algebraic Reasoning: Expressions and Equations (7.AEE)

7.AEE.A *Use properties of operations to generate equivalent expressions.*

7.AEE.A.1 Identify and write equivalent expressions with rational numbers by applying associative, commutative, and distributive properties.

7.AEE.A.2 Understand that rewriting an expression in different forms in a contextual problem can show how quantities are related.

7.AEE.B *Solve mathematical problems in authentic contexts using numerical and algebraic expressions and equations.*

7.AEE.B.3 Write and solve problems in authentic contexts using expressions and equations with positive and negative rational numbers in any form. Contexts can be limited to those that can be solved with one or two-step linear equations.

7.AEE.B.4 Use variables to represent quantities and construct one- and two-step linear inequalities with positive rational numbers to solve authentic problems by reasoning about the quantities.

Proportional Reasoning: Ratios and Proportions (7.RP)

7.RP.A *Analyze proportional relationships and use them to solve mathematical problems in authentic contexts.*

7.RP.A.1 Solve problems in authentic contexts involving unit rates associated with ratios of fractions.

7.RP.A.2 Recognize and represent proportional relationships between quantities in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Identify the constant of proportionality (unit rate) within various representations.

7.RP.A.3 Use proportional relationships to solve ratio and percent problems in authentic contexts.

7.RP.B *Investigate chance processes and develop, use, and evaluate probability models.*

7.RP.B.4 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Represent probabilities as fractions, decimals, and percents.

7.RP.B.5 Use experimental data and theoretical probability to make predictions. Understand the probability predictions may not be exact.

7.RP.B.6 Develop a probability model and use it to find probabilities of events. Compare theoretical and experimental probabilities and explain possible sources of discrepancy if any exists.

7.RP.B.7 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Numeric Reasoning: Number Systems (7.NS)

7.NS.A *Apply and extend previous understandings of operations with fractions.*

7.NS.A.1 Apply and extend previous understandings of addition, subtraction and absolute value to add and subtract rational numbers in authentic contexts. Understand subtraction as adding the additive inverse, $p – q = p + (–q)$.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. Interpret operations of rational numbers solving problems in authentic contexts.

7.NS.A.3 Understand that equivalent rational numbers can be written as fractions, decimals and percents.

Geometric Reasoning and Measurement (7.GM)

7.GM.A *Draw, construct, and describe geometrical figures and describe the relationships between them.*

7.GM.A.1 Solve problems involving scale drawings of geometric figures. Reproduce a scale drawing at a different scale and compute actual lengths and areas from a scale drawing.

7.GM.A.2 Draw triangles from three measures of angles or sides. Understand the possible side lengths and angle measures that determine a unique triangle, more than one triangle, or no triangle.

7.GM.B *Solve mathematical problems in authentic contexts involving angle measure, area, surface area, and volume.*

7.GM.B.3 Understand the relationship between area and circumference of circles. Choose and use the appropriate formula to solve problems with radius, diameter, circumference and area of circles.

7.GM.B.4 Apply facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to determine an unknown angle in a figure.

7.GM.B.5 Solve problems in authentic contexts involving two- and three-dimensional figures. Given formulas, calculate area, volume and surface area.

Data Reasoning (7.DR)

7.DR.A *Formulate Statistical Investigative Questions.*

7.DR.A.1 Formulate summary, comparative investigative questions to gain information about a population and that a sample is valid only if the sample is representative of that population.

7.DR.B *Collect and Consider Data.*

7.DR.B.2 Collect or consider data from a random sample to compare and draw inferences about a population with an unknown characteristic of interest.

7.DR.C *Analyze, summarize, and describe data.*

7.DR.C.3 Analyze two data distributions visually to compare multiple measures of center and variability.

7.DR.D *Interpret data and answer investigative questions.*

7.DR.D.4 Interpret measures of center and measures of variability for numerical data from random samples to compare between two populations, and to answer investigative questions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3I: Grade 8 Math Standards (2021)

Algebraic Reasoning: Expressions and Equations (8.AEE)

8.AEE.A *Expressions and Equations Work with radicals and integer exponents.*

8.AEE.A.1 Apply the properties of integer exponents using powers of 10 to generate equivalent numerical expressions.

8.AEE.A.2 Represent solutions to equations using square root and cube root symbols.

8.AEE.A.3 Estimate very large or very small quantities using scientific notation with a single digit times an integer power of ten.

8.AEE.A.4 Perform operations with numbers expressed in scientific notation.

8.AEE.B *Understand the connections between proportional relationships, lines, and linear equations.*

8.AEE.B.5 Graph proportional relationships in authentic contexts. Interpret the unit rate as the slope of the graph, and compare two different proportional relationships represented in different ways.

8.AEE.B.6 Write the equation for a line in slope intercept form y = mx + b, where m and b are rational numbers, and explain in context why the slope m is the same between any two distinct points.

8.AEE.C *Analyze and solve linear equations and pairs of simultaneous linear equations.*

8.AEE.C.7 Solve linear equations with one variable including equations with rational number coefficients, with the variable on both sides, or whose solutions require using the distributive property and/or combining like terms.

8.AEE.C.8 Find, analyze, and interpret solutions to pairs of simultaneous linear equations using graphs or tables.

Algebraic Reasoning: Functions (8.AFN)

8.AFN.A *Define, evaluate, and compare functions.*

8.AFN.A.1 Understand in authentic contexts, that the graph of a function is the set of ordered pairs consisting of an input and a corresponding output.

8.AFN.A.2 Compare the properties of two functions represented algebraically, graphically, numerically in tables, or verbally by description.

8.AFN.A.3 Understand and identify linear functions, whose graph is a straight line, and identify examples of functions that are not linear.

8.AFN.B *Use functions to model relationships between quantities.*

8.AFN.B.4 Construct a function to model a linear relationship in authentic contexts between two quantities.

8.AFN.B.5 Describe qualitatively the functional relationship between two quantities in authentic contexts by analyzing a graph.

Numeric Reasoning: Number Systems (8.NS)

8.NS.A *Know that there are numbers that are not rational, and approximate them by rational numbers.*

8.NS.A.1 Know that real numbers that are not rational are called irrational.

8.NS.A.2 Use rational approximations of irrational numbers to compare size and locate on a number line.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Geometric Reasoning and Measurement (8.GM)

8.GM.A *Understand congruence and similarity using physical models, transparencies, or geometry software.*

8.GM.A.1 Verify experimentally the properties of rotations, reflections, and translations.

8.GM.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.

8.GM.A.3 Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.

8.GM.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and/or dilations.

8.GM.A.5 Use informal arguments to establish facts about interior and exterior angles of triangles and angles formed by parallel lines cut with a transversal.

8.GM.B *Understand and apply the Pythagorean Theorem.*

8.GM.B.6 Distinguish between applications of the Pythagorean Theorem and its Converse in authentic contexts.

8.GM.B.7 Apply the Pythagorean Theorem in authentic contexts to determine unknown side lengths in right triangles.

8.GM.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

8.GM.C *Solve mathematical problems in authentic contexts involving volume of cylinders, cones, and spheres.*

8.GM.C.9 Choose and use the appropriate formula for the volume of cones, cylinders, and spheres to solve problems in authentic contexts.

Data Reasoning (8.DR)

8.DR.A *Formulate Statistical Investigative Questions.*

8.DR.A.1 Formulate statistical investigative questions to articulate research topics and uncover patterns of association seen in bivariate categorical data.

8.DR.B *Collect and Consider Data.*

8.DR.B.2 Collect or consider data using surveys and measurements to capture patterns of association, and critically analyze data collection methods.

8.DR.C *Analyze, summarize, and describe data.*

8.DR.C.3 Analyze patterns of association between two quantitative or categorical variables and reason about distributions to compare groups.

8.DR.D *Interpret data and answer investigative questions.*

8.DR.D.4 Interpret scatter plots for bivariate quantitative data to investigate patterns of association between two quantities to answer investigative questions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3J: High School Algebra Standards (2021)

Algebraic Reasoning: Expressions and Equations (HS.AEE)

HS.AEE.A *Use algebraic reasoning to rewrite expressions in equivalent forms.*

HS.AEE.A.1 Interpret an expression which models a quantity by viewing one or more of its parts as a single entity. Reason about how changes in parts of the expression impact the whole, and vice versa.

HS.AEE.A.2 Create and recognize an equivalent form of an expression to understand the quantity represented in an authentic context.

HS.AEE.A.3 Rearrange formulas and equations to highlight a specific quantity.

HS.AEE.B *Use algebraic reasoning to find solutions to an equation, inequality, and systems of equations or inequalities.*

HS.AEE.B.4 Define variables and create equations with two or more variables to represent relationships between quantities in order to solve problems in authentic contexts.

HS.AEE.B.5 Define variables and create inequalities with one or more variables and use them to solve problems in authentic contexts.

HS.AEE.B.6 Solve systems of linear equations and systems of linear inequalities in authentic contexts through reasoning, algebraic means, or strategically using technology.

HS.AEE.C *Analyze the structure of an equation or inequality to determine an efficient strategy to find and justify a solution.*

HS.AEE.C.7 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities; interpret solutions as viable or nonviable options in authentic contexts.

HS.AEE.C.8 Construct a viable argument to justify a method for solving equations or inequalities.

HS.AEE.D *Make predictions in different applications using expressions, equations, and inequalities to analyze authentic contexts.*

HS.AEE.D.9 Understand that the solution to an equation in two variables is a set of points in the coordinate plane that form a curve, which could be a line.

HS.AEE.D.10 Recognize and explain why the point(s) of intersection of the graphs of f(x) and g(x) are solutions to the equation f(x)=g(x). Interpret the meaning of the coordinates of these points in authentic contexts.

HS.AEE.D.11 Graph and explain why the points in a half plane are solutions to a linear inequality and the solutions to a system of inequalities are the points in the intersection of corresponding half planes. Interpret the meaning of the coordinates of these points in authentic contexts.

Algebraic Reasoning: Functions (HS.AFN)

HS.AFN.A *Describe functions by using both symbolic and graphical representations.*

HS.AFN.A.1 Understand a function as a rule that assigns a unique output for every input and that functions model situations where one quantity determines another.

HS.AFN.A.2 Use function notation and interpret statements that use function notation in terms of the context and the relationship it describes.

HS.AFN.A.3 Calculate and interpret the average rate of change of a function over a specified interval.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

HS.AFN.B *Compare and relate functions using common attributes.*

HS.AFN.B.4 Compare properties of two functions using multiple representations. Distinguish functions as members of the same family using common attributes.

HS.AFN.B.5 Relate the domain of a function to its graph and to its context.

HS.AFN.C *Represent functions graphically and interpret key features in terms of the equivalent symbolic representation.*

HS.AFN.C.6 Interpret key features of functions, from multiple representations, and conversely predict features of functions from knowledge of context.

HS.AFN.C.7 Graph functions using technology to show key features.

HS.AFN.D *Model a wide variety of authentic situations using functions through the process of making and changing assumptions, assigning variables, and finding solutions to contextual problems.*

HS.AFN.D.8 Model situations involving arithmetic patterns. Use a variety of representations such as pictures, graphs, or an explicit formula to describe the pattern.

HS.AFN.D.9 Identify and interpret the effect on the graph of a function when the equation has been transformed.

HS.AFN.D.10 Explain why a situation can be modeled with a linear function, an exponential function, or neither. In a given model, explain the meaning of coefficients and features of functions used, such as slope for a linear model.

Numeric Reasoning: Number and Quantity (HS.NQ)

HS.NQ.A *Understand and apply the real number system.*

HS.NQ.A.1 Use reasoning to establish properties of positive integer exponents. Extend the definition of exponentiation to include negative and rational exponents so as to be consistent with these properties. Utilize exponentiation to model authentic contexts.

HS.NQ.A.2 Compare real numbers presented through different representations, including both rational and irrational numbers. Apply comparisons in authentic contexts.

HS.NQ.B *Attend to units of measurement needed to solve problems through quantitative reasoning and mathematical modeling.*

HS.NQ.B.3 Use reasoning to choose and interpret measurement units consistently in formulas, graphs, and data displays, as a way to understand problems and to guide the solution of multi-step problems.

HS.NQ.B.4 Define, manipulate, and interpret appropriate quantities using rational and irrational numbers to authentically model situations and use reasoning to justify these choices.

HS.NQ.B.5 Use reasoning to choose a level of accuracy appropriate to limitations on measurement when reporting quantities in modeling situations.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3K: High School Geometry Standards (2021)

Geometric Reasoning and Measurement (HS.GM)

HS.GM.A *Apply geometric transformations to figures through analysis of graphs and understanding of functions.*

HS.GM.A.1 Apply definitions of rotations, reflections, and translations to transform a figure and map between two congruent figures in authentic contexts.

HS.GM.A.2 Verify experimentally the properties of a dilation given a center and a scale factor. Solve problems in authentic contexts involving similar triangles or dilations.

HS.GM.A.3 Use the slopes of segments and the coordinates of the vertices of triangles, parallelograms, and trapezoids to solve problems in authentic contexts.

HS.GM.A.4 Use definitions of transformations and symmetry relationships to justify the solutions of problems in authentic contexts.

HS.GM.B *Construct and communicate geometric arguments through use of proofs, logical reasoning, and geometric technology.*

HS.GM.B.5 Apply and justify triangle congruence and similarity theorems in authentic contexts.

HS.GM.B.6 Justify theorems of line relationships, angles, triangles, and parallelograms; and use them to solve problems in authentic contexts.

HS.GM.B.7 Perform geometric constructions with a variety of tools and methods.

HS.GM.C *Solve problems and interpret solutions of area and volume of shapes by applying concepts of congruence, similarity, symmetry in authentic contexts.*

HS.GM.C.8 Solve authentic modeling problems using area formulas for triangles, parallelograms, trapezoids, regular polygons, and circles.

HS.GM.C.9 Use volume and surface area formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems and apply to authentic contexts.

HS.GM.C.10 Use geometric shapes, their measures, and their properties to describe real world objects, and solve related authentic modeling and design problems.

HS.GM.C.11 Apply concepts of density based on area and volume in authentic modeling situations.

HS.GM.D *Apply concepts of right triangle trigonometry in authentic contexts to solve problems and interpret solutions.*

HS.GM.D.12 Apply sine, cosine, and tangent ratios, and the Pythagorean Theorem, to solve problems in authentic contexts.

HS.GM.D.13 Apply the Pythagorean Theorem in authentic contexts, and develop the standard form for the equation of a circle.

HS.GM.D.14 Use the coordinate plane to determine parallel and perpendicular relationships, and the distance between points.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3L: High School Data & Statistics Standards (2021)

Data Reasoning and Probability (HS.DR)

HS.DR.A *Formulate Statistical Investigative Questions.*

HS.DR.A.1 Formulate multivariable statistical investigative questions and determine how data from samples can be collected and analyzed to provide an answer.

HS.DR.A.2 Formulate summative, comparative, and associative statistical investigative questions for surveys, observational studies, and experiments using primary or secondary data.

HS.DR.A.3 Formulate inferential statistical investigative questions regarding causality and prediction from correlation.

HS.DR.A.4 Use mathematical and statistical reasoning to formulate questions about data to evaluate conclusions and assess risks.

HS.DR.B *Collect and Consider Data.*

HS.DR.B.5 Articulate what constitutes good practice in designing a sample survey, an experiment, and an observational study. Understand issues of bias and confounding variables in a study and their implications for interpretation.

HS.DR.B.6 Distinguish and choose between surveys, observational studies, and experiments to design an appropriate data collection that answers an investigative question of interest.

HS.DR.B.7 Apply an appropriate data collection plan when collecting primary data or selecting secondary data for the statistical investigative question of interest.

HS.DR.C *Analyze, summarize, and describe data.*

HS.DR.C.8 Identify appropriate ways to summarize and then represent the distribution of univariate and bivariate data multiple ways with graphs and/or tables. Use technology to present data that supports interpretation of tabular and graphical representations.

HS.DR.C.9 Use statistics appropriate to the shape of the data distribution to compare the center and spread of two or more different data sets.

HS.DR.C.10 Use data to compare two groups, describe sample variability, and decide if differences between parameters are significant based on the statistics.

HS.DR.D *Interpret data and answer investigative questions.*

HS.DR.D.11 Use statistical evidence from analyses to answer statistical investigative questions, and communicate the findings in a variety of formats (verbal, written, visual) to support informed data-based decisions.

HS.DR.D.12 Articulate what it means for an outcome or an estimate of a population characteristic to be plausible or not plausible compared to chance variation.

HS.DR.D.13 Use multivariate thinking to articulate how variables impact one another, and measure the strength of association using correlation coefficients for regression curves.

HS.DR.E *Understand independence and conditional probability and use them to interpret data.*

HS.DR.E.14 Describe the possible outcomes for a situation as subsets of a sample space.

HS.DR.E.15 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

SECTION FOUR – Grade/Course Level Guidance

4A: Grade K Math Standards and Guidance

Critical Areas of Focus

In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Domains and Clusters

K.OA - Algebraic Reasoning: Operations

- K.OA.A Understand addition and subtraction

K.NCC - Numeric Reasoning: Counting and Cardinality

- K.NCC.A Know number names and the count sequence
- K.NCC.B Count to tell the number of objects.
- K.NCC.C Compare numbers.

K.NBT - Numeric Reasoning: Base Ten Arithmetic

- K.NBT.A Work with numbers 11-19 to gain foundations for place value

K.GM - Geometric Reasoning and Measurement

- K.GM.A Identify and describe shapes
- K.GM.B Analyze, compare, create, and compose shapes.
- K.GM.C Describe and compare measurable attributes

K.DR - Data Reasoning

- K.DR.A Pose investigative questions and collect/consider data
- K.DR.B Analyze, represent, and interpret data.

Students should spend the large majority[1] of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

[1]*At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see the K-8 major work of the grade developed by Student Achievement Partners.*

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: K.OA.A - Understand addition and subtraction.

STANDARD: K.OA.A.1

Standards Statement (2021):

Represent addition as putting together and adding to and subtraction as taking apart and taking from using objects, drawings, physical expressions, numbers or equations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	K.OA.A.2, 1.OA.A.1	K.NCC.A.1	K.OA.A.1 K.OA.A Crosswalk

Standards Guidance:

Clarifications

- Practices combining, separating, and naming quantities.
- Uses simple strategies to solve mathematical problems and communicates how he/she solved it.
- Students should be able to represent real-life problems involving the addition and subtraction of whole numbers within 10 with objects and drawings.

Terminology

- Physical expressions can include, but not limited to, sounds (e.g., claps), acting out situations, or other types of physical movement.
- Pictorial drawings need not show details, but should show the mathematics in the problem.

Boundaries

- Exposure to equations is expected but mastery of equations is not required.
- Drawings do not need to show details but should show the mathematics in the problem.
- Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required. However, please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation.

Teaching Strategies

- Representations may include objects, fingers, mental images, drawings, expressions, or equations.
- Student drawings should show the mathematics of the solution from the given context. Equations should be derived from contexts.

Progressions

- Students may bring from home different ways to show numbers with their fingers and to raise (or lower) them when counting. The three major ways used around the world are starting with the thumb, the little finger, or the pointing finger (ending with the thumb in the latter two cases). Each way has advantages physically or mathematically, so students can use whatever is familiar to them. (Please reference page 8 in the Progression document)

Examples

- Representation can include objects, fingers, mental images, drawings, sounds, acting out, verbal explanations, expressions or equations. An example of representational sounds can be clapping.
- Illustrative Mathematics:[Ten Frame Addition] [Dice Addition 2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.OA.A.2

Standards Statement (2021):

Add and subtract within 10. Model authentic contexts and solve problems that use addition and subtraction within 10.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.OA.A.1	K.OA.A.3, 1.OA.A.1, 1.OA.B.3, 1.OA.B.4, 1.OA.C.6	N/A	K.OA.A.2 K.OA.A Crosswalk

Standards Guidance:

Clarifications

- Use addition and subtraction within 10 to solve and represent problems in authentic contexts involving situations of adding to, taking from, putting together, and taking apart.
- Practices combining, separating, and naming quantities.
- Uses simple strategies to solve mathematical problems and communicates how he/she solved it.

Terminology

- Students should be provided with a variety of problem types including Join: Result Unknown, Separate: Result Unknown, and Part-Part-Whole: Whole Unknown; however, students are not required to know or use this terminology.
    - Join: Result Unknown
        - Example: 3 birds were sitting in a tree and 2 more birds flew onto the tree. How many birds were in the tree then?
    - Separate: Result Unknown
        - Example: Toni had 8 guppies. She gave 3 guppies to Roger. How many guppies does Toni have now?
    - Part-Part-Whole: Whole Unknown
        - Example: 6 girls and 4 boys were playing soccer. How many children were playing soccer?

Boundaries

- Exposure to equations is expected but mastery of equations is not required in Kindergarten.

Teaching Strategies

- Use objects and drawings to represent the word problem. In order to solve word problems within 10, use numbers 0-9
- Students should be able to solve real-life problems involving the addition and subtraction of single-digit whole numbers, using a variety of strategies such as:
    - counting on
    - counting backward
    - making 10

Examples

- Illustrative Mathematics: [Ten Flashing Fireflies] [Dice Addition 1] [What's Missing?]
- Student Achievement Partners: [Teddy Bears] [Fly Away]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.OA.A.3

Standards Statement (2021):

Using objects or drawings, and equations, decompose numbers less than or equal to 10 into pairs in more than one way.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.OA.A.2	K.OA.A.4, K.OA.A.5, 1.OA.C.6	K.NBT.A.1, K.NCC.A.1	K.OA.A.3 K.OA.A Crosswalk

Standards Guidance:

Clarifications

- Students practice combining, separating, and naming quantities.

Terminology

- Decomposition is the process of breaking apart a number into a variety of parts that all equal the same whole. Example 9 = 6 +3; 9 = 5 + 4 both equations equal 9.
- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Compose – put together numbers
    - Decompose – break apart numbers

Teaching Strategies

- Use objects or drawings to decompose numbers in at least two different ways. Record each decomposition with a drawing, number bond, or equation.
- Teachers should use dot card images for students to explain how they see different number combinations.

Examples

- Illustrative Mathematics:
    - My Book of Five
    - Shake and Spill
    - Pick Two
    - Christina's Candies
    - Bobbie Bear's Buttons
    - Make 9

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.OA.A.4

Standards Statement (2021):

By using objects, drawings, or equations, find the unknown number that makes 10 when added to a given number from 1 - 9.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.OA.A.3	1.OA.C.6	N/A	K.OA.A.4 K.OA.A Crosswalk

Standards Guidance:

Clarifications

- This standard builds upon the understanding that a number can be decomposed into parts. (K.OA.A.3).
- Once students have had experiences breaking apart ten into various combinations, this asks students to find a missing part of 10.

Examples

- A full case of juice boxes has 10 boxes. There are only 6 boxes in this case. How many juice boxes are missing?
- Student Achievement Partners:
  - Cherries in a Bowl

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.OA.A.5

Standards Statement (2021):

Fluently add and subtract within 5 with accurate, efficient, and flexible strategies.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.OA.A.3	1.OA.C.6	N/A	K.OA.A.5 K.OA.A Crosswalk

Standards Guidance:

Clarifications

- Uses simple strategies to solve mathematical problems and communicates how he/she solved it.
- Students should be able to solve real-life problems involving the addition and subtraction of numbers within five.

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Fluently/Fluency -- To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Boundaries

- Fluency does not lend itself to timed tests or speed.
- Exposure to equations is expected but mastery of equations is not required.

Progressions

- Experience with decompositions of numbers and with Add To and Take From situations enables students to begin to fluently add and subtract within 5. (Please reference page 11 in the Progression document)

Examples

- Record the sum or difference with a drawing oral response, visual cue or equation. Can use an oral response to a verbal or visual cue to demonstrate fluency.
- When making toothpick designs to represent the various combinations of the number “5”, the student writes the numerals for the various parts (such as “4” and “1”) or selects a number sentence that represents that particular situation (such as 5 = 4 + 1).
- Illustrative Mathematics:
    - Many Ways to Do Addition 1
    - My Book of Five
- Student Achievement Partners:
    - Assessment example

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster K.NCC.A - Know number names and the count sequence.

STANDARD: K.NCC.A.1

Standards Statement (2021):

Orally count to 100 by ones and by tens in sequential order.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	K.NCC.A.2, K.NCC.A.3, K.NCC.B.4, 1.NBT.A.1	K.OA.A.2, K.OA.A.3	K.CC.A.1 K.NCC.A Crosswalk

Standards Guidance:

Clarifications

- Students should count for authentic purposes, which connect to their everyday experiences.
- Students should understand that each successive number name refers to a quantity that is one larger.
- When students are rote counting forward, start the count sequence at 1.
- When students are counting backward, start the count sequence beginning at 10 and progress to counting backward beginning at 20.

Boundaries

- When students count by tens, they are only expected to master counting by the decade (10, 20, ...).
- This expectation does not require recognition of numerals.

Teaching Strategies

- Beginning at number zero students can orally count in sequential order without skipping or repeating numbers to 100
- When students count backward from 20, they can use visual resources such as a number line, a 99-chart, or a 100-chart.

Examples

- Illustrative Mathematics:
  - Choral Counting
  - Counting Circles
  - Counting by Tens
  - Assessing Counting Sequences Part I
  - Find The Numbers 0-5 or 5-10
  - Teen Go Fish

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.NCC.A.2

Standards Statement (2021):

Count forward beginning from a given number within 100 of a known sequence.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NCC.A.1	K.NCC.B.4	N/A	K.CC.A.2 K.NCC.A Crosswalk

Standards Guidance:

Clarifications

- This learning objective builds on the skill of naming numbers up to 20 in sequence.
- Know sequence can start at a number of known sequence instead of having to begin at 1.
- Students should count forward and backward from a given number using the known number word sequence for authentic purposes.
- Students should be able to begin and end with any given number.

Boundaries

- The "known sequence" for this standard can be within 20. The intent is to build toward addition and subtraction in first grade.
- This expectation does not require recognition of numerals

Examples

- Counting forward can be demonstrated using manipulatives, or oral response. In written form would be beyond the intent of this standard.
- Given the number 54, a student will count "54, 55, 56, 57, 58…"
- Illustrative Mathematics:
  - Start-Stop Counting
  - Number After Bingo 1-15
  - Number Line Up
  - "One More" Concentration
  - Pick a Number, Counting On
  - Assessing Counting Sequences Part I
  - Assessing Counting Sequences Part II
- Student Acheivement Partners:
  - Count Sequence

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.NCC.A.3

Standards Statement (2021):

Identify number names, write numbers, and the count sequence from 0-20. Represent a number of objects with a written number 0-20.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NCC.A.1	K.NCC.B.4, K.NCC.C.6	N/A	K.CC.A.3 K.NCC.A Crosswalk

Standards Guidance:

Clarifications

- Recognizes numerals and uses counting as part of play and as a means for determining quantity.
- Students should be able to identify written numerals for authentic purposes.
- Students should be able to demonstrate the relationship between written numerals and a number of objects.

Teaching Strategies

- Common errors include transposing or skipping a written number. For example, transposing 13 in writing to 31.
- Instruction includes helping student understand that 0 represents a count of no objects.

Examples

- Know number names and the count sequence.
- Write numbers from 0-20 and represent a number of objects with a written number 0-20.
- Illustrative Mathematics:
  - Bags of Stuff
  - Assessing Writing Numbers
  - Rainbow Number Line
  - Assessing Reading Numbers
  - Assessing Sequencing Numbers
  - Number TIC TAC TOE
  - Race to the Top
- Student Acheivement Partners:
  - Dice Addition 1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: K.NCC.B - Count to tell the number of objects.

STANDARD: K.NCC.B.4

Standards Statement (2021):

Understand the relationship between numbers and quantities; connect counting to cardinality.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NCC.A.1, K.NCC.A.2, K.NCC.A.3	K.NCC.B.5, K.NCC.C.6	1.OA.C.5	K.CC.B.4 K.NCC.B Crosswalk

Standards Guidance:

Clarifications

- Counts at least 10 objects using one-to-one correspondence.
- Students should count objects using one-to-one correspondence saying the number names in the standard order and communicate quantities for authentic purposes. “Authentic purposes” refers to experiences students have in their everyday lives.

Terminology

- When counting objects, explain that the last number counted represents the total quantity in a set (cardinality), regardless of the arrangement and order.

Teaching Strategies

- Understand that objects are counted using 1:1 correspondence in sequential order to determine quantity with last number representing the total objects counted.
- Students should instantly see how many objects are in a group without counting (subitizing)

Examples

- Any type of counter, such as cereal, beads, rocks, and bears, are sample tools that can be used for counting objects.
- Dot cards, five-frames, ten-frames, and rekenreks can be used for subitizing
- Illustrative Mathematics:
  - Goody Bags
  - Counting Mat
  - Number Rods
  - The Napping House

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.NCC.B.5

Standards Statement (2021):

Count to answer "how many?" questions using up to 20 objects arranged in a variety of configurations or as 10 objects in a scattered configuration. Given a number from 1-20, count out that many objects.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NCC.B.4, K.NCC.C.6	1.NBT.B.3	N/A	K.CC.B.5 K.NCC.B Crosswalk

Standards Guidance:

Clarifications

- Count from 1 to 20 objects in sequential order in a variety of configurations. Configurations can include ten frames, arrays, circles or a line.
- Students should be able answer questions of "how many?" objects using one-to-one correspondence.

Teaching Strategies

- Dot cards, five-frames, ten-frames, rekenreks, dominoes, and playing cards are some tools that can be used for subitizing.

Progressions

- Students should be able to count to answer "how many?" questions with up to 20 objects arranged in a variety of ways (a line, a rectangular array, or a circle), or up to 10 objects arranged in a scattered configuration. (Please reference page 4 in the Progression document).

Examples

- Illustrative Mathematics:
  - Finding Equal Groups
  - Number Rods

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: K.NCC.C - Compare numbers.

STANDARD: K.NCC.C.6

Standards Statement (2021):

Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NCC.A.3, K.NCC.B.4	K.NCC.B.5, K.NCC.C.7, K.GM.C.7	K.DR.B.2	K.CC.C.6 K.NCC.C Crosswalk

Standards Guidance:

Clarifications

- Matches two equal sets using one- to-one correspondence and understands they are the same.
- Students should compare the number of objects in two groups in real-life situations and identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.
- Students should be able to explain that equal to is “the same” quantity.

Boundaries

- Group sizes limited to at most 10 objects each.
- The words greater than, less than, or the same as (equal to) should be used instead of the symbols.

Teaching Strategies

- Understand the intent is to use matching and counting strategies to reinforce vocabulary of greater than, less than or equal to.

Progressions

- The standards about comparing numbers focus on students identifying which of two groups has more than (or fewer than, or the same amount as) the other. Students first learn to match the objects in the two groups to see if there are any extra and then to count the objects in each group and use their knowledge of the count sequence to decide which number is greater than the other (the number farther along in the count sequence). (Please reference page 5 in the Progression document)

Examples

- Illustrative Mathematics:
  - Which number is greater? Which number is less? How do you know?
  - More or Less Handfuls
- Student Acheivement Partners:
  - Apples or Lemons

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.NCC.C.7

Standards Statement (2021):

Compare two numbers between 1 and 10 presented as written numerals.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NCC.C.6	1.NBT.B.3	N/A	K.CC.C.7 K.NCC.C Crosswalk

Standards Guidance:

Clarifications

- Students identify numbers that come before and after a given number up to 10.

Boundaries

- Students should have had the experience of comparing the number of physical objects prior to comparing and ordering written numerals.

Examples

- Compare numbers in a written format without manipulatives or visuals.
- Given the numerals 4 and 2, students would say, “Two is less than four, and four is greater than two.” Students would also put them in sequential order: “2, 4.”
- Illustrative Mathematics:
  - Guess the Marbles in the Bag

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: K.NBT.A - Work with numbers 11-19 to gain foundations for place value.

STANDARD: K.NBT.A.1

Standards Statement (2021):

Compose and decompose from 11 to 19 into groups of ten ones and some further ones using objects, drawings, or equations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	1.NBT.B.2	K.OA.A.2, K.OA.A.3	K.NBT.A.1 K.NBT.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to put together (compose) and break apart (decompose) numbers into a group of ten ones and some further ones to understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
- Students should use strategic thinking in order to communicate quantities for authentic purposes.

Terminology

- Composition refers to putting numbers together, such as 10 + 2 = 12
- Decomposition refers to breaking multi-digit numbers apart, which would be into groups of tens and ones at this grade level, such as 14 = 10 + 4.
- Math drawings are simple drawings that make essential mathematical features and relationships salient while suppressing details that are not relevant ot the mathematical ideas.

Teaching Strategies

- Compose and decompose numbers from 11 to 19 into ten and single ones (not “a ten;” this is in first grade).
- Use objects or drawings to record each composition or decomposition. An example would be a student recognizing that the number 13 is made up of ten single ones and three more. This is specific to teen numbers only.
- Students should be given the opportunity to use five frames, ten frames, and rekenreks with support to demonstrate each composition or decomposition.

Progressions

- The numerals 11, 12, 13, ..., 19 need special attention for children to understand them. The first nine numerals 1, 2, 3, ..., 9, and 0 are essentially arbitrary marks. These same marks are used again to represent larger numbers. Children need to learn the differences in the ways these marks are used. For example, initially, a numeral such as 16 looks like "one, six," not "1 ten and 6 ones." Layered place value cards can help children see the 0 "hiding" under the ones place and that the 1 in the tens place really is 10 (ten ones). (Please reference page 5 in the Progression document).

Examples

- Illustrative Mathematics: [What Makes a Teen Number?]
- Student Acheivement Partners: [Counters & Ten Frame]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: K.GM.A - Identify and describe shapes.

STANDARD: K.GM.A.1

Standards Statement (2021):

Describe objects in the environment using names of shapes and describe the relative positions of these objects in their environment.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	K.GM.B.4, K.GM.B.5, K.GM.B.6	N/A	K.G.A.1 K.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students use appropriate directional language to indicate where things are in their environment (for example: positions, distances, order).
- Kindergarten students should be able to explain the location of an object in relation to another object using positional language, such as “above,” “below,” “beside,” “in front of,” “behind,” or “next to.”

Teaching Strategies

- Use positional terms such as above, below, beside, in front of, behind, and next to when describing position of an object. For example, the box is under the chair or the bear is next to the table.

Boundaries

- Students can identify the following two-dimensional shapes based on attributes:
  - Square, circle, triangle, rectangle, hexagon, oval, rhombus
- Students can identify the following three-dimensional shapes based on attributes:
  - Cubes, cones, cylinders, spheres

Progressions

- Students refine their informal language by learning mathematical concepts and vocabulary so as to increasingly describe their physical world from geometric perspectives, e.g., shape, orientation, spatial relations (MP4).
- Students increase their knowledge of a variety of shapes, including circles, triangles, squares, rectangles, and special cases of other shapes such as regular hexagons, and trapezoids with unequal bases and non-parallel sides of equal length.
- Students also begin to name and describe three-dimensional shapes with mathematical vocabulary, such as “sphere,” “cube,” “cylinder,”and “cone.” Finally, in the domain of spatial reasoning, students discuss not only shape and orientation, but also the relative positions of objects, using terms such as “above,” “below,” “next to,” “behind,” “in front of,” and “beside.” (Please reference page 6 in the Progression document).

Examples

- “The cup is beside the pencil."
- “The boy is behind the girl in line.”
- In a sequence of pictures, the student would describe the position of a particular object.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.GM.A.2

Standards Statement (2021):

Correctly name common two-dimensional and three-dimensional geometric shapes regardless of their orientations or overall size.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	K.GM.B.4, K.GM.B.5, K.GM.B.6	N/A	K.G.A.2 K.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students should distinguish between defining attributes of two-dimensional shapes and three-dimensional figures versus non-defining attributes (e.g., triangles are closed and three-sided, a defining attribute versus triangles are red, non-defining attribute).
- Students should be able to build and draw shapes based on defining attributes. Two-dimensional shapes should be limited to triangles, squares, and rectangles.
- Students should be able to identify a shape's attributes, regardless of its orientation (i.e., flipped) or position (i.e., turned).

Boundaries

- Students can identify the following two-dimensional shapes based on attributes:
  - *Square, circle, triangle, rectangle, hexagon, oval (ellipse), rhombus*
- Students can identify the following three-dimensional shapes based on attributes:
  - *Cubes, cones, cylinders, spheres*

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
  - Attributes – characteristics of two-dimensional shapes and three-dimensional figures, including geometric properties.
  - Defining attributes – include number of sides, faces, vertices (corners), and angles.
  - Non-defining attributes – include size, orientation, texture, and color.

Progressions

- Students learn to name shapes such as circles, triangles, and squares, whose names occur in everyday language, and distinguish them from nonexamples of these categories, often based initially on visual prototypes. (Please reference page 6 in the Progression document).

Examples

- Illustrative Mathematics:
  - Shape Hunt Part 2
- Student Acheivement Partners:
  - Recognizing Squares
  - Naming Shapes

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.GM.A.3

Standards Statement (2021):

Identify shapes as two-dimensional or three-dimensional.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	K.GM.B.4, K.GM.B.5, K.GM.B.6	N/A	K.G.A.3 K.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students will name flat shapes as two-dimensional or solid shapes as three-dimensional. Example would be a square is flat but a cube is solid.
- In the domain of shape, students learn to match two-dimensional shapes even when the shapes have different orientations.

Teaching Strategies

- Have students explain their decisions about shape names or classifications prompts students to attend to and describe certain features of the shapes. That is, concept images and names they have learned for the shapes are the raw material from which they can abstract common features.

Progressions

- In the domain of shape, students learn to match two-dimensional shapes even when the shapes have different orientations.
- Students need to explain their decisions about shape names or classifications prompts students to attend to and describe certain features of the shapes. That is, concept images and names they have learned for the shapes are the raw material from which they can abstract common features.
- Students identify faces of three-dimensional shapes as two-dimensional geometric figures and explicitly identify shapes as two-dimensional (“flat” or lying in a plane) or three-dimensional ("solid"). (Please reference page 6 in the Progression document).

Examples

- Have students identify faces of three-dimensional shapes as two-dimensional geometric figures and explicitly identify shapes as two-dimensional (“flat” or lying in a plane) or three-dimensional ("solid").
- Illustrative Mathematics:
  - Shape Hunt Part 1
  - Shape Sequence Search

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: K.GM.B - Analyze, compare, create, and compose shapes.

STANDARD: K.GM.B.4

Standards Statement (2021):

Analyze and compare two and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts and attributes.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.GM.A.1, K.GM.A.2, K.GM.A.3	1.GM.A.1	N/A	K.G.B.4 K.GM.B Crosswalk

Standards Guidance:

Clarifications

- Students recognize and name common two-dimensional and three-dimensional shapes, their parts and attributes.
- Students should be able to identify basic shapes, including squares, circles, triangles, rectangles, hexagons, octagons, cubes, cones, cylinders, and spheres.
- Students begin to understand how three-dimensional figures are composed of two-dimensional shapes.

Teaching Strategies

- Develop spatial sense by connecting geometric shapes to students' everyday lives.
- Initiate natural conversations about shapes in the environment. Have students identify and name two and three-dimensional shapes in and outside of the classroom and describe their relative position.
- Ask students to find rectangles in the classroom and describe the relative positions of the rectangles they see, e.g. This rectangle (a poster) is over the sphere (globe).
- Teachers can use a digital camera to record these relationships.

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Attributes – characteristics (i.e., two- dimensional shapes (lying in a plane, "flat") and three-dimensional figures ("solid"), including geometric properties.). An example of an attribute is having sides of equal length.
    - Vertices – corners of a geometric figure

Examples

- Example a square has 4 equal sides and 4 corners/vertices and a cube has 8 equal sides with 8 corners/vertices and 6 faces. The intent is not for students to yet have the formal language, but teachers can start to guide the transition from informal to formal mathematical language.
- The base and top of a cylinder is a circle.
- Illustrative Mathematics:
    - Alike or Different Game

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.GM.B.5

Standards Statement (2021):

Represent shapes in the world by building shapes from components and drawing shapes.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.GM.A.1, K.GM.A.2, K.GM.A.3	1.GM.A.1	N/A	K.G.B.5 K.GM.B Crosswalk

Standards Guidance:

Clarification

- Standard includes student explanations about their decisions for shape names or classifications. That is, concept images and names they have learned for the shapes are the raw material from which they can abstract common features.

Teaching Strategies

- Prompt students to attend to and describe certain features of the shapes.
- Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
- Crate simple composite shapes using pattern blocks, such as a square and rectagle could represent a house.

Progressions

- Students need to explain their decisions about shape names or classifications prompts students to attend to and describe certain features of the shapes. That is, concept images and names they have learned for the shapes are the raw material from which they can abstract common features.
- Students represent shapes informally with drawings and by building them from components (e.g., manipulatives such as sticks). With repeated experiences such as these, students become more precise (MP6). (Please reference page 6 in the Progression document).

Examples

- Build 2-dimensional shapes or 3-dimensional shapes using manipulatives and other components. Example building a house using marshmallows and toothpicks or Legos.
- *Models* – sticks with clay balls, toothpicks with marshmallows, popsicle sticks, technology, etc.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.GM.B.6

Standards Statement (2021):

Compose common shapes to form larger shapes.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.GM.A.1, K.GM.A.2, K.GM.A.3	1.GM.A.2		K.G.B.6 K.GM.B Crosswalk

Standards Guidance:

Clarifications

- Students manipulate objects and describe the process for fitting objects together.
- Students combine simple shapes to form new shapes.

Teaching Strategies

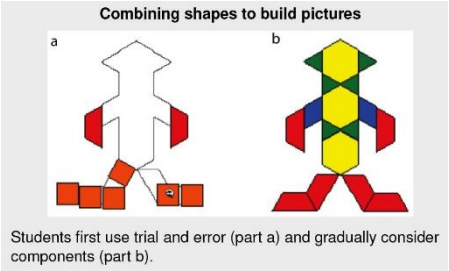
- Students should be able to form (compose) larger shapes by putting together smaller shapes through exploration and play.
- Have students not only build shapes from components, but also compose shapes to build pictures and designs.

Progressions

- An important area for kindergartners is the composition of geometric figures. Students not only build shapes from components, but also compose shapes to build pictures and designs. Initially lacking competence in composing geometric shapes, they gain abilities to combine shapes–first by trial and error and gradually by considering components–into pictures. At first, side length is the only component considered. Later experience brings an intuitive appreciation of angle size. (Please reference page 7 in the Progression document).

Examples

- What shapes can you create with these two triangles?"
- Use more than one shape to build a larger shape. Example two triangles make a rhombus or two trapezoids to make a hexagon.
- Students combine two-dimensional shapes and solve problems such as deciding which piece will fit into a space in a puzzle, intuitively using geometric motions (slides, flips, and turns, the informal names for translations, reflections, and rotations, respectively). Students can construct their own outline puzzles and exchange them, solving each other's.
- Exploring Tessellations



Students first use trial and error (part a) and gradually consider components (part b).

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: K.GM.C - Describe and compare measurable attributes.

STANDARD: K.GM.C.7

Standards Statement (2021):

Describe several measurable attributes of a single object using measurable terms, such as length or weight.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NCC.C.6	K.GM.C.8	N/A	K.MD.A.1 K.GM.C Crosswalk

Standards Guidance:

Clarifications

- Students use a variety of techniques and standard and non-standard tools to measure and compare length, volume (capacity) and weight
- Students independently orders objects using one characteristic and describes the criteria used.
- In Kindergarten, students should use language such as heavier, lighter, longer, taller, shorter, wider, larger, and smaller.
- In Kindergarten, students may use a variety of techniques and tools to compare, describe, and order objects. Students may use a referent object being compared as a tool to describe the other object(s).

Terminology

- Measurable attributes can be vocabulary that describe the length, weight or shape of an object.
- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Attributes – characteristics (i.e., length, height, width, or weight)
    - Referent object – an object used as the standard of comparison

Progressions

- Students often initially hold undifferentiated views of measurable attributes, saying that one object is "bigger" than another whether it is longer, or greater in area, or greater in volume, and so forth. For example, two students might both claim their block building is "the biggest."
- Conversations about how they are comparing—one building may be taller (greater in length) and another may have a larger base (greater in area)—help students learn to discriminate and name these measureable attributes. As they discuss these situations and compare objects using different attributes, they learn to distinguish, label, and describe several measureable attributes of a single object. (Please reference page 6 in the Progression document)

Examples

- Directly compare the heights of two objects and describe one object as taller/shorter.
- A student may describe a shoe as, "The red shoe is heavier than the blue shoe (the blue shoe is the referent in this case)! The red shoe is also longer!"
- Illustrative Mathematics:
    - How Heavy?
    - Longer and Heavier? Shorter and Heavier?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: K.GM.C.8

Standards Statement (2021):

Directly compare two objects with a measurable attribute in common, and describe which object has "more" or "less" of the attribute.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.GM.C.7	1.GM.B.4	K.DR.B.2, 1.DR.B.2	K.MD.A.2 K.GM.C Crosswalk

Standards Guidance:

Clarifications

- Students describe sets as having more, less, same as/equal.
- Students can tell numbers that come before and after a given number up to 10.
- Students should be able to understand that each successive number name refers to a quantity that is one larger and the previous number name is one less.

Teaching Strategies

- Use comparative vocabulary to directly compare two objects.
- Have students directly compare lengths in simple situations, such as comparing people's heights, because standing next to each other automatically aligns one endpoint.

Progressions

- Kindergartners easily directly compare lengths in simple situations, such as comparing people's heights, because standing next to each other automatically aligns one endpoint. However, in other situations they may initially compare only one endpoint of objects to say which is longer. Discussing such situations (e.g., when a child claims that he is "tallest" because he is standing on a chair) can help students resolve and coordinate perceptual and conceptual information when it conflicts. (Please reference page 6 in the Progression document).

Examples

- Example one child is shorter than the other child. Shorter being the identified attribute.
- Illustrative Mathematics:
  - Size Shuffle
  - Longer and Shorter
  - Which is Heavier?
  - Which is Longer?
  - Which weighs more? Which weighs less?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: K.DR.A - Pose investigative questions and collect/consider data.

STANDARD: K.DR.A.1

Standards Statement (2021):

Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by sorting and counting.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	1.DR.A.1	N/A	[new content] K.DR.A Crosswalk

Standards Guidance:

Clarifications

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.

Boundaries

- Limit category counts to be less than or equal to ten.
- At this grade level, more support is needed with formulating statistical questions. Students should be given guidance when developing statistical investigative questions.
- This standard should be taught throughout the year.

Terminology

- Collecting data would refer to student generating data sets, such as counting and recording the frequency of an event.
- Considering data refers to existing data sets given to students by a teacher for consideration.
- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - A statistical investigative question is one that requires data that will vary.

Teaching Strategies

- Students should be provided with support strategies for collecting and organizing their data.

Examples

- “How did you get to school today?”
- “What is your favorite _____?”

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: K.DR.B - Analyze, represent, and interpret data.

STANDARD: K.DR.B.2

Standards Statement (2021):

Analyze data sets by counting the number of objects in each category and interpret results by classifying and sorting objects by count.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	1.DR.B.2	K.GM.C.8	K.MD.B.3 K.DR.B Crosswalk

Standards Guidance:

Clarifications

- Students sort and classify objects using one or more attributes or relationships.
- Kindergarten students should have experience sorting objects by characteristics such as heavier, lighter, longer, and shorter (compare to benchmark item).

Terminology

- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
- Attributes – characteristics (i.e., length, height, width, or weight)

Boundaries

- Categories should have no more than 10 objects. In later grades, students will represent data in pictographs and bar graphs.
- In Kindergarten, students should be able to use friendly language to explain their data and answer the overall question.

Teaching strategies

- Students could display their data using objects and pictures.
- Represent the findings from generated questions using objects and pictures.
- Explain the findings based on the data collected and represented on graphs.

Progressions

- Students in Kindergarten classify objects into categories, initially specified by the teacher and perhaps eventually elicited from students. For example, in a science context, the teacher might ask students in the class to sort pictures of various organisms into two piles: organisms with wings and those without wings. Students can then count the number of specimens in each pile. Students can use these category counts and their understanding of cardinality to say whether there are more specimens with wings or without wings. (Please reference page 5 in the Progression document).

Examples

- Illustrative Mathematics:
  - Goodie Bags
  - Sort and Count I
  - Sort and Count II

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4B: Grade 1 Math Standards and Guidance

Critical Areas of Focus

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Domains and Clusters

1.OA - Algebraic Reasoning: Operations

- 1.OA.A Represent and solve problems involving addition and subtraction
- 1.OA.B Understand and apply properties of operations and the relationship between addition and subtraction.
- 1.OA.C Add and subtract within 20.
- 1.OA.D Work with addition and subtraction equations

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

1.NBT - Numeric Reasoning: Base Ten Arithmetic

- 1.NBT.A Extend the counting sequence
- 1.NBT.B Understand place value.
- 1.NBT.C Use place value understanding and properties of operations to add and subtract.

1.GM - Geometric Reasoning and Measurement

- 1.GM.A Reason with shapes and their attributes.
- 1.GM.B Describe and compare measurable attributes
- 1.GM.C Tell and write time.

1.DR - Data Reasoning

- 1.DR.A Pose investigative questions and collect/consider data.
- 1.DR.B Analyze, represent, and interpret data

Students should spend the large majority[1] of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

[1]At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see the K-8 major work of the grade developed by Student Achievement Partners.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.OA.A - Represent and solve problems involving addition and subtraction.

STANDARD: 1.OA.A.1

Standards Statement (2021):

Use addition and subtraction within 20 to solve and represent problems in authentic contexts involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.OA.D.8, K.OA.A.1, K.OA.A.2	1.OA.A.2, 2.OA.A.1	1.DR.B.2	1.OA.A.1 1.OA.A Crosswalk

Standards Guidance:

Clarifications

- Students should be given opportunities to use mental reasoning to solve problems involving number strings within 20.
- Students should also solve problem situations with an unknown in all positions.
- Students recognize and represent taking from, taking apart, and comparing situations as either subtraction or addition with a missing addend.

Boundaries

- Students should not be encouraged to use key/clue words because they will not work with subsequent problem types.
- The unknown quantity should be represented in all positions.

Terminology

- Addition and Subtraction Situations by Grade Level are presented in Table 1 pictured here, which include:
  - adding to,
  - taking from,
  - putting together, taking apart, and
  - comparing, with unknowns in all positions.
- Please reference pages 9 and 14 in the Progression document for additional information.

Table 1: Addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown[1]**	**Addend Unknown[2]**
Put Together /Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version.* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? *"More" suggests wrong operation.* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

This work is licensed under a Creative Commons Attribution Noncommercial NoDerivatives 4.0 International License.

Teaching Strategies

- Symbols can be used to represent unknown amounts in equations.
- Use the relationship between addition and subtraction within 20 (knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (6 + 7 is the same as 6 + 6 + 1 = 12 + 1 = 13).
- Students should be provided with learning experiences to develop strategies such as:
  - Advanced Counting; Counting On, Making Ten, Decomposing a number leading to a ten
  - *Counting All:* 5 + 2 = □. The student counts five counters. The student adds two more. The student counts 1, 2, 3, 4, 5, 6, 7 to get the answer.
  - *Counting Back:* 12 – 3 = □. The student counts twelve counters. The student removes a counter and says 11, removes another counter and says 10, and removes a third counter and says 9. The student knows the answer is 9 since they counted back 3.

Examples

- Represent addition and subtraction word problems using objects, drawings, and equations. Write an addition or subtraction equation with a symbol for the unknown number in different position, such as:
  - 13 + 5 = n, 13 - 5 = n, 13 + n= 18, 18 - n= 13.
- Recognize and represent adding to and putting together situations as addition.
- Illustrative Mathematics:
  - At the Park
  - Maria's Marbles
  - Peyton's Books
  - School Supplies
  - Sharing Markers
  - The Pet Snake
  - Field Day Scarcity
  - Finding a Chair
  - Link-Cube Addition
- Student Achievement Partners:
  - Pennies on the Table

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 1.OA.A.2

Standards Statement (2021):

Solve problems that call for addition of three whole numbers whose sum is less than or equal to 20 using objects, drawings or equations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.OA.A.1	N/A	1.DR.B.2	1.OA.A.2 1.OA.A Crosswalk

Standards Guidance:

Clarifications

- Students should understand subtraction as an unknown-addend problem.
- Students are not expected to know nor use the term inverse.

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
  - Addend – a number that is added to another number in an addition expression or equation. For example, in the expression 5 + 8, 5 and 8 are both addends.
  - An inverse relationship shows the relationship between addition and subtraction where addition can be used to find the quantity of a set after some in the set are removed. For example, 3+2 = 5 is related to 5 - 3 = 2 because of the inverse relationship.

Boundaries

- Problems should be within 20.

Examples

- Solve word problems by using objects, drawings or equations to represent the quantities in the problem.
- Solve word problems with an equation where a symbol stands for the unknown. For example, 5 + 4 + 6 = ___.
- Understand that objects, drawings, and equations are interchangeable representations of a story problem.
- There are 14 birds in the tree. 8 of them flew away. How many birds are left in the tree?
  - The student thinks of 14 – 8 = □ as 8 + □ = 14
- Jenny had 10 pencils and gave some to Eric. Jenny now has 8 pencils. How many pencils did she give to Eric?
  - The student thinks of 10 - □ = 8 as □ + 8 = 10
- Illustrative Mathematics:
  - Daisies in vases
  - The Very Hungry Caterpillar

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.OA.B - Understand and apply properties of operations and the relationship between addition and subtraction.

STANDARD: 1.OA.B.3

Standards Statement (2021):

Apply properties of operations as strategies to add and subtract.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.OA.A.2	1.OA.C.6	2.NBT.B.9, 3.NBT.A.2	1.OA.B.3 1.OA.B Crosswalk

Standards Guidance:

Clarifications

- Students should solve problem situations with an unknown in all positions.
- Understand that numbers can be added flexibly.
- Students do not necessarily have to justify their representations or solution using properties, but they can begin to learn to recognize these properties in action and discuss their use after solving. (Please reference page 15 in the Progression document)

Boundaries

- Students should not be encouraged to use key/clue words because they will not work with subsequent problem types.
- The unknown quantity should be represented in all positions.
- The terminology above is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.

Terminology

- Properties of operations used as strategies include:
  - Commutative property of addition: For example, if 8 + 3 = 11 is known, then 3 + 8 = 11 is also known.
  - Associative property of addition: For example, add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12.
- Addend – any number that is added to another number in an addition expression or equation. For example, in the expression 7 + 3, 7 and 3 are addends.

Examples

- Illustrative Mathematics:
  - Domino Addition
  - Fact Families
  - Fact Families with Pictures
  - Doubles?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 1.OA.B.4



Standards Statement (2021):

Understand subtraction as an unknown-addend problem.



Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.OA.A.2	1.OA.C.6	2.NBT.B.9, 3.NBT.A.2	1.OA.B.4 1.OA.B Crosswalk



Standards Guidance:

Teaching Strategies

- Restate a subtraction problem as a missing addend problem using the relationship between addition and subtraction.
- Recognize the inverse relationship between subtraction and addition within 20 and use this inverse relationship to solve real-life problems.

Progressions

- Put Together/Take Apart problems with Addend Unknown afford students the opportunity to see subtraction as the opposite of addition in a different way than as reversing the action, namely as finding an unknown addend.
- The meaning of subtraction as an unknown-addend addition problem is one of the essential understandings students will need in middle school in order to extend arithmetic to negative rational numbers. (Please reference page 13 in the Progression document).

Examples

- Subtract 10 – 8 by finding the number that makes 10 when added to 8.
- Understand that subtraction is equivalent to an unknown-addend problem because both ask for the unknown part in a situation where the total and another part are known.
- Illustrative Mathematics:
  - Cave Game Subtraction
- Student Achievement Partners:
  - Subtraction as an unknown-addend problem

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.OA.C - Add and subtract within 20.

STANDARD: 1.OA.C.5

Standards Statement (2021):

Relate counting to addition and subtraction.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NCC.B.4	1.OA.C.6	N/A	1.OA.C.5 1.OA.C Crosswalk

Standards Guidance:

Clarifications

- Students should be able to relate counting to addition and subtraction by counting all, counting on, and counting back when making sense of contextual addition and subtraction problems within 20.

Teaching Strategies

- Students should understand how addition and subtraction relate by solving situations in context.
- Students should use strategies to count up, count back, etc., to model this relationship on tools such as ten frames, rekenreks, number lines (predetermined and open), etc.
- Relate counting on to addition. For example, recognize counting on two after 15 as solving 15+2.
- Relate counting back to subtraction. For example, recognize counting back two from 15 as solving 15-2.
- Relate counting between two numbers to finding their difference. For example, recognize counting two number between 15 and 17 as solving 17-15.

Progression

- Unlike counting down, counting on reinforces that subtraction is an unknown-addend problem. Learning to think of and solve subtractions as unknown addend problems makes subtraction as easy as addition (or even easier), and it emphasizes the relationship between addition and subtraction. (Please reference page 20 in the Progression document).

Examples

- When students count on 3 from 4, they should write this as 4+3=7.
- When students count on for subtraction, 3 from 7, they should connect this to 7−3=4. Students write "7−3= ?" and think “I count on 3+ ?=7.”
- Illustrative Mathematics:
  - The Very Hungry Caterpillar

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 1.OA.C.6

Standards Statement (2021):

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10 with accurate, efficient, and flexible strategies.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.OA.A.2, K.OA.A.3, K.OA.A.4, K.OA.A.5, 1.OA.C.5, 1.OA.B.4, 1.OA.B.3	2.OA.B.2	1.NBT.C.4	1.OA.C.6 1.OA.C Crosswalk

Standards Guidance:

Terminology

- Fluently/Fluency – To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.
- Accuracy includes attending to precision.
- Efficiency includes using well-understood strategy with ease.
- Flexibility involves using strategies such as making 5 or making 10.

Boundaries

- Fluency does not lend itself to timed tests or speed.

Progression

- Students might use the commutative property of addition to change ? + 6 = 15 to 6 + ? = 15, then count on or use methods to compose 4 (to make ten) plus 5 (ones in the 15) to find 9.
- Students might reverse the action in the situation represented by ? - 6 = 9 so that is becomes 9 + 6 = ?. Or they might use their knowledge that the total is the first number in a subtraction equation and the last number in an addition equation to rewrite the situation equation as a solution equation: ? - 6 = 9 becomes 9 + 6 = ? or 6 + 9 = ?. (Please reference page 16 in the Progression document).

Examples

- Use strategies such as counting on; making ten, for example 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14; decomposing a number leading to a ten for example, 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9;
- Use the relationship between addition and subtraction, for example, knowing that 8 + 4 = 12, one knows 12 – 8 = 4;
- Create equivalent but easier or known sums, for example, adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13.
- Illustrative Mathematics:
  - Making a ten
  - $20 Dot Map
- Student Achievement Partners:
  - Addition and Subtraction Fluency Set of Tasks

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.OA.D - Work with addition and subtraction equations.

STANDARD: 1.OA.D.7

Standards Statement (2021):

Use the meaning of the equal sign to determine whether equations involving addition and subtraction are true or false.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	1.OA.D.8, 2.OA.C.3, 2.OA.C.4	N/A	1.OA.D.7 1.OA.D Crosswalk

Standards Guidance:

Clarifications

- Students should explore and explain the relationship of the equal sign to quantities and orally justify if equations involving addition and subtraction are "true" (equal) or "false" (not equal).

Teaching Strategies

- Use the meaning of the equal sign ("is the same as") to determine if two expressions involving a whole number and/or addition or subtraction expressions are equivalent.

Examples

- Determine if the equation is true or false, for example determining that 3-1 = 2+3 is false because the expressions do not have equal values.
- Which of the following equations are true and which are false? How do you know?
  - 6 = 6 (True/Correct Statement)
  - 7 = 8 – 1 (True/Correct Statement)
  - 5 + 2 = 2 + 5 (True/Correct Statement)
  - 4 + 1 = 5 + 2 (False/Incorrect Statement)
- Illustrative Mathematics:
  - 20 Tickets
  - Using lengths to represent equality
  - Valid Equalities?
  - Equality Number Sentences
  - The Very Hungry Caterpillar

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 1.OA.D.8

Standards Statement (2021):

Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.OA.D.7	1.OA.A.1	N/A	1.OA.D.8 1.OA.D Crosswalk

Standards Guidance:

Clarifications

- Determine the unknown whole number relating three whole numbers, with the unknown in any position.

Teaching Strategies

- Symbols can be used to represent unknown amounts in equations.

Progressions

- Students advancement of methods can be clearly seen in the context of situations with unknown addends. These are the situations that can be represented by an addition equation with one unknown addend, e.g., 9 + = 13. Students can start solving for some unknown addend problems by trial and error or by knowing the relevant decomposition of the total. But a more advanced counting on solution involves seeing the 9 as part of 13, and understanding that counting the 9 things can be "taken as done" if we begin the count from 9. (Please reference page 14 in the Progression document).

Examples

- Students should be given the opportunity to find missing part given a known part and total, such as:
  - A missing addend in an addition equation, for example 3+_=5.
  - A missing subtrahend in a subtraction equation, for example 5-_=2.
  - A missing difference in a subtraction equation, for example 5-3=_
- Students should be given the opportunity to find missing totals given known parts, such as:
  - A missing sum in an addition equation, for example 3+2=_.
  - A missing minuend in a subtraction equation, for example _-2=3.
- Determine the unknown number that makes the equation true in each of the equations: 8 + ? = 10, 5 = □ – 3, 3 + 4 = Δ. These are some possible ways to record equations that indicate an unknown number.
- Illustrative Mathematics:
  - Find the Missing Number
  - Kiri's Mathematics Match Game
- Student Achievement Partners:
  - Making a true equation

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.NBT.A - Extend the counting sequence.

STANDARD: 1.NBT.A.1

Standards Statement (2021):

Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NCC.A.1	1.NBT.B.2	N/A	1.NBT.A.1 1.NBT.A Crosswalk

Standards Guidance:

Clarifications

- Students should understand that as the counting sequence increases, the value of each number increases by one or ten. As the counting sequence decreases, the value of each number decreases by one or ten.
- Count forward and backward, starting at any number less than 120.

Teaching Strategies

- Students should count forwards and backwards by 1s and 10s from any number within 120.
- Skip count by twos to 20, by fives to 100, and by tens to 120.
- Students should have opportunities to explore the counting sequences using a variety of tools. These tools can include, but are not limited to 99 charts, hundred charts, number paths, number lines (predetermined and open), etc.

Examples

- Understand that two digit numbers are composed of tens and ones.
- Understand that 3 digit numbers are composed of hundreds, tens, and ones.
- Understand that numbers increase in consistent patterns because of the place value system.
- Illustrative Mathematics:
  - Choral Counting II
  - "Crossing the Decade" Concentration
  - Start/Stop Counting II
  - Where Do I Go?
  - Hundred Chart Digit Game
  - Number of the Day
- Student Achievement Partners
  - Continuing the pattern

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.NBT.B - Understand place value.

STANDARD: 1.NBT.B.2

Standards Statement (2021):

Understand 10 as a bundle of ten ones and that the two digits of a two-digit number represent amounts of tens and ones.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NBT.A.1, 1.NBT.A.1	1.NBT.B.3, 1.NBT.C.4, 1.NBT.C.5, 1.NBT.C.6, 2.NBT.A.1	N/A	1.NBT.B.2 1.NBT.B Crosswalk

Standards Guidance:

Clarifications

- Students should be able to recognize the relationship of a digit to its place indicates the number of groups represented in that place. For example: In the number 33, the digit "3" in the tens place has a value that is equivalent to three groups of ten. Students interpret the value of each digit. The number 33 has three tens and three remaining ones. They should also see this as equivalent to 33 ones.
- Students should understand the following as special cases:
  - 10 can be thought of as a bundle of ten ones — called a "ten." Bundles could include groups of pennies, bundles of straws, or other hands-on manipulatives.
  - The numbers from 11 to 19 are composed or decomposed as a ten and one, two, three, four, five, six, seven, eight, or nine ones.

Boundaries

- Students should be able to explain that the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

Teaching Strategies

- The numbers 11 to 19 can be represented on ten frames, double ten frames, rekenreks, and with pennies and dimes, etc.
- The numbers 10, 20, 30, 40, 50, 60, 70, 80, and 90, can be represented using a variety of tools (popsicle sticks, linking cubes, straws, etc.)

Progressions

- First graders learn that the two digits of a two-digit number represent amounts of tens and ones, e.g., 67 represents 6 tens and 7 ones. Saying 67 as "6 tens, 7 ones" as well as "sixty-seven" can help students focus on the tens and ones structure of written numerals. (Please reference page 6 in the Progression document).

Example

- Illustrative Mathematics:
  - Roll & Build
  - The Very Hungry Caterpillar
- Student Achievement Partners
  - How many ones and tens?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 1.NBT.B.3

Standards Statement (2021):

Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.NCC.B.5, K.NCC.C.7, 1.NBT.B.2	2.NBT.A.4	N/A	1.NBT.B.3 1.NBT.B Crosswalk

Standards Guidance:

Clarifications

- Students should understand whole numbers to 100 based on meanings of the tens and ones and record the results of comparisons with the symbols >, =, and <.

Boundaries

- Students should have ample experiences communicating their comparisons using words, models AND context before using only symbols in the learning objective.
- Students need practice justifying comparisons with words and models, prior to exposure and use of the comparison symbols.

Teaching Strategies

- Representations should include the use of physical materials such as number paths, base-ten materials, number lines (predetermined and open), dimes and pennies, etc.

Progressions

- Grade 1 students use their base-ten work to help them recognize that the digit in the tens place is more improtant for determining the size of a two-digit number.
- Correctly placing the < and > symbols is a challenge for early learners. Accuracy can improve if students think of putting the wide part of the symbol next to the larger number. (Please reference page 6 in the Progression document).

Examples

- Students should be given the opportunity to provide explanations of their results based on their understanding of place value.
    - 2 tens + 9 ones < 3 tens + 2 ones
    - 2 tens and 9 ones < 92
- Understand that a greater value in a given place supersedes any amount in a place with a smaller value.
- Illustrative Mathematics:
    - Comparing Numbers
    - Ordering Numbers

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.NBT.C - Use place value understanding and properties of operations to add and subtract.

STANDARD: 1.NBT.C.4

Standards Statement (2021):

Add within 100 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain why sometimes it is necessary to compose a ten.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.NBT.B.2	2.NBT.B.5	1.OA.C.6, 2.OA.A.1	1.NBT.C.4 1.NBT.C Crosswalk

Standards Guidance:

Clarifications

- Students should be able to interpret and manipulate concrete mathematical models.
- Students should be given opportunities to justify their solutions to meet this learning objective.
- Students should use estimation as a strategy to find numbers that are close to the numbers they are using to add and subtract.
- The numerical reasoning developed should include an understanding of the base-ten structure and properties of operations.
- Students should reason that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to put together (compose) or break apart (decompose) a ten.

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
  - Compose – put together numbers
  - Decompose – break apart numbers
  - Estimate – find a value that is close

Boundaries

- The properties of operation that should be explored in this objective are the commutative and associative properties. Students are not expected to identify properties.

Teaching Strategies

- Students should use concrete models, drawings, estimation, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction to explain their reasoning.
- Strategies may include reasoning involving making a ten, doubles and near-doubles, think addition, and using benchmark numbers.

Examples

- Illustrative Mathematics:
  - Ford and Logan Add 45+36
- Student Achievement Partners: [Addition within 100] [Making a true equation]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 1.NBT.C.5

Standards Statement (2021):

Without having to count, mentally find 10 more or 10 less than a given two-digit number and explain the reasoning used.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.NBT.B.2	2.NBT.B.5	2.OA.A.1	1.NBT.C.5 1.NBT.C Crosswalk

Standards Guidance:

Boundaries

- This expectation requires students to apply this mental strategy and become fluent through purposeful practice. The goal is automaticity built on a deep understanding of the patterns of tens within our base-ten system.

Examples

- Find 10 more than a given two digit number, for example 34 + 10.
- Find 10 less than a given two digit number, for example 34-10.
- Understand that adding or subtracting multiples of 10 from a number changes only the tens digit because the addition or subtraction changes only the quantity of tens.
- There were 74 birds in the park. 10 of the birds flew away. How many birds are in the park, now?
    - I pictured 7 ten-frames and 4 left over in my head. Since 10 birds flew away, I took one of the ten-frames away. That left 6 ten-frames and 4 left over. So, there are 64 birds left in the park.
- Illustrative Mathematics:
    - Number Square

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 1.NBT.C.6

Standards Statement (2021):

Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy and model used to a written method and explain the reasoning used.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.NBT.B.2	2.NBT.B.5	2.OA.A.1	1.NBT.C.6 1.NBT.C Crosswalk

Standards Guidance:

Boundaries

- By the end of first grade, students should be able to state and write their justifications showing the relationship between their solution path and their reasoning. The focus of this standard is on thought processes, not merely on computational accuracy.

Teaching Strategies

- Students should use concrete models; drawings, and strategies based on place value, properties of operations, and or/the relationship between addition and subtraction to explain their reasoning.
- Students should describe sums and differences, using concrete models (tools and manipulatives), drawings, and strategies based on place value, properties of operations and/or the relationship between addition and subtraction to explain (verbally and/or written) the reasoning used.

Progressions

- Differences of multiples of 10, such as 70 – 40 can be viewed as 7 tens minus 4 tens and represented with concrete models such as objects bundled in tens or drawings.
- Students use the relationship between subtraction and addition when they view 80 – 70 as an unknown addend addition problem, 70 + <box> = 80, and reason that 1 ten must be added to 70 to make 80, so 80 –70 = 10. (Please reference page 7 in the Progression document).

Examples

- Represent subtraction of multiples of 10 with concrete and/or visual models based on place value. For example, represent 30 as 3 groups of ten and no ones.
- Understand that the inverse relationship between subtraction and addition exists because both are different representations of the same part-part-whole relationship. For example, understand that both 20+30=50 and 50-20=30 represent the same parts and whole.
- Student Achievement Partners:
  - Place Value Tens
  - Subtracting multiples of 10

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.GM.A - Reason with shapes and their attributes.

STANDARD: 1.GM.A.1

Standards Statement (2021):

Distinguish between defining attributes versus non-defining attributes for a wide variety of shapes. Build and draw shapes to possess defining attributes.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.GM.B.4, K.GM.B.5	2.GM.A.1	N/A	1.G.A.1 1.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students should distinguish between defining attributes of two-dimensional shapes and three-dimensional figures versus non-defining attributes (e.g., triangles are closed and three-sided, a defining attribute versus triangles are red, non-defining attribute).
- Students should be able to build and draw shapes based on defining attributes. Two-dimensional shapes should be limited to triangles, squares, and rectangles.
- Students should be able to identify a shape's attributes, regardless of its orientation (i.e., flipped) or position (i.e., turned).

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Attributes – characteristics of two-dimensional shapes and three-dimensional figures, including geometric properties.
    - Defining attributes – include number of sides, faces, vertices (corners), and angles.
    - Non-defining attributes – include size, orientation, texture, and color.
- Students should identify these two-dimensional shapes based on attributes:
    - half circles, quarter circles, circles, triangles, squares, rectangles (Students should know that a square is a type of rectangle, based on its attributes.), hexagons
- Students should identify these three-dimensional shapes based on attributes:
    - Cubes, cones, cylinders, spheres, rectangular prisms

Examples

- Students differentiate between geometrically defining attributes (e.g., "hexagons have six straight sides") and nondefining attributes (e.g., color, overall size, or orientation). For example, they might say of this shape, "This has to go with the squares, because all four sides are the same, and these are square corners. It doesn't matter which way it's turned". (Please reference page 8 in the Progression document).
- Illustrative Mathematics:
    - Shape Sort
    - All vs. Only some
- Student Achievement Partners:
    - What Is a Hexagon?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 1.GM.A.2

Standards Statement (2021):

Compose common two-dimensional shapes or three-dimensional shapes to create a composite shape, and create additional new shapes from composite shapes.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.GM.B.6	1.GM.A.3, 3.GM.C.5, 4.GM.A.3	N/A	1.G.A.2 1.GM.A Crosswalk

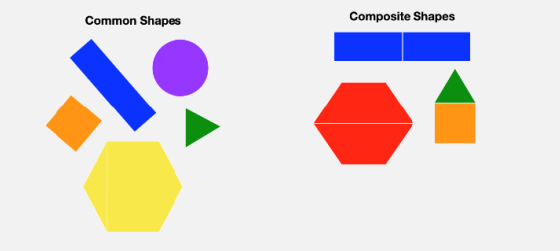
Standards Guidance:

Clarifications

- It is important to note that the size of the shape does not necessary distinguish between common and composite. Students do not need to learn formal names, such as, “right rectangular prism”.

Terminology

- Shapes that are made up of two or more common shapes are called composite shapes.
- Students will be working with shapes to compose and decompose shapes to form new shapes.
    - Compose – put together
    - Decompose – break apart



Boundaries

- Students should use these common two-dimensional shapes to create composite shapes:
    - Circles, half-circles, quarter-circles, triangles, squares, rectangles (Students should know that a square is a type of rectangle, based on its attributes.), hexagons, trapezoids
- Students should use these common three-dimensional shapes to create composite shapes:
    - Cubes, cones, cylinders, spheres, rectangular prisms, right circular cones, right circular cylinders

Progressions

- From the early beginnings of informally matching shapes and solving simple shape puzzles, students learn to intentionally compose and decompose plane and solid figures (e.g., putting two congruent isosceles triangles together with the explicit purpose of making a rhombus), building understanding of part-whole relationships as well as the properties of the original and composite shapes. In this way, they learn to perceive a combination of shapes as a single new shape (e.g., recognizing that two isosceles triangles can be combined to make a rhombus, and simultaneously seeing the rhombus and the two triangles). (Please reference page 8 in the Progression document).

Examples

- Illustrative Mathematics:
    - Counting Squares
    - Overlapping Rectangles
    - Grandfather Tang's Story
    - Make Your Own Puzzle

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 1.GM.A.3

Standards Statement (2021):

Partition circles and rectangles into two and four equal shares. Describe the equal shares and understand that partitioning into more equal shares creates smaller shares.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.GM.A.2	2.GM.A.3	N/A	1.G.A.3 1.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students should explore and justify reasoning about the relationship of parts to the whole.
- Students should describe the shares using the words “halves,” “fourths” or “quarters.”
- Students should describe the whole as “two of” or “four of” the shares.
- Students should reason that partitioning a shape into more equal shares creates smaller shares.

Boundaries

- No shading of the shares is needed for this learning objective because the student is only required to partition the whole shape into equal shares.
- Students are not expected to write the fraction using fraction notation in first grade.

Examples

- Describe the equal shares created using the words halves, fourths, and quarters.
- Relate the equal shares to the whole using the phrases half of, fourth of, and quarter of.
- Describe the whole as two of, or four of the shares.
- Understand that halves and fourths are equal parts of a partitioned whole.
- Illustrative Mathematics:
  - Equal Shares
- Student Achievement Partners:
  - Shapes divided into quarters

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.GM.B - Describe and compare measurable attributes.

STANDARD: 1.GM.B.4

Standards Statement (2021):

Order three objects by length; compare the lengths of two objects indirectly by using a third object.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.GM.C.8	1.GM.B.5	N/A	1.MD.A.1 1.GM.B Crosswalk

Standards Guidance:

Clarifications

- Students should express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end, by using non- standard units.
- Students should explore this concept with objects found in the real world to develop solid measurement reasoning.

Terminology

- Length measurement of an object is the number of same- sized length units that span an object with no gaps or overlaps (iteration).
- Iteration –the process of repeating a unit length end to end along an object to obtain a measurement.
- Transitivity can be explicitly discussed: If A is longer than B and B is longer than C, then A must be longer than C as well. (Please reference page 8 in the Progression document).

Boundaries

- Students should learn through exploration that the length measurement of an object is the number of same-sized length units that span it with no gaps or overlaps (iteration). For example, when students are measuring the height of a vegetable plant in their classroom garden, they may use snap cubes put together to determine how tall the plant is.

Teaching Strategies

- Students should use terminology such as, but not limited to, “longer than”, “shorter than”, “same length as”, “taller than”, and “equal to”.
- Appropriate tools to measure non-standard units can be items such as one-inch paper clips, one-inch tiles, centimeter cubes, etc. The units need to correspond to standard units of measurement.

Examples

- Determine when an object is longer or shorter than another object.
- Compare two objects to a third and use those comparisons against the third object to compare the two objects.
- Students at an elementary school are maintaining an aquaponics garden. To measure the heights of the plants growing in their garden, they use snap cubes to determine how many cubes high the plant have grown.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 1.GM.B.5

Standards Statement (2021):

Express the length of an object as a whole number of non-standard length units, by laying multiple copies of a shorter object (the length unit) end to end.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.GM.B.4	2.GM.B.4	N/A	1.MD.A.2 1.GM.B Crosswalk

Standards Guidance:

Boundaries

- Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.
- Include use of standard units such as inch-tiles or centimeter tiles.

Teaching Strategies

- Estimate, measure, and record lengths of objects using non-standard units, and compare and order up to three objects using the recorded measurements.
- Use a shorter object to measure the length of a longer object.
- Record the length of an object as the total number of shorter objects it takes to span the longer object without gaps or overlaps.

Examples

- Illustrative Mathematics:
  - Measure Me!
  - Measuring Blocks
  - Growing Bean Plants
  - How Long
- Student Achievement Partners:
  - Measuring Length

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.GM.C - Tell and write time.

STANDARD: 1.GM.C.6

Standards Statement (2021):

Tell and write time in hours and half-hours using analog and digital clocks.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	2.GM.D.10	N/A	1.MD.B.3 1.GM.C Crosswalk

Standards Guidance:

Clarifications

- The familiarity of the number line provides students with an opportunity to make sense of the concept of elapsed time. The connection to the traditional clock can be made by bending the clock number line into a circle.

Boundaries

- Students should tell and write time to the hour and half hour in everyday settings, paying attention to a.m. and p.m.
- Problems presented to students should avoid crossing over a.m. and p.m.
- Students are not required to know the term elapsed time at this grade level.

Teaching Strategies

- Begin with a one-handed clock (just the hour hand) and use a lot of approximate language such as:
  - "It's close to 10:00."
  - "It's half-way between 11:00 and 12:00."
  - "It's just a little after 1:00."
- Connect using a number line to tell time with how the number line can be curved to look like a circular clock.

Examples

- Tell time in hours and half hours using an analog clock.
- Tell time in hours and half hours using a digital clock.
- Write time in hours and half-hours.
- At 3:00 PM we are going to the trampoline park. We will be there for 4 hours. What time will we be leaving the trampoline park? Represent this on a number line. (It will be 7:00 when we leave the trampoline park).
- Illustrative Mathematics:
  - Making a clock

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.DR.A - Pose investigative questions and collect/consider data.

STANDARD: 1.DR.A.1

Standards Statement (2021):

Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by representing data visually.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.DR.A.1	2.DR.A.1	N/A	[new content] 1.DR Crosswalk

Standards Guidance:

Clarifications

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.

Terminology

- Collecting data would refer to student generating data sets, such as counting and recording the frequency of an event.
- Considering data refers to existing data sets given to students by a teacher for consideration.
- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - A statistical investigative question is one that requires data that will vary.

Boundaries

- Determine strategies for collecting and organizing data within 20 to answer a statistical investigative question.
- This standard should be taught throughout the year.

Teaching Strategies

- Students should use tally marks and numerical values within 20 to organize and represent the data.
- Developing strategies for collecting data include students collaborating to determine ways to collect data.
- Data can be gathered from a variety of sources to answer the statistical investigative question posed.

Progressions

- Students should formulate a statistical investigative question to explore a real-life situation in their classroom.

Examples

- “How many pets do you have?” is a statistical investigative question because it anticipates variability in students’ responses.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 1.DR.B - Analyze, represent, and interpret data.

STANDARD: 1.DR.B.2

Standards Statement (2021):

Analyze data sets with up to three categories by representing data visually, such as with graphs and charts, and interpret information presented to answer investigative questions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
K.DR.B.2	2.DR.B.2	1.OA.A.1, 1.OA.A.2, K.GM.C.8	1.MD.C.4 1.DR Crosswalk

Standards Guidance:

Clarifications

- Students should be able to organize the data collected, represent the data on a table, and ask questions about the data generated."
- Understand that the sum of the data points in all categories is the total number of data points.
- Understand that data representations make data points easier to read, count, and compare.
- Understand that the number of data points in different categories can be compared using subtraction, counting on, or counting back between the quantities.
- Students should be able to organize the data collected, represent the data on a table, and ask questions about the data generated.

Boundaries

- This expectation is limited to data with up to three categories presented in graphs and charts.
- Students should be using tally marks and numerical values to organize and represent data.
- Students should be able to summarize the number of tally marks in each category.

Teaching Strategies

- Create a picture graph and a bar graph (with single-unit scale) to represent a data set with up to three categories. Analyze the information by asking and answering questions about the data.
- Interpret categorical data to answer the statistical investigative question created, including total number of data points, how many in each category, and how many more or less are in one category than another.

Progressions

- Students' data work in Grade 1 has important connections to addition and subtraction. Students can ask and answer questions about categorical data based on a representation of the data.
- Students can also ask and answer questions leading to other kinds of addition and subtraction problems (1.OA), such as compare problems or problems involving the addition of three numbers (for situations with three categories). (Please reference page 5 in the Progression document).

Examples:

- Illustrative Mathematics:
  - [Weather Graph Data] [Favorite Ice Cream Flavor]
- Student Achievement Partners:Organize, represent, and interpret data

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4C: Grade 2 Math Standards and Guidance

Critical Areas of Focus

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Domains and Clusters

2.OA - Algebraic Reasoning: Operations

- 2.OA.A Represent and solve problems involving addition and subtraction.
- 2.OA.B Add and subtract within 20.
- 2.OA.C Work with equal groups of objects to gain foundations for multiplication.

2.NBT - Numeric Reasoning: Base Ten Arithmetic

- 2.NBT.A Understand place value
- 2.NBT.B Use place value understanding and properties of operations to add and subtract.

2.GM - Geometric Reasoning and Measurement

- 2.GM.A Reason with shapes and their attributes.
- 2.GM.B Measure and estimate lengths in standard units.
- 2.GM.C Relate addition and subtraction to length.
- 2.GM.D Work with time and money.

2.DR - Data Reasoning

- 2.DR.A Pose investigative questions and collect/consider data.
- 2.DR.B Analyze, represent, and interpret data.

Students should spend the large majority[1] of their time on the major work of the grade (■). Supporting work (□) and, where appropriate, additional work (○) can engage students in the major work of the grade.

[1]*At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see the K-8 major work of the grade developed by Student Achievement Partners.*

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.OA.A - Represent and solve problems involving addition and subtraction.

STANDARD: 2.OA.A.1

Standards Statement (2021):

Use addition and subtraction within 100 to solve one- and two-step problems in authentic contexts by using drawings and equations with a symbol for the unknown.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.OA.A.1	3.OA.D.8	1.NBT.C.4, 1.NBT.C.5, 1.NBT.C.6, 2.NBT.B.5, 2.GM.C.8	2.OA.A.1 2.OA.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to solve one and two-step mathematical problems within 100 and represent the problem by using concrete materials, drawings, and equations with a symbol for the unknown number.

Boundaries

- Students should work with contextual, mathematical problems involving standard units of linear measurement (inches). Note: This is an ongoing process that will take much of the year.
- The sum of the numbers should be no greater than 1000.
- Because some grade 2 students are still developing proficiency with addition and subtraction, two-step problems should not involve more difficult situations, and be limited to single-digit addends.

Terminology

- Addition and Subtraction Situations by Grade Level are presented in Table 1 pictured here, which include:
  - adding to,
  - taking from,
  - putting together, taking apart, and
  - comparing, with unknowns in all positions.
- Please reference pages 9 and 18 in the Progression document for additional information.

Table 1: Addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown[1]**	**Addend Unknown[2]**
Put Together /Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version.* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? *"More" suggests wrong operation.* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Teaching Strategies

- Opportunities to engage with problem types should include adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.
- Students should be given the opportunity to explore and develop a variety of flexible strategies and algorithms.
- When solving problems, students should be given the opportunity to use concrete materials, drawings, tools, and part-whole reasoning strategies.
- Students should be able to solve contextual, mathematical problems involving the addition of up to four two-digit numbers using strategies based on place value, properties of operations and the relationship between addition and subtraction.

Progressions

- Problems should be presented through contexts to provide students with the opportunity to make sense of the mathematics.
- Students should be able to use strategies that are based on a deep understanding of place-value in order to meet this expectation.
- Problems presented may include money as a context.

Examples

- In the morning, there are 25 students in the cafeteria. 18 more students come in. After a few minutes, some students leave. If there are 14 students still in the cafeteria, how many students left the cafeteria? Write an equation for your problem.
- Illustrative Mathematics:
  - A Pencil and a Sticker
- Student Achievement Partners:
  - Two Snakes
  - Seashells
  - Representing and Solving Addition and Subtraction Problems Mini-Assessment

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.OA.B - Add and subtract within 20.

STANDARD: 2.OA.B.2

Standards Statement (2021):

Fluently add and subtract within 20 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.OA.C.6	2.NBT.B.5	N/A	2.OA.B.2 2.OA.B Crosswalk

Standards Guidance:

Clarification

- The word "fluent" is used in the Standards to mean " accurate, efficient, and flexible."
- Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., "adding 0 yields the same number") and knowing some answers from the use of strategies.
- Fluency is not a matter of instilling facts divorced from their meanings, but rather as an outcome of a multi-year process that heavily involves the interplay of practice and reasoning.

Terminology

- Fluently/Fluency – To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.
- Accuracy includes attending to precision.
- Efficiency includes using well-understood strategy with ease.
- Flexibility involves using strategies such as making 5 or making 10.

Boundaries

- Students build on strategies within 10 from first grade to build towards fluency to 20.
- This standard does not require timed assessments.

Teaching Strategies

- Students add and subtract within 20 using a variety of mental, part-whole strategies.
- Students should explain their approaches and produce accurate answers efficiently and appropriately using mental strategies that include counting on, making ten, decomposing a number leading to a ten, using the relationship between addition and subtraction, creating equivalent but easier or known sums.

Examples

- Illustrative Mathematics:
  - Hitting The Target Number
  - Building toward fluency
- Student Achievement Partners:
  - Addition Facts
  - Addition and Subtraction within 20 Mini-Assessment

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.OA.C - Work with equal groups of objects to gain foundations for multiplication.

STANDARD: 2.OA.C.3

Standards Statement (2021):

Determine whether a group up to 20 objects has an odd or even number by pairing objects or counting them by 2s; record using drawings and equations including expressing an even number as a sum of two equal addends.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.OA.D.7	3.OA.A.1, 3.OA.D.9	2.NBT.A.2	2.OA.C.3 2.OA.C Crosswalk

Standards Guidance:

Terminology

- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Addend – any number that is added to another number in an addition expression or equation. For example, in the expression 16 + 4, 16 and 4 are addends.

Teaching Strategies

- Students should explore strategies such as pairing objects, counting by 2s, and drawing arrays to express doubles.
- Students should write an equation to express an even number as a sum of equal addends and as a sum of repeated pairings
- Students can group by pairing objects or counting them by 2s.
- Students may also use doubles to determine if a quantity is even. For example, 18 is even because adding two nines equals 18 or 9 + 9 =18.

Examples

- 12 is even because 6+6= 12 and also because 2+2+2+2+2+2=12 so that 12 is being represented as two groups of six or six groups of two.
- Illustrative Mathematics:
    - Red and Blue Tiles
    - Buttons odd and even

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.OA.C.4

Standards Statement (2021):

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.OA.D.7	3.OA.A.1	N/A	2.OA.C.4 2.OA.C Crosswalk

Standards Guidance:

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
  - Rectangular array – an arrangement of objects into rows and columns that form a rectangle
  - Addend – any number that is added to another number in an addition expression or equation. For example, in the expression 2 + 7 + 5=14, (2, 7 and 5) are addends.

Boundaries

- The intent of the standard is to provide students the opportunities to work with arrays and connect them to repeated addition and equal groupings as a foundation to multiplication.

Teaching Strategies

- Students should model using rectangular arrays to determine the number of objects and discuss their reasoning.

Examples

- Students should have the opportunity to recognize that the total in the array is the same whether adding by row or adding by column. For example, an array with 5 rows and 4 columns could be represented as 5+5+5+5 and 4+4+4+4+4 and results in the same total of 20.
- Illustrative Mathematics:
  - Counting Dots in Arrays
  - Delayed Gratification
- Student Achievement Partners:
  - Total number of circles

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.NBT.A - Understand place value.

STANDARD: 2.NBT.A.1

Standards Statement (2021):

Understand 100 as a bundle of ten tens and that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.NBT.B.2, 2.NBT.A.2	2.NBT.A.3, 2.NBT.A.4, 2.NBT.B.6, 2.NBT.B.7, 2.NBT.B.8, 3.NBT.A.1, 3.NBT.A.3, 4.NBT.A.1	N/A	2.NBT.A.1 2.NBT.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to put together (compose) and break apart (decompose) three-digit numbers.
- Students should have multiple opportunities use concrete materials to develop an understanding of the place value structures, the relationship between numbers, and the value of quantities.

Teaching Strategies

- Students should be given the opportunity to discover base-ten units can be broken down and built back up in different ways. For example, understand the number 706 can be represented as:
  - 7 hundreds, 0 tens, and 6 ones where a 0 is used as a placeholder.
  - 70 tens and 6 ones.
  - 706 ones.
- Students should be able to explain that a bundle of ten 10s is equal to 100.

Progressions

- This content lays the groundwork for understanding the structure of the base-ten system as based in repeated bundling in groups of 10 and understanding that the unit associated with each place is 10 of the unit associated with the place to its right. (Please reference page 8 in the Progression document).

Examples

- Illustrative Mathematics:
  - [Bundling and Unbundling] [Boxes and Cartons of Pencils] [Counting Stamps] [Largest Number Game] [Making 124] [One, Ten, and One Hundred More and Less] [Regrouping] [Three Composing/Decomposing Problems] [Ten $10s Make $100] [Party Favors]
- Student Achievement Partners:
  - Understanding Place Value within 1000 Mini-Assessment
  - Place Value Units

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.NBT.A.2

Standards Statement (2021):

Count within 1000; skip-count by 5's, 10's, and 100's.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	2.NBT.A.1	2.OA.C.3	2.NBT.A.2 2.NBT.A Crosswalk

Standards Guidance:

Teaching Strategies

- Students need to be provided the opportunity to count and skip count both forward and backward starting from any number within 1000 to notice patterns within the number system.
- Students should explore patterns on a hundred-chart, starting from a given number 10-90.
- Students should be able to use coins to count, including nickels, dimes, quarters, and dollars. Half-dollars may also be used, if available.

Examples

- Illustrative Mathematics:
  - Saving Money 2
- Student Achievement Partners:
  - Understanding Place Value within 1000 Mini-Assessment

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.NBT.A.3

Standards Statement (2021):

Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.NBT.A.1	4.NBT.A.2	N/A	2.NBT.A.3 2.NBT.A Crosswalk

Standards Guidance:

Boundaries

- Students should be able to represent a quantity from word form.

Teaching Strategies

- Representations should include concrete materials (i.e., base ten blocks, counters, etc.), base ten numerals, words, expanded form, and pictures.

Progressions

- Representations such as manipulative materials, math drawings, and layered three-digit place value cards afford connections between written three-digit numbers and hundreds, tens, and ones...
- Unlayering three-digit place value cards... reveals the expanded form of the number.

Examples

- The number 706 in base-ten numerals is represented as 7 hundreds, 0 tens, and 6 ones, in number names is represented as "seven hundred six" and in expanded form is represented as 700 + 6.
- The number two-hundred forty-one written in standard form is 241 and in expanded form is 200+40+1.
- Illustrative Mathematics:
  - Looking at Numbers Every Which Way
- Student Achievement Partners:
  - Understanding Place Value within 1000 Mini-Assessment
  - Recognize numbers in different forms

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.NBT.A.4

Standards Statement (2021):

Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.NBT.B.3, 2.NBT.A.1	4.NBT.A.2	N/A	2.NBT.A.4 2.NBT.A Crosswalk

Standards Guidance:

Teaching Strategies

- Tools such as a hundred chart and visual number lines may be used to help students compare three digit numbers.

Progressions

- Comparing magnitude of three-digit numbers uses the understanding that 1 hundred (the smallest three-digit number) is greater than any amount of tens and ones represented by a two-digit number. For this reason, three-digit numbers are compared by first inspecting the hundreds place (e.g., 845 > 799; 849 < 855). Drawings help support these understandings. (Please reference page 8 in the Progression document).

Examples

- Students should be given the opportunity to provide explanations of their results based on their understanding of place value, for example:
  - 2 hundreds + 3 ones > 5 tens + 9 ones
  - 9 tens + 2 hundreds + 4 ones < 924
  - 456 < 5 hundreds
- Illustrative Mathematics:
  - Comparisons 1
  - Comparisons 2
  - Number Line Comparisons
  - Ordering 3-digit numbers
  - Digits 2-5-7
  - Using Pictures to Explain Number Comparisons
- Student Achievement Partners:
  - Compare two three-digit numbers

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.NBT.B - Use place value understanding and properties of operations to add and subtract.

STANDARD: 2.NBT.B.5

Standards Statement (2021):

Fluently add & subtract within 100 using accurate, efficient, & flexible strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.NBT.C.4, 1.NBT.C.5, 1.NBT.C.6, 2.OA.B.2	3.NBT.A.2	2.OA.A.1	2.NBT.B.5 2.NBT.B Crosswalk

Standards Guidance:

Terminology

- This standard uses the word fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property or partial sums).
- Fluently/Fluency – To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Boundaries

- Students should be given multiple opportunities to solve contextual, mathematical problems as they work to build fluency.
- The sum of the number should be no greater than 100.

Progressions

- Students should be able to use numerical reasoning to solve contextual, mathematical problems involving all problem types.

Examples

- Students should move from count all toward strategies that are efficient, accurate, and flexible based on the math situation presented. For example:
  - 56+48=50+40+6+8=90+14=104
  - 56+48=54+2+48=54+50=104
  - 56-48 can be thought of as 48+x=56
- Student Achievement Partners:
  - Curious Subtraction Task
  - Subtracting within 100

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.NBT.B.6

Standards Statement (2021):

Add up to four two-digit numbers using strategies based on place value and properties of operations and describe how two different strategies result in the same sum.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.NBT.A.1, 2.NBT.B.7	3.NBT.A.2	N/A	2.NBT.B.6 2.NBT.B Crosswalk

Standards Guidance:

Clarifications

- Students should investigate repeating patterns to make predictions and build algebraic reasoning.
- Patterns may include exposure to even and odd.
- Students should be using any tools available such as a number line, hundred-chart, 99-chart, etc., to create and analyze the patterns.
- Patterns should be extended from 1st grade, where they explore intervals of 1s, 2s, 5s, and 10s, to also include intervals of 25s and 100s.

Boundaries

- Patterns involving addition and subtraction should include sums within 1,000 through models and representations.
- Problems presented may include money as a context.

Teaching Strategies

- Students should be given the opportunity to use a variety of strategies to identify, describe, and create numerical patterns.
- Students describe how two different strategies result in the same sum

Progressions

- Problems should be presented through contexts to provide students with the opportunity to make sense of the mathematics.
- This work affords opportunities for students to see that they may have to compose more than one ten, and as many as three new tens. (Please reference page 11 in the Progression document).

Examples

- Students should be given the opportunity to connect representations. For example:
  - 42 + 31 + 12 + 83 may be decomposed into tens and ones to add 40 + 30 + 10 + 80 and then 2 + 1 + 2 + 3.
  - 42+31= 73 and 12+83= 95 so 73+95= 168.
- Start with 3 and jump by 5s to create a pattern. Change the start number and create another pattern. What do you notice about the two patterns? How did they change?
- Illustrative Mathematics:
  - Toll Bridge Puzzle

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.NBT.B.7

Standards Statement (2021):

Add and subtract within 1000 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain why sometime

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.NBT.A.1	2.NBT.B.6, 2.NBT.B.8, 2.NBT.B.9, 3.NBT.A.2	N/A	2.NBT.B.7 2.NBT.B Crosswalk

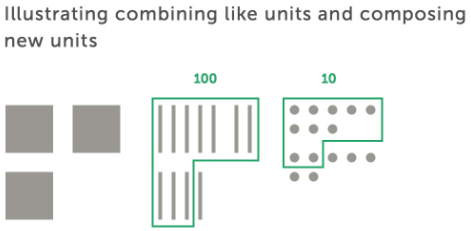
Standards Guidance:

Teaching Strategies

- Students should be encouraged to use place value language such as hundreds, tens and ones, when connecting their representation to their explanation.
- Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Progressions

- Drawings can support students in explaining [methods for addition within 1000] how addends can be decomposed into their base-ten units (e.g. hundreds, tens, and ones).
- The drawing below shows the base-ten units of 278 and 147. Like units are shown together, with boundaries drawn around the newly composed hundred and the newly composed ten. The newly composed units could also be indicated by crossing out grouped units and drawing the next highests unit (e.g. crossing out the group of ten ones and drawing a single ten).



- The putting together of quick drawings can illustrate adding adding like units as specified in 2.NBT.[B.]7: add ones to ones, tens to tens, and hundreds to hundreds. (Please reference pages 9 and 10 in the Progression document)

Examples:

- Students may use equations to represent their strategies based on place value such as: 324+515=(300+500)+(20+10)+(4+5)=839.
- Illustrative Mathematics:
  - How Many Days Until Summer Vacation?
  - Many Ways to do Addition 2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.NBT.B.8

Standards Statement (2021):

Without having to count, mentally find 10 more or 10 less and 100 more or 100 less than a given three-digit number.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.NBT.A.1, 2.NBT.B.7	3.NBT.A.2	N/A	2.NBT.B.8 2.NBT.B Crosswalk

Standards Guidance:

Boundaries

- Mental addition and subtraction is limited to adding or subtracting by 10 or 100 for numbers between 100-900.

Teaching Strategies

- Add and subtract within 1000 using properties of operations and/or the relationship between addition and subtraction, including mentally adding or subtracting 10 or 100 to a given number;
- Relate the strategies used to a written method.
- Tools such as a hundred chart and visual number lines may be used to help students discover the patterns of ten more and ten less.

Examples

- Illustrative Mathematics:
  - Choral Counting

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.NBT.B.9

Standards Statement (2021):

Explain why strategies to add and subtract work using properties of operations and the relationship between addition and subtraction.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.NBT.B.7	3.NBT.A.2	1.OA.B.3	2.NBT.B.9 2.NBT.B Crosswalk

Standards Guidance:

Clarification

- Provide many activities that will help students develop a strong understanding of number relationships, addition and subtraction so they can develop, share and use efficient strategies for computation.
- Students gain computational fluency, using efficient and accurate methods for computing, as they come to understand the role and meaning of arithmetic operations in number systems.

Teaching Strategies

- Explanations may be supported by drawings or objects.
- Make anchor charts/posters for student-developed mental strategies for addition and subtraction within 20.
- Use names for the strategies that make sense to the students and include examples of the strategies (e.g. making ten, doubling, etc).

Examples

- A student uses number talk to say “I know that 9 plus 4 equals 13. So 13 minus 9 equals 4”.
- When presented the problem, 4 + 8 + 6, the student uses number talk to say “I know 6 + 4 = 10, so I can add 4 + 8 + 6 by adding 4 + 6 to make 10 and then add 8 to make 18.”

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.GM.A - Reason with shapes and their attributes.

STANDARD: 2.GM.A.1

Standards Statement (2021):

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.GM.A.1	3.GM.A.1	N/A	2.G.A.1 2.GM.A Crosswalk

Standards Guidance:

Clarifications

- Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
- Sizes are compared directly or visually, not compared by measuring.

Terminology

- Attributes – characteristics of a two-dimensional or three-dimensional shape
- Vertices – corners of a geometric figure

Progressions

- Students should be able to use spatial reasoning to analyze shapes in the environment.
- Students learn to name and describe the defining attributes of categories of two-dimensional shapes,including circles, triangles, squares, rectangles, rhombuses, trapezoids, and the general category of quadrilateral.
- Students describe pentagons, hexagons, septagons, octagons and other polygons by their number of sides. They recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Please reference page 10 in the Progression document).

Examples

- Describe a shape based on its attributes and compare and sort a collection of shapes based on the number of angles, vertices, sides, and equal faces.
- Illustrative Mathematics:
  - Polygons
- Student Achievement Partners:
  - Identifying Quadrilaterals

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.GM.A.2

Standards Statement (2021):

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	2.GM.A.3, 3.GM.C.6	N/A	2.G.A.2 2.GM.A Crosswalk

Standards Guidance:

Clarifications

- As a foundation for multiplication and meaning of area, students should draw and build these arrays.

Boundaries

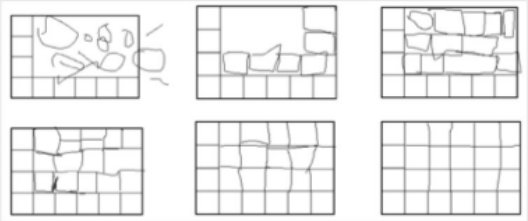
- Rectangles should be no larger than 5 rows and 5 columns.

Progressions

- Another type of composition and decomposition is essential to students' mathematical development- *spatial structuring*. Students need to understand how a rectangle can be tiled with squares lined up in rows and columns.
- Students learn to compose this two-dimensional shape as a collection of rows of squares and as a collection of columns of squares (MP7). Spatial structuring is thus the mental operation of constructing an organization or form for an object or set of objects in space. (Please reference pages 10 & 11 in the Progression document).

2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Levels of thinking in spatial structuring



Levels of thinking portrayed by different students as they attempted to complete a drawing of an array of squares, given one column and row. This was an assessment, not an instructional task.

Examples

- Illustrative Mathematics:
  - Partitioning a Rectangle into Unit Squares

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.GM.A.3

Standards Statement (2021):

Partition circles and rectangles into two, three, or four equal parts. Recognize that equal parts of identical wholes need not have the same shape.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.GM.A.3, 2.GM.A.2	3.GM.A.2, 3.GM.C.5	3.NF.A.1	2.G.A.3 2.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students have explored quarters and halves in first grade and are extending their understanding of fractions to thirds.
- As a foundation of the meaning of fractions, students should describe the shares using the words halves, thirds, fourths, half of, a third of, a fourth of and describe the whole as two halves, three thirds, four fourths.

Boundaries

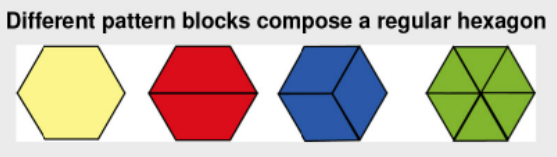
- Partitioning shapes prepares students to reason about fractions in upper grades.
- No shading should occur within images for this grade because the student is only required to partition the whole shape into equal shares.

Teaching Strategies

- Students are not expected to precisely partition circles into thirds, but rather partition circles and rectangles into thirds close enough to be described as three equal parts.

Progressions

- Students learn to combine their composition and decomposition competencies to build and operate on composite units (units of units), intentionally substituting arrangements or composites of smaller shapes or substituting several larger shapes for many smaller shapes, using geometric knowledge and spatial reasoning to develop foundations for area, fraction, and proportion. They recognize that the hexagonal faces of these constructions have equal area, that each trapezoid has half of that area, and each rhombus has a third of that area. (Please reference page 10 in the Progression document).



Examples

- Illustrative Mathematics:
  - Which Pictures Represent One Half?
  - Representing Half of a Rectangle
- Student Achievement Partners:
  - A fourth of a rectangle

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.GM.B - Measure and estimate lengths in standard units.

STANDARD: 2.GM.B.4

Standards Statement (2021):

Measure the length of an object by selecting and using appropriate measurement tools.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.GM.B.5	2.GM.B.5, 2.GM.B.6, 3.GM.B.4	N/A	2.MD.A.1 2.GM.B Crosswalk

Standards Guidance:

Boundaries

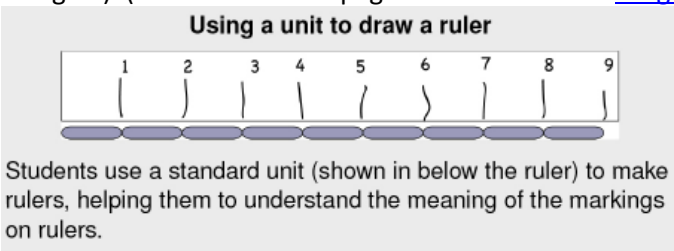
- Content expectation includes grade appropriate use of the customary measurement system of units (e.g. feet, inches, yards) and metric system of units (e.g. meters, centimeters, millimeters).

Teaching Strategies

- Appropriate standardized measurement tools include rulers, yardsticks, meter sticks, and measuring tapes.
- Students should determine which measuring tool is appropriate for a given object.
- Instruction would need to include simple conversions between common imperial units to comparable metric units of measurement.

Progressions

- It is vital that students learn that "one" represents the space from the beginning of the ruler to the hash mark, not the hash mark itself.
- To learn measurement concepts and skills, students might use both simple rulers (e.g., having only whole units such as centimeters or inches) and physical units (e.g., manipulatives that are centimeter or inch lengths). (Please reference pages 12 and 13 in the Progression document)



Students use a standard unit (shown in below the ruler) to make rulers, helping them to understand the meaning of the markings on rulers.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.GM.B.5

Standards Statement (2021):

Measure the length of an object using two different length units and describe how the measurements relate to the size of the unit chosen.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.B.4, 2.GM.B.6	3.GM.C.5, 3.NF.A.1	N/A	2.MD.A.2 2.GM.B Crosswalk

Standards Guidance:

Boundaries

- In Grade 1, students used one-inch items as non-standard units of measure for length. In Grade 2, students compare constructed ruler with standard rulers and compare the use of the devices.

Teaching Strategies

- Students may use objects but tools from different systems of measurement, such as inches and centimeters, will allow them to begin to compare these systems.
- Students should discuss how measurement with iterating individual one-inch units, such as one-inch tiles, compares with measurement using an instrument such as a standard ruler.

Progressions

- Students can learn that the larger the unit, the fewer number of units in a given measurement. That is, for measurements of a given length there is an inverse relationship between the size of the unit of measure and the number of those units. This is the time that measuring and reflecting on measuring the same object with different units, both standard and nonstandard, is likely to be most productive.
- Students can also use the concept of unit to make inferences about the relative sizes of objects; for example, if object A is 10 regular paperclips long and object B is 10 jumbo paperclips long, the number of units is the same, but the units have different sizes, so the lengths of A and B are different. (Please reference page 13 in the Progression document)

Examples

- Student Achievement Partners:
  - Use a ruler to measure the eraser

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.GM.B.6

Standards Statement (2021):

Estimate lengths using units of inches, feet, yards, centimeters, and meters.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.B.4	2.GM.B.5, 2.GM.B.7	N/A	2.MD.A.3 2.GM.B Crosswalk

Standards Guidance:

Teaching Strategies

- Students should be encouraged to use real world objects and body benchmarks for estimations.

Progressions

- Although "guess and check" experiences can be useful, research suggests explicit teaching of estimation strategies (such as iteration of a mental image of the unit or comparison with a known measurement) and prompting students to learn reference or benchmark lengths (e.g., an inch-long piece of gum, a 6-inch dollar bill), order points along a continuum, and build up mental rulers. (Please reference page 15 in the Progression document)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.GM.B.7

Standards Statement (2021):

Measure two objects and determine the difference in their lengths in terms of a standard length unit.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.B.6	2.GM.C.8	N/A	2.MD.A.4 2.GM.B Crosswalk

Standards Guidance:

Clarifications

- This is the first time students are introduced to a standard-length unit such as an inch.

Teaching Strategies

- Students should use tools such as rulers, measuring tapes, and yardsticks to obtain measurements.
- Comparisons in length are recorded in standard length units such as inches, feet or yards, as well as metric length units such as meters, centimeters, or millimeters.

Progressions

- Second graders learn to combine and compare lengths using arithmetic operations. That is, they can add two lengths to obtain the length of the whole and subtract one length from another to find out the difference in lengths. (Please reference page 14 in the Progression document).

Examples

- I measured my two pet parakeets. One was 7 inches long and one was 15 inches long. The larger one is 8 inches longer than the smaller one.
- Student Achievement Partners:
    - Use a ruler to measure a paper clip

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.GM.C - Relate addition and subtraction to length.

STANDARD: 2.GM.C.8

Standards Statement (2021):

Use addition and subtraction within 100 to solve problems in authentic contexts involving lengths that are given in the same units.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.B.7	2.GM.C.9, 2.GM.D.11	2.OA.A.1, 2.DR.A.1	2.MD.B.5 2.GM.C Crosswalk

Standards Guidance:

Teaching Strategies

- Students should represent the problem using drawings and equations with a symbol for the unknown number.

Progressions

- As an arithmetic example, students might measure all the sides of a table with unmarked (foot) rulers to measure how much ribbon they would need to decorate the perimeter of the table. (Please reference page 14 in the Progression document).

Examples

- Student Achievement Partners:
  - How tall is Kelsey's sister?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.GM.C.9

Standards Statement (2021):

Represent whole number lengths on a number line diagram; use number lines to find sums and differences within 100.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.C.8	2.GM.D.11, 3.GM.B.4, 3.NF.A.2	2.DR.A.1	2.MD.B.6 2.GM.C Crosswalk

Standards Guidance:

Clarifications

- Students should be able to represent sums and differences presented in contextual, mathematical problems on a number line diagram.

Boundaries

- This prepares students to use number lines for fractions in higher grades

Teaching Strategies

- Students should understand length as the distance on a number line where equally spaced points correspond to the numbers 0, 1, 2 and so on.
- The length of an object is the amount of space on this diagram.
- Students will use a number line to show how to move up and down the number system while representing sums and difference (100-28 means you would jump down 20 and 8 to land on 72).

Progressions

- To use a number line diagram to understand number and number operations, students need to understand that number line diagrams have specific conventions: the use of a single position to represent a whole number and the use of marks to indicate those positions.
- Students need to understand that a number line diagram is like a ruler in that consecutive whole numbers are 1 unit apart, thus they need to consider the distances between positions and segments when identifying missing numbers. (Please reference page 14 in the Progression document).

Examples

- Illustrative Mathematics:
  - Frog and Toad on the number line

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.GM.D - Work with time and money.

STANDARD: 2.GM.D.10

Standards Statement (2021):

Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.GM.C.6	3.GM.B.3	N/A	2.MD.C.7 2.GM.D Crosswalk

Standards Guidance:

Clarifications

- Students should be able to categorize daily activities by a.m. and p.m.

Boundaries

- Problems involving elapsed time in second grade should be written so as to avoid crossing over a.m. and p.m.

Teaching Strategies

- Connect using a number line to tell time with how the number line can be curved to look like a circular clock.

Examples

- Denise had soccer practice after school today. Practice began at 3:30 and ended at 6:00. How much time did she spend at soccer practice?
- Illustrative Mathematics:
  - Ordering Time

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 2.GM.D.11

Standards Statement (2021):

Solve problems in authentic contexts involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and c (cents) symbols appropriately.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.C.8, 2.GM.C.9	3.GM.C.6, 4.GM.B.4	3.NF.A.2, 2.DR.A.1	2.MD.C.8 2.GM.D Crosswalk

Standards Guidance:

Clarifications

- Students should be able to identify the values of pennies, nickels, dimes, and quarters. Half-dollars may also be investigated, if available.

Boundaries

- This is the first time students are required to find the value of a group of coins.
- The total quantity should be based on cents and the value of a group of coins should be less than 100 cents.
- Use of written decimal numbers is not an expectation for this grade level.
- The $ symbol should only be used when referring to whole dollar amounts at this grade level.
- Students should be able to solve contextual, mathematical problems that either have only dollars or only cents.
- Dollar bills may include $1, $5, $10, $20, and $100.

Teaching Strategies

- Students should be given opportunities to explore this concept using hands-on manipulatives. Virtual manipulatives may also be used.

Examples

- If you have 2 dimes and 3 pennies, how many cents do you have?
- If you have $3 and 4 quarters, how many dollars or cents do you have?
- Illustrative Mathematics:
  - Choices, Choices, Choices
  - Alexander, Who Used to be Rich Last Sunday
  - Jamir's Penny Jar
  - Pet Shop
  - Saving Money 1
  - Susan's Choice

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.DR.A - Pose investigative questions and collect/consider data.

STANDARD: 2.DR.A.1

Standards Statement (2021):

Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by using measurements with whole-number units.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.DR.A.1	3.DR.A.1	2.GM.C.8, 2.GM.C.9, 3.NF.A.2	2.MD.D.9 2.DR Crosswalk

Standards Guidance:

Clarification

- Students should formulate a statistical investigative question to explore a real-life situation in their classroom.
- Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object.
- Data could be organized and recorded on a line plot (dot plot) where the horizontal scale is marked off in whole-number units.

Terminology

- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - A statistical investigative question is one that requires data that will vary.

Boundaries

- The data collection can occur through the use of surveys and scientific observations.
- Tables and tally marks can be used to organize data.
- Developing strategies for collecting data include students collaborating to determine ways to collect data.
- Data can be gathered from a variety of sources to answer the statistical investigative question posed.

Teaching Strategies

- Students should display data set with up to four categories and solve problems that put-together, take-apart, and compare the information presented in the graph.

Progressions

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.

Examples

- Illustrative Mathematics:
    - Hand Span Measures
    - Growing Bean Plants

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 2.DR.B - Analyze, represent, and interpret data.

STANDARD: 2.DR.B.2

Standards Statement (2021):

Analyze data with a single-unit scale and interpret information presented to answer investigative questions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.DR.B.2	3.DR.B.2	N/A	2.MD.D.10 2.DR Crosswalk

Standards Guidance:

Clarification

- Investigative question identified can include those created by students or posed by the teacher.

Terminology

- Single unit scale -each unit represents one data point
- Pictograph – uses a symbol to represent the items being graphed. A pictograph has one picture represented in each category and has a key to give a numerical value to each picture.
- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Categorical data take on values that are names or labels, such as colors, foods, and types of pets
    - Pictograph – uses a symbol to represent the items being graphed. A pictograph has one picture represented in each category and has a key to give a numerical value to each picture.

Boundaries

- Pictographs and bar graphs used at this grade level should represent a data set with no more than four categories.

Teaching Strategies

- Students should solve simple join, separate, and compare problems using information presented.
- Students should use addition and subtraction to create and obtain information from tables, pictographs, bar graphs, and tally charts.

Examples

- Create a picture graph and a bar graph (with single- unit scale) to represent a data set with up to four categories.
- Student Achievement Partners:
    - Park Animals

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4D: Grade 3 Math Standards and Guidance

Critical Areas of Focus

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Domains and Clusters

3.OA - Algebraic Reasoning: Operations

- 3.OA.A Represent and solve problems involving multiplication and division.
- 3.OA.B Understand properties of multiplication and the relationship between multiplication and division.
- 3.OA.C Multiply and divide within 100.
- 3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3.NBT - Numeric Reasoning: Base Ten Arithmetic

- 3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.

3.NF - Numeric Reasoning: Fractions

- 3.NF.A Develop understanding of fractions as numbers.

3.GM - Geometric Reasoning and Measurement

- 3.GM.A Reason with shapes and their attributes.
- 3.GM.B Solve problems involving measurement and estimation.
- 3.GM.C Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- 3.GM.D Geometric measurement: recognize perimeter

3.DR - Data Reasoning (3.DR)

- 3.DR.A Pose investigative questions and collect/consider data.
- 3.DR.B Analyze, represent, and interpret data.

Students should spend the large majority[1] of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

[1]*At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see the K-8 major work of the grade developed by Student Achievement Partners.*

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.OA.A - Represent and solve problems involving multiplication and division.

STANDARD: 3.OA.A.1

Standards Statement (2021):

Represent and interpret multiplication of two factors as repeated addition of equal groups.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.OA.C.3, 2.OA.C.4	3.OA.B.5, 3.OA.A.3, 3.OA.B.6, 3.OA.A.2, 5.NF.B.4	5.NF.B.5, 5.NF.B.6	3.OA.A.1 3.OA.A Crosswalk

Standards Guidance:

Boundaries:

- Interpret the factors as representing the number of equal groups and the number of objects in each group. Describe a context in which a total number of objects can be expressed as __ x __.
- This standard does not include calculating products. It is about understanding the meaning of each of the factors in 5 x 7, not the product of 5 x 7.

Progressions

- The equation 3 x 6 = ? means how many are in 3 groups of 6 things each: three sixes. But in many other countries the equation 3 x 6 = ? means how many are 3 things taken 6 times (6 groups of 3 things each): six threes. Some students bring this interpretation of multiplication equations into the classroom. So it is useful to discuss the different interpretations and allow students to use whichever is used in their home. (Please reference page 25 in the Progression document).

Examples

- Student Achievement Partners:
  - Plums
  - Foundations of Multiplication and Division Mini-Assessment
  - Smarter Balanced Assessment Item Illustrating 3.OA.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.OA.A.2

Standards Statement (2021):

Represent and interpret whole-number quotients as dividing an amount into equal sized groups.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.A.1, 3.OA.A.3, 3.OA.B.6	3.OA.B.5	5.NF.B.3, 5.NF.B.5, 5.NF.B.6	3.OA.A.2 3.OA.A Crosswalk

Standards Guidance:

Clarifications

- Students should solve multiplication problems including single-digit factors and division problems including single- digit divisors and quotients.

Terminology

- This standard focuses on two models of division: partition models and measurement (repeated subtraction) models.
  - Partition models focus on "How many in each equal-sized group?"
  - Measurement (repeated subtraction) models focus on "How many groups can you make?".
- This standard does not include calculating. It is about understanding the meaning of what does 56 ÷ 8 mean, not the quotient of what does 56 ÷ 8 equal.

Boundaries

- Students should be able to use numerical reasoning to learn multiplication and division facts through playing games and solving contextual, mathematical problems.
- Fluency does not lend itself to timed tests or speed. Students should be given opportunities to choose flexibly among strategies to solve mathematical problems accurately and efficiently.

Teaching Strategies

- Multiplication strategies may include repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line and skip counting. Multiplication tables may be used to help students discover patterns and relationships.
- Division strategies may include repeated subtraction, equal sharing, and forming equal groups.

Progressions

- In Equal Groups, the roles of the factors differ. One factor is the number of objects in a group (like any quantity in addition and subtraction situations), and the other is a multiplier that indicates the number of groups. So, for example, 4 groups of 3 objects is arranged differently than 3 groups of 4 objects. Thus there are two kinds of division situations depending on which factor is the unknown (the number of objects in each group or the number of groups). (Please reference page 24 in the Progression document).

Examples

- Illustrative Mathematics:
  - Fish Tanks
  - Markers in Boxes

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.OA.A.3

Standards Statement (2021):

Use multiplication and division within 100 to solve problems in authentic contexts involving equal groups, arrays, and/or measurement quantities.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.A.1, 3.OA.A.2	3.OA.A.4, 3.OA.D.8, 4.OA.A.1, 4.OA.A.2	4.NF.B.4	3.OA.A.3 3.OA.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to solve practical, real-life division problems including "how many in each group" and "how many groups" using efficient and flexible strategies.
- 7 x 3 is known, then 3 x 7 is also known (Commutative Property)
- 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or 5 x 2 = 10, then 3 x 10 = 30 (Associative Property)
- Knowing 8 x 5 = 40 and 8 x 2 = 16, 8 x 7 can be found as the sum of these partial products: 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive Property)

Boundaries

- Solve multiplication word problems with factors up to and including 10.
- Solve division word problems with a divisor and quotient up to and including 10.
- Students at this grade level are not expected to formally name or identify the specific properties (e.g. commutative, associative, and distributive).

Teaching Strategies

- Students should use a variety of representations for creating and solving one-step word problems, including using drawings and equations with a symbol for the unknown number.
- Some problems should include reading bar graphs, pictographs, and dot plots, as well as measurements in grams, kilograms, liters. Dot plots and line plots can be used interchangeably.

Progressions

- Relating Equal Group situations to Arrays, and indicating rows or columns within arrays, can help students see that a corner object in an array (or a corner square in an area model) is not double counted: at a given time, it is counted as part of a row or as a part of a column but not both.
- Problems in terms of "rows" and "columns," e.g., "The apples in the grocery window are in 3 rows and 6 columns," are difficult because of the distinction between the number of things in a row and the number of rows. There are 3 rows but the number of columns (6) tells how many are in each row. There are 6 columns but the number of rows (3) tells how many are in each column. (Please reference page 24 in the Progression document).

Examples

- Illustrative Mathematics: [Gifts from Grandma, Variation 1] [Classroom Supplies] [Two Interpretations of Division] [Analyzing Word Problems Involving Multiplication]
- Student Achievement Partners: [Water Balloons]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.OA.A.4

Standards Statement (2021):

Determine the unknown number in a multiplication or division equation relating three whole numbers by applying the understanding of the inverse relationship of multiplication and division.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.A.3	N/A	4.NBT.B.6, 4.GM.B.6	3.OA.A.4 3.OA.A Crosswalk

Standards Guidance:

Boundaries

- The focus of 3.OA.4 goes beyond the traditional notion of fact families by having students explore the inverse relationship of multiplication and division.

Examples

- Determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = __ \div 3$, $6 \times 6 = ?$.
- Illustrative Mathematics:
  - Finding the unknown in a division equation
- Student Achievement Partners:
  - Foundations of Multiplication and Division Mini-Assessment

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.OA.B - Understand properties of multiplication and the relationship between multiplication and division.

STANDARD: 3.OA.B.5

Standards Statement (2021):

Apply properties of operations as strategies to multiply and divide.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.A.1, 3.OA.A.2	3.OA.C.7, 3.OA.D.9	4.NBT.B.5, 4.NBT.B.6	3.OA.B.5 3.OA.B Crosswalk

Standards Guidance:

Boundaries

- Students need not use formal terms for these properties.

Progressions

- In the Array situations, the roles of the factors do not differ. One factor tells the number of rows in the array, and the other factor tells the number of columns in the situation. But rows and columns depend on the orientation of the array. If an array is rotated 90º, the rows become columns and the columns become rows. This is useful for seeing the commutative property for multiplication in rectangular arrays and areas. (Please reference page 24 in the Progression document).

Examples

- If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.)
- If 3 × 5 × 2 can be found by 3 × 5 = 15 then 15 × 2 = 30, or by 5 × 2 = 10 then 3 × 10 = 30. (Associative property of multiplication.)
- Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)
- Illustrative Mathematics:
  - Valid Equalities? (Part 2)
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 3.OA.B.5

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.OA.B.6

Standards Statement (2021):

Understand division as an unknown-factor in a multiplication problem.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.A.1	3.OA.C.7, 3.OA.A.2	4.NBT.B.6, 5.NF.B.3, 5.NF.B.7	3.OA.B.6 3.OA.B Crosswalk

Standards Guidance:

Boundaries

- Solve an unknown factor problem, by using division strategies or changing the division problem to an equivalent multiplication problem.
- Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems.

Examples

- Divide 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. (8 x ? = 32)
- Student Achievement Partners:
  - Foundations of Multiplication and Division Mini-Assessment
  - Smarter Balanced Assessment Item Illustrating 3.OA.B.6

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.OA.C - Multiply and divide within 100.

STANDARD: 3.OA.C.7

Standards Statement (2021):

Fluently multiply and divide within 100 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.B.5, 3.OA.B.6	4.OA.B.4	4.NBT.B.5, 4.NBT.B.6, 4.GM.B.4	3.OA.C.7 3.OA.C Crosswalk

Standards Guidance:

Terminology

- This standard uses the word fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property).
- Fluently/Fluency – To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Boundaries

- By the end of Grade 3, know from memory all products of one-digit numbers. "Know from memory" should not focus only on timed tests and repetitive practice.
- This standard does not require timed assessments. Ample opportunity to develop efficient, accurate, and flexible strategies is essential.
- Students should be allowed to choose an appropriate strategy to demonstrate fluency.
- Finding and using key words is not an appropriate strategy.

Teaching Strategies

- Ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9 x 9).
- Some problems should include reading bar graphs, pictographs, and dot plots. Some problems should involve grams, kilograms, and liters. Dot plots and line plots can be used interchangeably.

Progressions

- All of the understandings of multiplication and division situations, of the levels of representation and solving, and of patterns need to culminate by the end of Grade 3 in fluent multiplying and dividing of all single-digit numbers and 10.
- Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning. To achieve this by the end of Grade 3, students must begin working toward fluency for the easy numbers as early as possible. (Please reference pages 26 & 27 in the Progression document)

Examples

- Student Achievement Partners:
- [Multiplication and Division Fluency Set of Tasks] [Multiplication Facts] [Multiplication and Division within 100 Mini-Assessment]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.OA.D - Solve problems involving the four operations, and identify and explain patterns in arithmetic.

STANDARD: 3.OA.D.8

Standards Statement (2021):

Solve two-step problems in authentic contexts that use addition, subtraction, multiplication, and division in equations with a letter standing for the unknown quantity.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.OA.A.1, 3.OA.A.3	4.OA.A.3	3.GM.B.4	3.OA.D.8 3.OA.D Crosswalk

Standards Guidance:

Clarifications

- Students should represent problems using equations with a variable standing for the unknown quantity and justify their answers.

Boundaries

- This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
- This is limited to problems posed with whole numbers and having whole-number answers. Situations involving money should not include decimal numbers.

Teaching Strategies

- Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- Some problems should include reading bar graphs, pictographs, and dot plots, as well as measurements in grams, kilograms, liters. Dot plots and line plots can be used interchangeably.
- Represent these problems using equations with a letter standing for the unknown quantity.
- Students should use numerical reasoning to assess the reasonableness of answers.

Progressions

- Use of two-step problems involving easy or middle difficulty adding and subtracting within 1,000 or one such adding or subtracting with one step of multiplication or division can help to maintain fluency with addition and subtraction while giving the needed time to the major Grade 3 multiplication and division standards. (Please reference page 28 in the Progression document).

Examples

- At the movies, tickets cost $11 each, popcorn costs $7 each, and drinks costs $4 each. If I have $25, do I have enough to purchase 1 ticket, 1 popcorn, and 2 drinks?
- Illustrative Mathematics:
  - [The Class Trip] [The Stamp Collection]
- Student Achievement Partners:
  - [Two-Step Problems Using the Four Operations] [SBAC Item Illustrating 3.OA.D.8]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.OA.D.9

Standards Statement (2021):

Identify and explain arithmetic patterns using properties of operations, including patterns in the addition table or multiplication table.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.OA.C.3, 3.OA.B.5	4.OA.C.5	N/A	3.OA.D.9 3.OA.D Crosswalk

Standards Guidance:

Boundaries

- Identifying patterns can help students derive and automatize multiplication facts.
- Multiplication tables may be used to help students discover patterns and relationships.
- A student looking at a multiplication table may discover that multiples of even numbers (2, 4, 6, and 8) are always even; the products in each row and column increase by the same amount (skip counting); the multiples of 6 are double the multiples of 3; the multiples of any number fall on a horizontal and a vertical line due to the commutative property, etc.
- Patterns may include exposure to even and odd extending from previous work in 2nd grade.

Teaching Strategies

- Opportunities for students to examine numerical patterns.
- The ability to recognize and explain patterns in mathematics leads students to developing the ability to make generalizations, a foundational concept in algebraic thinking.
- Students investigate multiplication tables in search of patterns and explain why these patterns make sense mathematically.
    - The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
    - The doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
    - On a multiplication chart, the products in each row and column increase by the same amount (skip counting).
    - All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Examples

- Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.
- A student highlighting the multiples of 9 on a hundreds chart might notice 2 x 9 is 2 away from 20, 3 x 9 is 3 away from 30, and so forth.
- Illustrative Mathematics:
    - Making a ten
    - Addition Patterns
    - Patterns in the Multiplication Table
    - Symmetry of the Addition Table

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.NBT.A - Use place value understanding and properties of operations to perform multi-digit arithmetic.

STANDARD: 3.NBT.A.1

Standards Statement (2021):

Use place value understanding to round whole numbers within 1000 to the nearest 10 or 100.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.NBT.A.1	4.NBT.A.3	1.OA.B.3, 1.OA.B.4	3.NBT.A.1 3.NBT.A Crosswalk

Standards Guidance:

Clarifications

- Students should be given opportunities to build understanding by exploring the concept within 100 first and then progressing to applying the same mathematical thinking within 1000.

Teaching Strategies

- Students should locate numbers on a number line to determine the nearest multiple of 10 or 100.
- Students should be able to use place value understanding to round whole numbers for an authentic purpose within contextual situations.

Progressions

- Students need to understand that when moving to the right across the places in a number (e.g., 456), the digits represent smaller units. When rounding to the nearest 10 or 100, the goal is to approximate the number by the closest number with no ones or no tens and ones (e.g., so 456 to the nearest ten is 460; and to the nearest hundred is 500).
- Rounding to the unit represented by the leftmost place is typically the sort of estimate that is easiest for students and often is sufficient for practical purposes.
- Rounding to the unit represented by a place in the middle of a number may be more difficult for studetns (the surrounding digits are sometimes distracting). Rounding two numbers before computing can take as long as just computing their sum or difference.

Examples

- On a road trip, there is a gas station at the 700-mile mark and the 800-mile mark. You have about 50 miles left in the tank when you hit the 765-mile mark, which gas station is the closest for you to go to?
- Illustrative Mathematics:
  - Rounding to 50 or 500
  - Rounding to the Nearest Ten and Hundred
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 3.NBT.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.NBT.A.2

Standards Statement (2021):

Fluently add and subtract within 1000 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.NBT.B.5, 2.NBT.B.7, 2.NBT.B.8, 2.NBT.B.9	4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6	1.OA.B.3, 1.OA.B.4	3.NBT.A.2 3.NBT.A Crosswalk

Standards Guidance:

Clarifications

- Students should add and subtract multi-digit whole numbers within 10,000 to solve contextual, mathematical problems using efficient and generalizable procedures, based on knowledge of place value and properties of operations.

Teaching Strategies

- Students will have opportunities to use strategies based on place value and properties of operations.
- This standard uses the word fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies).
- This standard does not require timed assessments. Ample opportunity to develop efficient, accurate, and flexible strategies is essential.
- Students should be given opportunities to use variety of models and representations when extending their understanding of part-whole reasoning strategies.
- Students should be given the choice of which strategy they can use.

Progressions

- At Grade 3, the major focus is multiplication, so students' work with addition and subtraction is limited to maintenance of fluency within 1000 for some students and building fluency to within 1000 for others...They focus on methods that generalize readily to larger numbers so that these methods can be extended to 1,000,000 in Grade 4 and fluency can be reached with such larger numbers.
- Fluency within 1000 implies that students use written methods without concrete models or drawings, though concrete models or drawings can be used with explanations to overcome errors and to continue to build understanding as needed. (Please reference page 12 in the Progression document)

Examples

- Students will use estimation strategies to assess reasonableness of answers.
- Use expanded form to decompose numbers and then find sums and differences
- Illustrative Mathematics:
  - Classroom Supplies
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 3.NBT.A.2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.NBT.A.3

Standards Statement (2021):

Find the product of one-digit whole numbers by multiples of 10 in the range 10-90, such as 9 x 80. Students use a range of strategies and algorithms based on place value and properties of operations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.NBT.A.1	4.NBT.B.5	N/A	3.NBT.A.3 3.NBT.A Crosswalk

Standards Guidance:

Boundaries

- Students should be given an opportunity to explore that when a number is 10 times larger than another number, this does not come from adding zero.
- Students should understand that adding zero does not change the overall quantity.
- Students should explore the patterns of multiplying by ten and notice how the magnitude of the number changes. Exploring the pattern, students should uncover as numbers are multiplied by a multiple of 10, the digit shifts left, making the value ten times more with each shift.

Teaching Strategies

- Students extend their work in multiplication by applying understanding of place value. The special role of 10 in the base-ten system is important in understanding multiplication of one-digit numbers with multiples of 10.
- Using the properties of operations (commutative, associative, and distributive) and place value, students are able to explain their reasoning.
- Use concrete and pictorial models, based on place value and the properties of operations, to find the product of a one-digit whole number by a multiple of 10 in the range 10–90.

Examples

- For example, the product 3 x 50 can be represented as 3 groups of 5 tens, which is 15 tens, which is 150. This reasoning relies on the associative property of multiplication: 3 x 50 = 3 x (5 x 10) = (3 x 5) x 10 = 15 x 10 = 150. It is an example of how to explain an instance of a calculation pattern for these products: calculate the product of the non-zero digits, then shift the product one place to the left to make the result ten times as large.
- Illustrative Mathematics:
  - How Many Colored Pencils?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.NF.A - Develop understanding of fractions as numbers.

STANDARD: 3.NF.A.1

Standards Statement (2021):

Understand the concept of a unit fraction and explain how multiple copies of a unit fraction form a non-unit fraction.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.A.3, 2.GM.B.5	3.NF.A.3, 4.NF.B.3, 4.NF.B.4, 5.NF.B.7	3.GM.A.2	3.NF.A.1 3.NF.A Crosswalk

Standards Guidance:

Clarifications

- Understand a unit fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts;
- Understand a fraction a/b as the quantity formed by a parts of size 1/b.

Boundaries

- Grade 3 expectations are limited to denominators of 2, 3, 4, 6, and 8 as quantities formed when a whole is partitioned into equal parts. Students should explore the relationship between halves, fourths, and eighths, as well as thirds and sixths to generate simple equivalent fractions.

Teaching Strategies

- Students should investigate unit fractions using area models, parts of a set, linear models, and points on a number line.
- Students should be given the opportunity to explore this concept using a variety of visual tools such as Cuisenaire rods, fraction tiles, fraction strips, fraction bars, number lines, etc.
- Students should determine that two fractions are equal when they are the same size or on the same location on a number line.
- Students should express whole numbers as fractions recognize fractions that are equivalent to whole numbers.

Progressions

- Grade 3 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and taking one part, e.g., if a whole is partitioned into 4 equal parts then each part is 1/4 of the whole, and 4 copies of that part make the whole.
- Next, students build fractions from unit fractions, seeing the numerator 3 of 3/4 as saying that 3/4 is the quantity you get by putting 3 of the 1/4's together. They read any fraction this way, and in particular there is no need to introduce "proper fractions" and "improper fractions" initially; 5/3 is the quantity you get by combining 5 parts together when the whole is divided into 3 equal parts. (Please reference page 3 in the Progression document)

Examples

- Understand that 3/4 is composed of three pieces, each with a size of ¼.
- Student Achievement Partners: [Are Fractions Numbers?] [Understanding a Fraction as a Number]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.NF.A.2

Standards Statement (2021):

Understand a fraction as a number on the number line; Represent fractions on a number line diagram.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.C.9	3.NF.A.3, 4.NF.B.3, 5.GM.A.1, 5.GM.A.2	2.DR.A.1, 6.NS.C.6, 2.GM.D.11	3.NF.A.2 3.NF.A Crosswalk

Standards Guidance:

Clarification

- Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts.
- Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
- Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0.
- Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

Boundaries

- Grade 3 expectations are limited to fractions with denominators 2, 3, 4, 6, and 8.
- Set sizes should not exceed 24.

Teaching Strategies

- Students should investigate unit fractions using area models, set models (parts of a set), linear models, and points representing distances on a number line.
- Students should be given the opportunity to explore this concept using a variety of visual tools such as Cuisenaire rods, fraction tiles, fraction strips, fraction bars, fraction towers, number lines, etc.

Progressions

- The number line reinforces the analogy between fractions and whole numbers. Just as 5 is the point on the number line reached by marking off 5 times the length of the unit interval from 0, so 5/3 is the point obtained in the same way using a different interval as the basic unit of length, namely the interval from 0 to 1/3. (Please reference page 3 in the Progression document).

Examples

- It snowed 6 inches overnight. By dinner time, 2 inches have melted. What fraction of snowfall melted during the day?"?
  - Possible Solution: She gave 1/3 of her collection.
  - A number line could be used to visulaize the total snowfall and the fraction that melted.
- Illustrative Mathematics: [Find 2/3] [Find 1/4 Starting from 1, Assessment Variation] [Find 1 Starting from 5/3, Assessment Variation] [Find 7/4 starting from 1, Assessment Variation]
- Student Achievement Partners: [Are Fractions Numbers?] [Locating Fractions Less than One on the Number Line] [Smarter Balanced Assessment Item Illustrating 3.NF.A.2] [Find 1 with commentary]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.NF.A.3

Standards Statement (2021):

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NF.A.1, 3.NF.A.2	4.NF.A.1	N/A	3.NF.A.3 3.NF.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to recognize that comparisons are valid only when the two fractions refer to the same whole.
- Students should record the results of comparisons with symbols >, =, or <, and justify the conclusions.

Terminology

- Equivalence of fractions in special cases include:
  - Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
  - Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3),
  - Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Teaching Strategies

- Student should have opportunity to compose and decompose fractions into equivalent fractions using related fractions: halves, fourths and eighths; thirds and sixths.
- Students can also use fractions strips to see fraction equivalence.

Progressions

- As students experiment on number line diagrams they discover that many fractions label the same point on the number line, and are therefore equal; that is, they are equivalent fractions. For example, the fraction 1/2 is equal to 2/4 and also to 3/6..
- Previously, in Grade 2, students compared lengths using a standard measurement unit. In Grade 3, they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions. (Please reference page 4 in the Progression document)

Examples

- The segment from 0 to 3/4 is shorter than the segment from 0 to 5/4 because it measures 3 units of 1/4 as opposed to 5 units of 1/4. Therefore 3/4 < 5/4.
- Illustrative Mathematics: [Snow Day] [Jon and Charlie's Run] [Halves, thirds, and sixths]
- Student Achievement Partners: [Fraction Comparisons] [Are Fractions Numbers?] [Fraction Comparisons with Pictures, Assessment Variation] [Smarter Balanced Assessment Item Illustrating 3.NF.A.3] [Understanding a Fraction as a Number]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.GM.A - Reason with shapes and their attributes.

STANDARD: 3.GM.A.1

Standards Statement (2021):

Understand that shapes in different categories may share attributes and that shared attributes can define a larger category.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.A.1	4.GM.A.1, 5.GM.B.3, 5.GM.D.6	N/A	3.G.A.1 3.GM.A Crosswalk

Standards Guidance:

Clarifications

- Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).
- Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- There should be a focus on the investigation of quadrilaterals, specifically, but other polygons should also be explored.

Progressions

- Students should be able to analyze, compare, and classify two dimensional shapes by their properties. Because they have built a firm foundation of several shape categories, these categories can be the raw material for thinking about the relationships between the classes. (Please reference page 13 in the Progression document).

Examples

- Compare and classify shapes by their sides and angles.
- Recognize rhombi, rectangles, squares, and trapezoids as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.GM.A.2

Standards Statement (2021):

Partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.A.3	3.GM.C.5	3.NF.A.1	3.G.A.2 3.GM.A Crosswalk

Standards Guidance:

Progressions

- Students [continue to] develop competence in the composition and decompostion of rectangular regions, that is spatially structuring rectangular arrays. They learn to partition a rectangle into identical squares by anticipating the final structure and thus forming the array by drawing rows and columns. (Please reference page 13 in the Progression document).

Examples

- This could include partitioning a shape into 4 parts with equal area and describe each part as 1/4 of the area of the total shape.
- Draw lines to separate a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.
- Illustrative Mathematics:
  - Representing Half of a Circle
  - Geometric pictures of one half
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 3.GM.A.2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.GM.B - Solve problems involving measurement and estimation.

STANDARD: 3.GM.B.3

Standards Statement (2021):

Tell, write, and measure time to the nearest minute. Solve problems in authentic contexts that involve addition and subtraction of time intervals in minutes.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.D.10	N/A	N/A	3.MD.A.1 3.GM.B Crosswalk

Standards Guidance:

Clarifications

- Students should be given opportunities to determine relative time and predict time to the nearest fifteen minutes using only the hour hand of an analog clock.

Boundaries

- Students may use tools such as clocks, number line diagrams, and tables to solve problems involving time intervals.

Teaching Strategies

- Students will have opportunities to representing the problems in different ways, including using a number line diagram.
- Problems should include am/pm, start unknown, end unknown, and change unknown and addition/subtraction of time intervals.
- Students should be given opportunities to use number lines to find unknowns.

Examples

- The bus comes at 7:00 a.m. It takes me 15 minutes to eat breakfast and 30 minutes to get ready. What time do I need to wake up? (e.g., start unknown)
- I went to the movies at 3:15 p.m. The movie lasted 1 hour 45 minutes. What time did the movie end? (e.g., end unknown)
- After school I went to the park at 2:30 p.m. and left to go home at 3:45 p.m. How long was I at the park? (e.g., change unknown)
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 3.GM.B.3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.GM.B.4

Standards Statement (2021):

Measure, estimate and solve problems in authentic contexts that involve liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.B.4, 2.GM.C.9	4.GM.B.4	3.OA.D.8	3.MD.A.2 3.GM.B Crosswalk

Standards Guidance:

Clarifications

- Students should have an opportunity to compare capacity by filling one container with something and then pouring this amount into the comparison container.
- Students may use drawings (such as a beaker with a measurement scale) to represent the problem. This standard does not include conversions between units.
- The standard includes making reasonable estimates, use benchmarks to measure weight, and capacity.

Terminology

- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Customary measurement units include weight (oz., lbs., tons) capacity (fl. oz, cups, pints, quarts, gallons), length (in., ft., yds., miles).

Boundaries

- Excludes compound units such as cm^3 and finding the geometric volume of a container.
- Excludes multiplicative comparison problems (problems involving notions of “times as much”).
- Students are not required to memorize the conversion factors.
- Students extend understanding of measuring length in different measurement systems (e.g. grams (g) to ounces (oz), kilograms to pounds (lb), liters (l) to quarts (qt)/gallons (gal), etc).

Progressions

- Identify and use the appropriate tools and units of measurement, both customary and metric, to solve one-step word problems using the four operations involving weight, mass, liquid volume, and capacity (within the same system and unit).

Teaching Strategies

- Add, subtract, multiply or divide to solve one-step word problems involving masses or volumes that are given in the same units.
- Students should have opportunities to physically measure objects.
- Record measurement equivalents in a two-column table and/or double number line.

Examples

- Student Achievement Partners:
    - Smarter Balanced Assessment Item Illustrating 3.GM.B.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.GM.C - Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

STANDARD: 3.GM.C.5

Standards Statement (2021):

Recognize area as an attribute of plane figures and understand concepts of area measurement presented in authentic contexts by tiling and counting unit squares.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.GM.A.2, 2.GM.A.3, 2.GM.B.5, 3.GM.A.2	3.GM.C.6, 3.GM.D.8	N/A	3.MD.C.5 3.GM.C Crosswalk

Standards Guidance:

Clarifications

- Concept of area is measured with unit squares tiling a plane without gaps or overlaps.
    - A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
    - A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

Teaching Strategies

- Students should use numerical and spatial reasoning to determine the area of rectangles in contextual, mathematical problems by counting or tiling.
- Instruction could include grade appropriate use of different measurement systems for area (e.g. in^2/ft^2 and cm^2/m^2, etc).

Progressions

- Students need to learn to conceptualize area as the amount of two dimensional space in a bounded region and to measure it by choosing a unit of area, often a square. A two-dimensional geometric figure that is covered by a certain number of squares without gaps or overlaps can be said to have an area of that number of square units. (Please reference page 17 in the Progression document).

Examples

- A laptop cover is being made with square vinyl stickers. There are four rows of stickers. There are 9 stickers in each row. How many square stickers were used to create the laptop cover?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.GM.C.6

Standards Statement (2021):

Measure areas by counting standard and non-standard unit squares.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.A.2, 2.GM.D.11, 3.GM.C.5	3.GM.C.7	N/A	3.MD.C.6 3.GM.C Crosswalk

Standards Guidance:

Terminology

- Standard unit squares use standard measurement units, such as square feet or square meters.
- Non-standard unit squares could be improvised unit measurements such as sticky notes or floor/countertop tiles.

Teaching Strategies

- Area can be counted in square cm, square m, square in, square ft, and improvised units.
- Students should use numerical and spatial reasoning to determine the area of rectangles in contextual, mathematical problems.

Progressions

- To begin an explicit focus on area, teachers might then ask students which of three rectangles covers the most area. Students may first solve the problem with decomposition (cutting and/or folding) and re-composition, and eventually analyses with area-units, by covering each with unit squares (tiles). (Please reference page 17 in the Progression document).

Examples

- Students can determine the area of the top of their desk or other rectangle outlined by tape on the desk by covering it using non-standard units, such as index cards, sticky notes, tiles, etc.
- Illustrative Mathematics:
  - Finding the Area of Polygons
- Student Achievement Partners:
  - A Walk in the Park

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 3.GM.C.7

Standards Statement (2021):

Relate area to multiplication and addition. Use relevant representations to solve problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.GM.C.6	4.GM.B.6, 5.NF.B.4	4.NBT.B.6	3.MD.C.7 3.GM.C Crosswalk

Standards Guidance:

Terminology

- The dimensions of a rectangle can be referred to as length and width OR base and height.
- A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area (e.g., square cm, square m, square in, square ft).

Teaching Strategies

- Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
- Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving problems, and represent whole-number products as rectangular areas in mathematical reasoning.
- Use tiles and/or arrays to illustrate and explain that the area of a rectangle can be found by partitioning it into two smaller rectangles and that the area of the larger rectangle is the sum of the two smaller rectangles.
- Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding.

Progressions

- Students can be taught to multiply length measurements to find the area of a rectangular region. But, in order that they make sense of these quantities, they first learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows.
- Students learn to understand and explain that the area of a rectangular region of, for example, 12 length-units by 5 length-units can be found either by multiplying 12 x 5 or by adding two products, e.g., 10 x 5 and 2 x 5, illustrating the distributive property. (Please reference pages 17-18 in the Progression document)

Examples

- The area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c; 4 x 7 is the same as 4 x (2 + 5) and is the sum of 4 x 2 and 4 x 5.
- In a rectangular garden, you have four rows of peanut plants. There are 9 peanut plants in each row. How many peanut plants are there in the garden?
- Student Achievement Partners:
    - Smarter Balanced Assessment Item Illustrating 3.GM.C.7

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.GM.D - Geometric measurement: recognize perimeter.

STANDARD: 3.GM.D.8

Standards Statement (2021):

Solve problems involving authentic contexts for perimeters of polygons.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.GM.C.5	4.GM.B.6	N/A	3.MD.D.8 3.GM.D Crosswalk

Standards Guidance:

Clarifications

- Students should be given opportunities to develop a conceptual understanding of perimeter of all types of polygons including regular and irregular.
- Students should investigate perimeters of polygons with a focus on quadrilaterals.
- Students should be able to find the perimeter given the side lengths.
- Students should be able to find the unknown side length given the perimeter.

Terminology

- The focus of this learning objective should be on developing the conceptual understanding of perimeter, rather than on terminology.
- A polygon is a closed figure with at least three straight sides and angles; a polygon is regular only when all sides are equal and all angles are equal; and a polygon is irregular when all sides are not equal or all angles are not equal.

Teaching Strategies

- Finding the perimeter given the side lengths;
- Finding an unknown side length;
- Showing rectangles with the same perimeter and different area;
- Showing rectangles with the same area and different perimeters.
- Students should solve contextual, mathematical problems involving perimeter and area of rectangles.

Progressions

- Perimeter problems for rectangles and parallelograms often give only the lengths of two adjacent sides or only show numbers for these sides in a drawing of the shape. The common error is to add just those two numbers. Having students first label the lengths of the other two sides as a reminder is helpful. Students then find unknown side lengths in more difficult "missing measurements" problems and other types of perimeter problems. (Please reference page 16 in the Progression document).

Examples

- I have eighteen 1-foot panels to build a raised garden bed. How many different ways can I put these eighteen panels together to build a rectangular raised garden bed? Which rectangle will have the greatest area?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.DR.A - Pose investigative questions and collect/consider data.

STANDARD: 3.DR.A.1

Standards Statement (2021):

Generate questions to investigate situations within the classroom, school or community. Collect or consider measurement data that can naturally answer questions by using information presented in a scaled picture and/or bar graph.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.DR.A.1	4.DR.A.1	N/A	3.MD.B.4 3.DR Crosswalk

Standards Guidance:

Clarifications

- Expectations in this domain should be taught throughout the year and applied contextually as students should formulate investigative questions to explore a real-life situations.

Terminology

- A statistical investigative question is one that requires data that will vary.
- Numerical data: How many siblings does each student in the class have?
- Categorical data: Out of football, basketball, baseball, soccer, none of these, what is your favorite sport to watch?
- Dot plots and line plots can be used interchangeably.

Boundaries

- Rulers measurement can be limited to those marked with halves and fourths of an inch.
- Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.
- Some problems should include reading bar graphs, pictographs, and dot plots, as well as measurements in grams, kilograms, liters.

Teaching Strategies

- Students should be provided with learning experiences to collect and analyze both numerical data and categorical data.
- Data can be gathered from a variety of sources to answer the statistical investigative question posed.

Progressions

- In Grade 3, students are beginning to learn fraction concepts (3.NF). They understand fraction equivalence in simple cases, and they use visual fraction models to represent and order fractions. Grade 3 students also measure lengths using rulers marked with halves and fourths of an inch. They use their developing knowledge of fractions and number lines to extend their work from the previous grade by working with measurement data involving fractional measurement values. (Please reference page 10 in the Progression document).

Examples

- Questions can be created to help students get to know each other better in the first weeks of school.
- Student Achievement Partners: Smarter Balanced Assessment Item Illustrating 3.DR.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 3.DR.B - Analyze, represent, and interpret data.

STANDARD: 3.DR.B.2

Standards Statement (2021):

Analyze measurement data with a scaled picture graph or a scaled bar graph to represent a data set with several categories. Interpret information presented to answer investigative questions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.DR.B.2	4.DR.B.2	N/A	3.MD.B.3 3.DR Crosswalk

Standards Guidance:

Clarification

- Dot plots and line plots can be used interchangeably.
- Dot plots should be used for numerical data representation on a number line.

Terminology

- Numerical data - data that can be expressed in numbers rather than natural language. An example of numerical data that could be collected is the number of people who attended the movie theater over the course of a month.
- Categorical data - a type of data used to group information with similar characteristics. Examples of categorical data that could be collected might be marital status, favorite sport, or favorite type of movie.

Boundaries

- Students should use a ruler that is marked at halves and fourths only to create an evenly spaced number line for the dot plot.
- Data sets for categorical data created by students may include several categories.
- The scales of the pictographs, bar graphs, and dot plots should depend on the data collected.

Teaching Strategies

- Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve problems using information presented in these graphs.
- Collect data by asking a question that yields data in up to four categories.
- The scales of the pictographs, bar graphs, and dot plots should depend on the data collected.

Progressions

- For dot plots and bar graphs, students may analyze the data presented making connections with single-digit multiplication or division in their explanations. On a pictograph, one symbol may stand for a value greater than 1 to allow students to apply their understanding of single digit multiplication and division facts. On a pictograph, one symbol may stand for a value greater than 1.
- In Grade 3, the most important development in data representation for categorical data is that students now draw picture graphs in which each picture represents more than one object, and they draw bar graphs in which the height of a given bar in tick marks must be multiplied by the scale factor in order to yield the number of objects in the given category. These developments connect with the emphasis on multiplication in this grade. (Please reference page 7 in the Progression document).

Examples

- Illustrative Mathematics: [Classroom Supplies] [SBAC Example]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4E: Grade 4 Math Standards and Guidance

Critical Areas of Focus

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Domains and Clusters

4.OA - Algebraic Reasoning: Operations

- 4.OA.A Use the four operations with whole numbers to solve problems.
- 4.OA.B Gain familiarity with factors and multiples.
- 4.OA.C Generate and analyze patterns.

4.NBT - Numeric Reasoning: Base Ten Arithmetic

- 4.NBT.A Generalize place value understanding for multi-digit whole numbers.
- 4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4.NF - Numeric Reasoning: Fraction

- 4.NF.A Extend understanding of fraction equivalence and ordering.
- 4.NF.B Build fractions from unit fractions.
- 4.NF.C Understand decimal notation for fractions, and compare decimal fractions.

4.GM - Geometric Reasoning and Measurement

- 4.GM.A Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
- 4.GM.B Solve problems involving measurement and conversion of measurements.
- 4.GM.C Geometric measurement: understand concepts of angle and measure angles.

4.DR - Data Reasoning

- 4.DR.A Pose investigative questions and collect/consider data.
- 4.DR.B Analyze, represent, and interpret data.

Students should spend the large majority[1] of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

[1]At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see the K-8 major work of the grade developed by Student Achievement Partners.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.OA.A - Use the four operations with whole numbers to solve problems.

STANDARD: 4.OA.A.1

Standards Statement (2021):

Interpret a multiplication equation as comparing quantities. Represent verbal statements of multiplicative comparisons as equations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.A.3	4.OA.A.2, 5.NF.B.4	5.NF.B.3, 5.NF.B.5, 5.NF.B.6	4.OA.A.1 4.OA.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to solve contextual, mathematical problems involving multiplicative comparison.
- Students should be able to distinguish multiplicative comparison from additive comparison.

Terminology

- The terms below are used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
    - Multiplicative comparison – a comparison situation based on one set of a quantity being a particular multiple of the other set within the comparison.
    - Additive comparison – involves two distinct quantities and the difference between them.

Teaching Strategies

- Students should be able to demonstrate an understanding of simple multiplicative relationships by using concrete materials, drawings, and equations with a variable for the unknown number to represent the problem.

Progressions

- In a multiplicative comparison, the underlying question is 'what factor would multiply one quantity' in order to result in the other (Please reference page 29 in the Progression document).

Examples

- Interpret 35 = 5 x 7 as 35 is the same value as 5 times as many as 7 and 7 times as many as 5.
- MP2 - Represent verbal statements of multiplicative comparisons as multiplication equations.
- Mara has four pencils. Josh has three times as many pencils as Mara. How many pencils does Josh have?
- Illustrative Mathematics
    - Thousands and Millions of Fourth Graders
    - Threatened and Endangered

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.OA.A.2

Standards Statement (2021):

Multiply or divide to solve problems in authentic contexts involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.A.3, 4.OA.A.1,	4.NF.A.1, 4.NF.B.4, 5.NF.B.6	5.NF.B.5, 6.RP.A.1, 6.RP.A.2, 8.AEE.A.3	4.OA.A.2 4.OA.A Crosswalk

Standards Guidance:

Clarification

- Students may use drawings and equations with a symbol for the unknown number to represent the problem.
- Distinguish between multiplicative comparison from additive comparison.

Terminology

- Students should recognize that additive comparison refers to the difference between two numbers and multiplicative comparison refers to how much or how many times larger the bigger number is from the smaller number as a comparison.

Examples

- If the cost of a red hat is three times more than a blue hat that costs $5 then a red hat cost $15.
- MP6 - Use drawings and equations with a symbol for the unknown number to represent the problem.
- Illustrative Mathematics:
  - Comparing Money Raised
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 4.OA.A.2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.OA.A.3

Standards Statement (2021):

Solve multistep problems in authentic contexts using whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.D.8, 4.NBT.B.6,	N/A	4.NBT.A.3, 4.GM.B.5, 7.NS.A.3	4.OA.A.3 4.OA.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to use the four operations with whole numbers to solve contextual, mathematical problems.
- Students should assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Boundaries

- Problems should include solutions in which remainders must be interpreted.

Teaching Strategies

- Students should represent and model problems using equations and diagrams with a variable for the unknown quantity.

Progressions

- Numbers can be those in the Grade 4 standards, but the number of steps should be no more than three and involve only easy and medium difficulty addition and subtraction problems (Please reference page 29 in the Progression document).

Examples

- "How many busses are needed to transport 250 students, if each bus holds 36 students?" In which the remainder of 34 would be interpreted to include an additional bus.
- MP2 - Interpret and use remainders, and assess the reasonableness of answers using mental computation and estimation strategies.
- MP4 - Represent these problems using equations with a letter standing for the unknown quantity.
- MP6 - Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- Illustrative Mathematics:
  - Carnival Tickets
  - Karl's Garden
- Student Achievement Partners:
  - How Many Teams?
  - Multi-Step Problems Using the Four Operations Mini-Assessment
  - Smarter Balanced Assessment Item Illustrating 4.OA.A.3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.OA.B - Gain familiarity with factors and multiples.

STANDARD: 4.OA.B.4

Standards Statement (2021):

Find all factor pairs for a whole number in the range 1-100. Determine whether a given whole number in the range of 1-100 is a multiple of a given one-digit number, and whether it is prime or composite.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.C.7	6.AEE.A.1	6.NS.B.4	4.OA.B.4 4.OA.B Crosswalk

Standards Guidance:

Clarifications

- Students should recognize that a whole number is a multiple of each of its factors.
- Students should be able to determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number.
- Students should be able to determine whether a given whole number in the range 1–100 is prime or composite.

Terminology

- Prime number – A whole number greater than 1 that with two unique factors, 1 and itself.
- Composite number – A whole number greater than 1 that has at least one whole-number factor other than 1 and itself.

Examples

- The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.
- MP8 - Recognize that a whole number is a multiple of each of its factors (e.g., 24 is a multiple of 1, 2, 3, 4, 6, 8, 12, and 24).
- If there are 24 students in a class, how many unique ways can they be arranged into equal-sized groups?
- Every 8th person of the first hundred people in line for a concert will get a free T-shirt. Which places in the line will get a T-shirt?
- Illustrative Mathematics:
  - The Locker Game
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 4.OA.B.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.OA.C - Generate and analyze patterns.

STANDARD: 4.OA.C.5

Standards Statement (2021):

Analyze a number, visual, or contextual pattern that follows a given rule.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.D.9	5.OA.B.3	N/A	4.OA.C.5 4.OA.C Crosswalk

Standards Guidance:

Clarifications

- Within numeric patterns, students should be able to connect each term in a growing or shrinking pattern with its term number (e.g., in the sequence 1, 4, 7, 10, …, the first term is 1, the second term is 4, the third term is 7, and so on), and record the patterns in a table of values that shows the term number.
- Students should be provided with opportunities to explore and extend growing patterns using shapes.
- Students should be provided with opportunities to explore and extend numerical patterns using a given rule.
- Students should be able to identify features of the pattern that were not explicit in the rule itself.
- Students should be able to explain, informally, why a pattern will continue to develop as it does.

Boundaries

- Students are not expected to determine the rule but instead are expected to extend the pattern or complete a pattern.
- Patterns are limited to 8 elements.

Progressions

- Connection to MP3 - Identify and describe features of the rule not explicit in the rule itself.

Examples

- Given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
- Use square tiles to generate a growing pattern that shows multiples of four.
- Use the rule, multiply by 3 and add 1 to find the next two stages in the following growing pattern:
    - Where does the pattern multiply by 3? Where is the “1” that is being added as this pattern grows? Create a different growing pattern using this rule. Identify where it multiplies by three and where one is added.
    - Illustrative Mathematics: [Double Plus One] [Multiples of nine]
- Student Achievement Partners:
    - Smarter Balanced Assessment Item Illustrating 4.OA.C.5

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.NBT.A - Generalize place value understanding for multi-digit whole numbers.

STANDARD: 4.NBT.A.1

Standards Statement (2021):

Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.NBT.A.1	4.NBT.A.2, 4.NBT.A.3, 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6, 5.NBT.A.1	N/A	4.NBT.A.1 4.NBT.A Crosswalk

Standards Guidance:

Clarifications

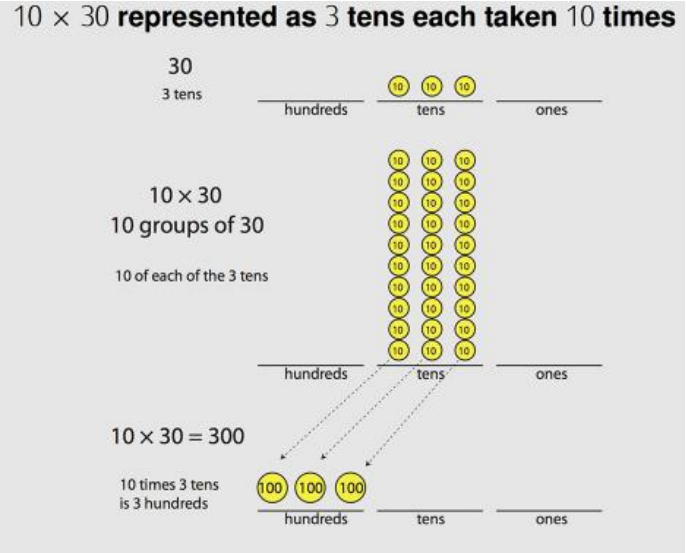
- Students should be able to use numerical reasoning to represent and explain using concrete materials, the relationship among the numbers 1, 10, 100, and 1,000. Students should be able to extend the pattern to the hundred-thousands place.
- Students should be able to recognize the relationship of same digits located in different places in a whole number.

Boundaries

- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Progressions

- In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left.
- Each of the 3 [groups of] tens becomes a hundred and moves to the left. In the product, the three in the tens place of 30 is shifted one place to the left to represent three hundreds. In 300 divided by 10 the 3 is shifted one place to the right in the quotient to represent three tens. (Please reference page 13 in the Progression document).



Examples

- Recognize that 700 ÷ 70 = 10 by applying concepts of place value and division
- The population of Atlanta is about 500,000 people and the population of Valdosta is about 50,000 people. How many times greater is the population of Atlanta than Valdosta?
- Illustrative Mathematics: [Threatened and Endangered] [Thousands and Millions of 4th Graders]
- Student Achievement Partners: [Smarter Balanced Assessment Item Illustrating 4.NBT.A.1]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.NBT.A.2

Standards Statement (2021):

Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Use understandings of place value within these forms to compare two multi-digit numbers using >, =, and < symbols.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.NBT.A.3, 2.NBT.A.4, 4.NBT.A.1	4.NBT.A.3, 5.NBT.A.1, 5.NBT.A.3	N/A	4.NBT.A.2 4.NBT.A Crosswalk

Standards Guidance:

Boundaries

- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
- Students are not expected to write numbers in word form.

Teaching Strategies

- Make connections across representations of multi-digit whole numbers using base ten numerals, number names, and expanded form.
- Develop rules for comparing the multi-digit numbers.

Progressions

- To read numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (thousand, million, billion, trillion, etc.). Thus, 457,000 is read "four hundred fifty seven thousand." (Please reference page 13 in the Progression document).

Examples

- The number two hundred seventy-five thousand eight hundred two written in standard form is 275,802 and in expanded form is 200,000+70,000+5,000+800+2 or (2×100,000)+(7×10,000)+(5×1,000)+(8×100)+(2×1).
- Illustrative Mathematics:
  - Ordering 4-digit numbers
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 4.NBT.A.2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.NBT.A.3

Standards Statement (2021):

Use place value understanding to round multi-digit whole numbers to any place.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NBT.A.1, 4.NBT.A.1, 4.NBT.A.2	5.NBT.A.4	4.OA.A.3	4.NBT.A.3 4.NBT.A Crosswalk

Standards Guidance:

Boundaries

- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
- Grade 4 students should explore rounding within contextual situations.

Teaching Strategies

- Students rounding to 348 to the nearest hundred may mistakenly round initially to 350 and then 400 by applying rules such as if the digit is 0-4 then round down and 5-9 and round up. Models can help them see that 348 is closer to 300 than 400.
- Students should locate numbers on a number line to determine the nearest multiple of 1,000s, 10,000s or 100,000s.

Examples

- Illustrative Mathematics:
  - Rounding to the Nearest 1000
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 4.NBT.A.3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.NBT.B - Use place value understanding and properties of operations to perform multi-digit arithmetic.

STANDARD: 4.NBT.B.4

Standards Statement (2021):

Fluently add and subtract multi-digit whole numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NF.A.1, 3.NBT.A.2, 4.NBT.A.1	5.NBT.B.5, 5.NBT.B.7	N/A	4.NBT.B.4 4.NBT.B Crosswalk

Standards Guidance:

Clarifications

- Students should fluently (flexibly, accurately, and efficiently) add and subtract multi-digit whole numbers, to solve contextual, mathematical problems using efficient and flexible procedures, based on knowledge of place value and properties of operations.
- Students should use efficient algorithms that make sense for the given numbers and draw upon their understanding of multi-digit whole numbers, the properties of operations, and place value.

Terminology

- Efficiency in mathematics is the ability to produce answers relatively easily with a minimal number of steps. An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and making use of intermediate results to solve the problem. Efficiency does not mean students should be timed.
- Flexibility is the ability to think about a problem in more than one way and to adapt or adjust thinking, if necessary.
- Accuracy is the ability to produce mathematically precise answers.
- Appropriateness is the ability to select and apply a strategy that is appropriate for solving a given problem efficiently.

Boundaries

- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.
- Students should be given the choice of which procedure they can use.
- Students should add and subtract multi-digit whole numbers within 100,000, to solve math problems using generalizable procedures, based on place value and properties of operations.

Progressions

- Because students in Grade 2 and Grade 3 have been using at least one method that readily generalizes to 1,000,000, this extension does not have to take a long time. Thus, students will have time for the major NBT focus for this grade: multiplication and division. (Please reference page 14 in the Progression document)

Examples

- Student Achievement Partners: [Subtraction Accuracy] [Multi-Digit Add./Sub. Mini-assessment]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.NBT.B.5

Standards Statement (2021):

Use representations and strategies to multiply a whole number of up to four digits by a one-digit number, and a two-digit number by a two-digit number using strategies based on place value and the properties of operations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NBT.A.2, 3.NBT.A.3, 4.NBT.A.1	4.NBT.B.6, 5.NBT.B.5	3.OA.C.7, 3.OA.B.5	4.NBT.B.5 4.NBT.B Crosswalk

Standards Guidance:

Boundaries

- Students should be familiar with multiple strategies but should be able to select and use the strategy with which they most closely connect and understand, with the ultimate goal of supporting students to use more efficient strategies.
- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
- A range of efficient algorithms may be used.

Teaching Strategies

- Illustrate and explain calculations using rectangular arrays, area models, and/or equations, along with strategies based on place value and properties of operations.
- Students should be able to solve contextual, mathematical problems involving the multiplication of a number with up to four digits by a 1-digit whole number.
- Students should be able to illustrate and explain their calculations using equations, rectangular arrays, and/or area models for all numbers included in the learning objective.

Progressions

- In fourth grade, students compute products of one-digit numbers and multi-digit numbers (up to four digits) and products of two two-digit numbers. They divide multi-digit numbers (up to four digits) by one-digit numbers.
- As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose. (Please reference pages 14 & 15 in the Progression document).

Examples

- Connect numeric and visual models such as those created by representing 285 with base 10 pieces and repeating three times. Use this area model with dimensions of 285 and 3 to find partial products.
- There are 7 boxes of chocolates. Each box contains 16 chocolates. How many chocolates are there all together?
- The school bought thirty-nine cases of popcorn for the school carnival. Each case contained 15 bags of popcorn. How many bags of popcorn is that all together?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.NBT.B.6

Standards Statement (2021):

Use representations and strategies to find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NBT.A.2, 4.NBT.A.1, 4.NBT.B.5	5.NBT.B.6	3.OA.B.5, 3.OA.B.6, 3.OA.C.7, 4.OA.A.3, 3.GM.C.7	4.NBT.B.6 4.NBT.B Crosswalk

Standards Guidance:

Clarifications

- Students should be able to solve contextual, mathematical problems involving division of whole numbers.
- Students should be familiar with multiple strategies but should be able to select and use the strategy with which they most closely connect and understand, with the ultimate goal of supporting students to use more efficient strategies.

Content Boundaries

- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.
- Long division is not an expectation at this grade level.

Teaching Strategies

- Students should be able to illustrate and explain their calculations using equations, rectangular arrays, and/or area models.

Progressions

- General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understandings as for multiplication, but cast in terms of division. One component is quotients of multiples of 10, 100, or 1000 and one-digit numbers. For example, 42 ÷ 6 is related to 420 ÷ 6 and 4200 ÷ 6. Students can draw on their work with multiplication and they can also reason that 4200 ÷ 6 means partitioning 42 hundreds into 6 equal groups, so there are 7 hundreds in each group. (Please reference pages 16 & 17 in the Progression document).

Examples

- Apply knowledge of decomposing whole numbers into divisible parts. Such as, connect numeric and visual models such as those created by representing 136 with base 10 pieces and dividing into groups of 4 to determine either the size of the group or the number of groups.
- Antonio won a jar of 373 jellybeans in a school contest. He wants to share them. He and his 7 friends will share them. How many jellybeans will each of the friends get?
    - Possible solution: 373 ÷ 8 = (368 ÷ 8) + (5 ÷ 8) = 46 with 5 jellybeans left over.
- Illustrative Mathematics: [Mental Division Strategy]
- Student Achievement Partners: [How Many Teams?] [SBAC Item Illustrating 4.NBT.B.6]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.NF.A - Extend understanding of fraction equivalence and ordering.

STANDARD: 4.NF.A.1

Standards Statement (2021):

Use visual fraction representations to recognize, generate, and explain relationships between equivalent fractions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NF.A.3	4.NF.A.2, 4.NF.B.3, 4.NF.C.5, 5.NF.A.1, 5.NF.B.5	4.OA.A.2	4.NF.A.1 4.NF.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to describe how the number and size of the parts differ even though the fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- Students should be able to explain fraction equivalence as a multiplicative relationship, not additive.
- Students should be able to explain why $ab = (n \times a)(n \times b)$ is a true mathematical statement, whereas $ab = (n+a)(n+b)$ is NOT a true mathematical statement.

Boundaries

- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.
- This expectation includes fractions greater than 1.
- Fractions should be limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Teaching Strategies

- Concrete materials may include fraction circles, fraction strips, pattern blocks.
- Students may represent their problems and explain their reasoning with drawing and number lines.
- Students should be able to discover, explain, and generalize the relationship between the identity property of multiplication and equivalent fractions (i.e., paper folding activities, number lines, etc.).

Progressions

- Students can use area models and number line diagrams to reason about equivalence. They see that the numerical process of multiplying the numerator and demonimator of a fraction by the same number, n, corresponds physically to partitioning each unit fraction piece into n smaller equal pieces. The whole is then partitioned into n times as many pieces, and there are ntimes as many smaller unit fraction pieces as in the original fraction. (Please reference page 6 in the Progression document).

Examples

- Peter is giving half of his candy bar to four friends. Provide a mathematical representation to show this scenario.
- Illustrative Mathematics:
  - Explaining Fraction Equivalence with Pictures
  - Fractions and Rectangles
- Student Achievement Partners: [Odd Fraction Out] [Fraction Concepts Mini-Assessment]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.NF.A.2

Standards Statement (2021):

Compare two fractions with different numerators and/or different denominators, record the results with the symbols >, =, or <, and justify the conclusions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NF.A.1	4.NF.C.7, 5.NF.A.2	4.OA.A.2	4.NF.A.2 4.NF.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to recognize that comparisons are valid only when the two fractions refer to the same whole.
- Students should record the results of comparisons with symbols >, =, or <, and justify the conclusions.

Boundaries

- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.
- Students should be given fractions with common numerators to compare.

Teaching Strategies

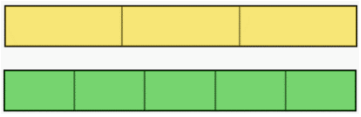
- MP3 - Justify using conceptual and procedural strategies. Conceptual strategies should include using visual models; comparing benchmark fractions such as 0, ½, 1; and attending to the size of the piece for the like numerators or number of pieces for like denominators. Procedural strategies should include finding a common denominator to directly compare the number of pieces.

Progressions

- Grade 4 students use their understanding of equivalent fractions to compare fractions with different numerators and different denominators. For example, to compare 5/8 and 7/12 [students] rewrite both fractions as 60/96 (= 12x5/12x8) and 56/96 (= 7x8/12x8). Because 60/96 and 56/96 have the same denominator, students can compare them using Grade 3 methods and see that 56/96 is smaller, so 7/12 < 5/8. (Please reference page 6 in the Progression document)

Examples

- Jamie and Kendra each had the same grid to color using any pattern they wished. Jamie colored 23 of the yellow grid pattern and Kendra colored 25 of the green grid pattern. Who colored more?
  - Jamie colored more because thirds are bigger than fifths, so 23 thirds is more than 25 fifths.



- Kennedy ran 5/8 of a mile during practice and Alice ran 7/8 of a mile. Who ran farther?
- Illustrative Mathematics:
  - Listing fractions in increasing size
  - Using Benchmarks to Compare Fractions
- Student Achievement Partners: [Fraction Concepts Mini-Assessment] [Assessment Item 4.NF.A.2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.NF.B - Build fractions from unit fractions.

STANDARD: 4.NF.B.3

Standards Statement (2021):

Understand a fraction (a/b) as the sum (a) of fractions of the same denominator (1/b). Solve problems in authentic contexts involving addition and subtraction of fractions referring to the same whole and having like denominators.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NF.A.1, 3.NF.A.2, 4.NF.A.1	4.NF.C.5, 5.NF.A.1, 5.NF.B.3	2.OA.A.1, 4.GM.B.5	4.NF.B.3 4.NF.B Crosswalk

Standards Guidance:

Clarifications

- Students should be able to break apart (decompose) whole numbers and fractions as the sum of unit fractions.
- Break apart (decompose) a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.

Content Boundaries

- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.
- Extend understanding addition and subtraction to include fractions.
- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.
- Add and subtract mixed numbers with like denominators.

Teaching Strategies

- Students should be able to add and subtract fractions and mixed numbers with the same (like) denominators by joining and separating parts referring to the same whole while solving contextual, mathematical problems.
- Tools include fraction concrete materials, such as Cuisenaire rods, drawings, and number lines.
- Students should be flexible in their choice of strategy when subtracting fractions. Reasoning about the sizes of the fractions and their relationships is the expectation here rather than memorizing regrouping procedures.
- Students can justify their work using a visual fraction representation.

Examples

- MP2 - Decompose and recompose 3/8 as 1/8 + 1/8 + 1/8 or 1/8 + 2/8 and 2 1/8 as 1 + 1 + 1/8 or 8/8 + 8/8 + 1/8.
- MP4 - Use visual fraction models and equations to represent problems.
- MP7 - Replace mixed numbers with equivalent fractions and/or use properties of operations and the relationship between addition and subtraction to solve problems.

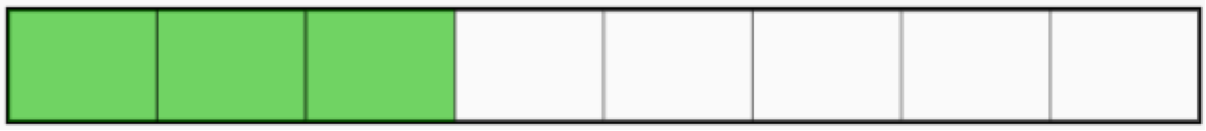
This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

- Alex has a whole pizza. How can it be cut so that it can be shared with (4, 6, 8, 12) people? What fraction of the whole pizza will each person get?
- Express 1 in the form 1 = 44 (1 whole is equal to four fourths 14+ 14+ 14+ 14= 44 = 1) recognize that additional wholes cut into fourths can also be written as the sum of

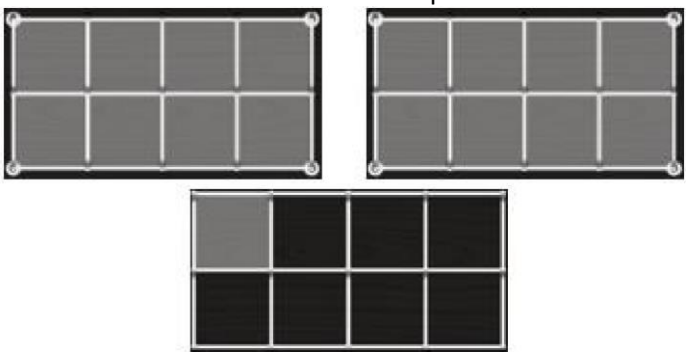
- Locate 44 and 1 at the same point of a number line diagram.



- A piece of ribbon was cut into eighths for a classroom art project. Three pieces were left at the end of the day. Show a mathematical representation of the ribbon that is left.
    - Possible student response: 38 = 18 + 18 + 18 ; 38 = 18 + 28

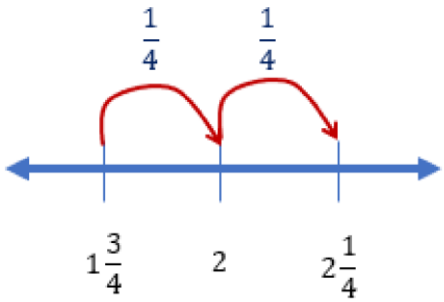


- Three pans of brownies were cut into eighths to sell at a school function. 78 of one pan were sold. How many eighths are left to sell? Show a mathematical representation of the ribbon that is left.
    - Possible student response: 2 18 = 1 + 1 + 18 = 88 + 88 + 18



Example (4.4.6)

- Luisa needs to know how much bigger her 214 inch piece of cardstock is than her 134 inch piece of cardstock in order to finish her project. o Possible student response: The 214 inch piece is 24 inch bigger than the 134 inch piece.



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.NF.B.4

Standards Statement (2021):

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. Represent and solve problems in authentic contexts involving multiplication of a fraction by a whole number.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NF.A.1	5.NF.B.3, 5.NF.B.4, 5.NF.B.7	3.OA.A.3, 4.OA.A.2, 4.GM.B.5	4.NF.B.4 4.NF.B Crosswalk

Standards Guidance:

Clarifications

- Extend understanding multiplication to include fractions.
  - Understand a fraction a/b as a multiple of 1/b.
  - Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.

Boundaries

- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Examples

- MP2 - Use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
- MP3 - Use a visual fraction model to demonstrate 3 × (2/5) is the same as 6 × (1/5), recognizing this product as 6/5. Justify the general idea n × (a/b) = (n × a)/b.
- Illustrative Mathematics:
  - Extending Multiplication From Whole Numbers to Fractions
- Student Achievement Partners:
  - The Penny
  - Fraction Concepts Mini-Assessment
  - Smarter Balanced Assessment Item Illustrating 4.NF.B.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.NF.C - Understand decimal notation for fractions, and compare decimal fractions.

STANDARD: 4.NF.C.5

Standards Statement (2021):

Demonstrate and explain the concept of equivalent fractions with denominators of 10 and 100, using concrete materials and visual models. Add two fractions with denominators of 10 and 100.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NF.A.1, 4.NF.B.3	4.NF.C.6, 5.NF.A.2, 5.NF.B.5	4.GM.B.5, 5.NBT.A.1	4.NF.C.5 4.NF.C Crosswalk

Standards Guidance:

Clarifications

- Students should also use mixed numbers and fractions greater than 1.
- Students should express fractions such as 310 as 30100, and add fractions such as 310 + 4100 = 34100.

Boundaries

- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Teaching Strategies

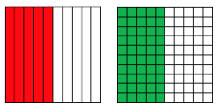
- Students should be able to solve contextual, mathematical problems involving the addition of two fractions with denominators of 10 and 100.
- Students should be given multiple opportunities to use visual models to develop part-whole reasoning when building an understanding of equivalent fractions.

Progressions

- Grade 4 students learn to add decimal fractions by converting them to fractions with the same denominator, in preparation for general fraction addition in Grade 5. (Please reference page 9 in the Progression document).

Examples

- Colin wants to use 5/10 of a board for a project. He is wondering how he can cut his whole board into pieces that are equivalent to 5/10. What fraction(s) of the whole board can Colin cut the board that are equivalent to 5/10 ? Possible student response: the board could be divided into 10ths (e.g. 5/10) or 100ths (e.g. 50/100).



- Illustrative Mathematics:
  - Adding Tenths and Hundredths
  - How Many Tenths and Hundredths?
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 4.NF.C.5

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.NF.C.6

Standards Statement (2021):

Use and interpret decimal notation for fractions with denominators 10 or 100.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NF.C.5	4.NF.C.7	4.GM.B.5, 5.NBT.A.1	4.NF.C.6 4.NF.C Crosswalk

Standards Guidance:

Clarifications

- Represent decimal number values on a place value chart.

Boundaries

- Students are not expected to write word names of decimal numbers at this grade level.
- To the hundredths place

Teaching Strategies

- Concrete materials could include base ten block where the "flat" or hundred square is considered one whole or a ten frame where the whole frame is considered one whole.

Examples

- Rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.
    - Eric overpaid his medical bill by $0.62. When businesses write refund checks, they often write the cents as a fraction. What fraction will the doctor's office use to represent the $0.62 on the check?
    - Possible student response: I wrote 62 cents ($0.62) as 62100 because $0.62 is sixty-two hundredths of a dollar. If I place $0.62 on a number line, it would be between $0.50 and $0.75.



- Illustrative Mathematics:
    - Expanded Fractions and Decimals
    - Dimes and Pennies
- Student Achievement Partners:
    - Smarter Balanced Assessment Item Illustrating 4.NF.C.6

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.NF.C.7

Standards Statement (2021):

Use decimal notation for fractions with denominators 10 or 100. Compare two decimals to hundredths place by reasoning about their size, and record the comparison using the symbols >, =, or <.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NF.A.2, 4.NF.C.6	5.NBT.A.3	5.NBT.A.1	4.NF.C.7 4.NF.C Crosswalk

Standards Guidance:

Clarifications

- Recognize that comparisons are valid only when the two decimal numbers refer to the same whole.
- Students should be able to order up to 5 whole numbers less than 1,000,000 through the hundred-thousands place.

Boundaries

- Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.
- Students are not expected to use more than two inequality symbols when recording comparisons (< or >) to the hundredths place.

Teaching Strategies

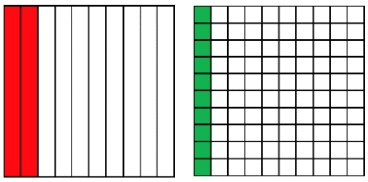
- Decimal quantities should be presented within a context.
- Students should be given multiple opportunities to use visual models to develop part-whole reasoning when comparing decimal numbers.
- Students should be able to determine and explain, through investigation, the relationship between decimal numbers, using a variety of tools (e.g., concrete materials, drawings, number lines) and strategies.

Progressions

- Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator. For example, to compare 0.2 and 0.09, students think of them as 0.20 and 0.09 and see that 0.20 > 0.09 because 20/100 > 9/100. (Please reference page 10 in the Progression document).

Examples

- What do you notice about the fractions 2/10 and 10/100? Write a comparison statement about the two fractions and use visual models to support your reasoning.
    - Possible student response: I know that 2-10ths is greater than 10-100ths because 2-10ths takes up more space in the decimal squares to the right. So, 2-10ths>10-100ths.



- Illustrative Mathematics: [Using Place Value]
- Student Achievement Partners: [Smarter Balanced Assessment Item Illustrating 4.NF.C.7]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.GM.A - Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

STANDARD: 4.GM.A.1

Standards Statement (2021):

Explore, investigate, and draw points, lines, line segments, rays, angles, and perpendicular and parallel lines. Identify these in two-dimensional figures.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.GM.A.1	4.GM.A.2, 4.GM.C.7, HS.GM.A.4	N/A	4.G.A.1 4.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students should draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular lines.
- Expectation that drawing and identifying right, acute, and obtuse angles are included in this standard.

Terminology

- Right angle – An angle measuring exactly 90°.
- Acute angle – An angle larger than 0° and smaller than 90°.
- Obtuse angle – An angle larger than 90° and smaller than 180°.
- Perpendicular lines – Two lines that meet to form an intersection at a right angle

Teaching Strategies

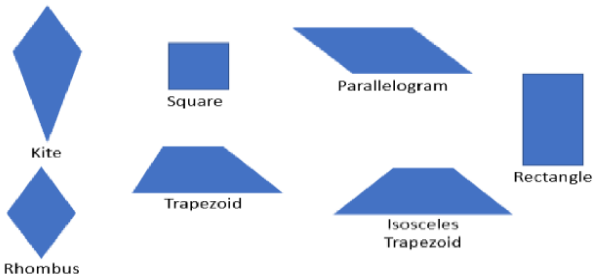
- Students should investigate lines of symmetry in two dimensional figures as a property. This is an extension from work in third grade.

Progressions

- Students see points and lines as abstract objects: Lines are infinite in extent and points have location but no dimension. Grids are made of points and lines and do not end at the edge of the paper. (Please reference page 15 in the Progression document).

Examples

- How many lines of symmetry do each of the quadrilaterals to the right have?



- Illustrative Mathematics:
  - The Geometry of Letters
  - What's the Point?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.GM.A.2

Standards Statement (2021):

Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.GM.A.1, 4.GM.C.7	5.GM.B.3, 5.GM.D.6, HS.GM.A.4	N/A	4.G.A.2 4.GM.A Crosswalk

Standards Guidance:

Clarifications

- Right angles should be indicated with a square symbol.
- Polygons should include triangles, quadrilaterals including kites, trapezoids, rectangles, squares, rhombuses, and other parallelograms, and pentagons.

Terminology

- A polygon is a closed figure with at least three straight sides and angles; a polygon is regular only when all sides are equal and all angles are equal; and a polygon is irregular when all sides are not equal or all angles are not equal.
- Isosceles triangle – A triangle containing at least two equal length sides and two equal interior angle measures. Sub- class includes equilateral triangles.
- Equilateral triangle – A triangle with three equal-length sides and three 60-degree interior angles. Also known as an equiangular triangle.
- Scalene triangle – A triangle containing three unequal side lengths and three unequal angle measures.
- Right triangle – a triangle with one right angle.
- Acute triangle – a triangle containing three acute angles.
- Obtuse triangle – a triangle containing one obtuse angle.

Boundaries

- This objective does not require students to create a hierarchy.

Progressions

- Students can use side length to classify triangles as equilateral, equiangular, isosceles, or scalene; and can use angle size to classify them as acute, right, or obtuse. They then learn to cross-classify, for example, naming a shape as a right isosceles triangle. Thus, students develop explicit awareness of and vocabulary for many concepts they have been developing, including points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. (Please reference page 15 in the Progression document).

Examples

- Illustrative Mathematics: [Defining Attributes of Rectangles and Parallelograms] [Are these right?] [What is a Trapezoid? (Part 1)] [What shape am I?]
- Student Achievement Partners: [Smarter Balanced Assessment Item Illustrating 4.GM.A.2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.GM.A.3

Standards Statement (2021):

Recognize and draw a line of symmetry for a two dimensional figure.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
1.GM.A.2	HS.GM.A.4	N/A	4.G.A.3 4.GM.A Crosswalk

Standards Guidance:

Clarifications

- A line of symmetry is a line across the figure such that the figure can be folded along the line into matching parts.

Teaching Strategies

- Students should be provided multiple opportunities to investigate symmetry through paper folding and/or the use of mirrors.
- Students should develop an understanding of what a line of symmetry is through exploration with real-world objects.
- Identify or create line-symmetric figures by drawing and testing proposed lines of symmetry and sketching the second half of a symmetrical figure. (MP.7)

Examples

- Identify lines of symmetry seen in real-life objects, such as a butterfly, stop sign, flower, or dragonfly. Identify lines of symmetry seen and how they connect to the object.
- Illustrative Mathematics:
  - Finding Lines of Symmetry
  - Lines of symmetry for quadrilaterals
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 4.GM.A.3 [Example 1] [Example 2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.GM.B - Solve problems involving measurement and conversion of measurements.

STANDARD: 4.GM.B.4

Standards Statement (2021):

Know relative sizes of measurement units and express measurements in a larger unit in terms of a smaller unit.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
2.GM.D.11, 3.GM.B.4	4.GM.B.5, 5.GM.C.4	3.OA.C.7, 6.RP.A.1	4.MD.A.1 4.GM.B Crosswalk

Standards Guidance:

Clarifications

- Students can convert within a single system of measurement, express measurements in a larger unit in terms of a smaller unit.
- Students can record measurement equivalents in a two-column table.

Boundaries

- Measurement units within one system a student should be familiar with include km, m, cm, kg, g, lb, oz, l, hr, min, sec.
- Instruction could familiarity of like units in different measurement systems, but does not need to include formal conversions between similar units (e.g. kilometer (km) and miles (mi), kilograms (kg) and pounds (lb), liters (l) to gallons (g), etc.)

Progressions

- Justify conversions using understanding that larger units can be partitioned into smaller equal sized units (MP.3)
- Relating units within the metric system is another opportunity to think about place value. For example, students might make a table that shows measurements of the same lengths in centimeters and meters. (Please reference page 20 in the Progression document).

Examples

- Know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 inches. Generate a conversion table for feet and inches listed as number pairs (1, 12), (2, 24), (3, 36), ….
- Illustrative Mathematics:
  - Who is the tallest?
- Student Achievement Partners:
  - Cobra vs. Iguana
  - Smarter Balanced Assessment Item Illustrating 4.GM.B.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.GM.B.5

Standards Statement (2021):

Apply knowledge of the four operations and relative size of measurement units to solve problems in authentic contexts that include familiar fractions or decimals.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.GM.B.4	5.GM.C.4	4.OA.A.3, 4.NF.B.3, 4.NF.B.4, 4.NF.C.5, 4.NF.C.6, 5.NF.B.3, 5.NF.B.5, 5.NF.B.6	4.MD.A.2 4.GM.B Crosswalk

Standards Guidance:

Clarifications

- Students should express larger units in terms of smaller units within the same measurement system
- Students should express smaller units in terms of larger units within the same measurement system.
- Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
- Student should experience problems involving distances, intervals of time, liquid volumes, masses of objects, and money.

Terminology

- Metric measurement units include weight (grams and kilograms) capacity (milliliters and liters), length (centimeter, meter, and kilometer).

Boundaries

- Word problems should involve simple fractions or decimals and expressing measurements given in a larger unit in terms of a smaller unit.
- Fractions should be limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Multiplication and division of fractions is not a requirement of this grade level.
- Contexts could include distance, intervals of time, liquid volumes, mass and money.

Teaching Strategies

- Represent measurement quantities using number line diagrams that feature a measurement scale.
- Students should reason about the relative sizes of measurement units within the metric system.
- Students should be able to accurately record measurement equivalents in a two-column table.

Progressions

- Students combine competencies from different domains as they solve measurement problems using all four arithmetic operations: addition, subtraction, multiplication, and division. For example, “How many liters of juice does the class need to have at least 35 cups if each cup takes 225 ml?” Students may use tape or number line diagrams for solving such problems (MP1). (Please reference page 20 in the Progression document).

Examples

- What time does Eric have to leave his house to get to the concert by quarter after nine, if the trip takes 90 minutes?
- Illustrative Mathematics: [Margie Buys Apples]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.GM.B.6

Standards Statement (2021):

Apply the area and perimeter formulas for rectangles in authentic contexts and mathematical problems.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.GM.C.7, 3.GM.D.8	6.GM.A.1	3.OA.A.4	4.MD.A.3 4.GM.B Crosswalk

Standards Guidance:

Clarifications

- Students should express their answers in linear (perimeter) and square (area) units. Students are not expected to use the 1 cm^2 notation.

Boundaries

- Students should not be expected to find unknown side lengths when exploring composite rectangles.

Progressions

- Such abstraction and use of formulas underscores the importance of distinguishing between area and perimeter in Grade 3 (3.MD.8 3) and maintaining the distinction in Grade 4 and later grades, where rectangle perimeter and area problems may get more complex and problem solving can benefit from knowing or being able to rapidly remind oneself of how to find an area or perimeter.
- By repeatedly reasoning about how to calculate areas and perimeters of rectangles, students can come to see area and perimeter formulas as summaries of all such calculations (MP8). (Please reference page 21 in the Progression document)

Examples

- Find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.
- Illustrative Mathematics:
  - Karl's Garden

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.GM.C - Geometric measurement: understand concepts of angle and measure angles.

STANDARD: 4.GM.C.7

Standards Statement (2021):

Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. Understand and apply concepts of angle measurement.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.GM.A.1	4.GM.A.2, 4.GM.C.8, 4.GM.C.9, 7.GM.B.4, HS.GM.A.4	N/A	4.MD.C.5 4.GM.C Crosswalk

Standards Guidance:

Clarifications

- An angle can be viewed as a wedge of a circle or a turn through a circular arc where 1/360 of the wedge or turn is one-degree.
- An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
- Draw right, acute, and obtuse angles

Teaching Strategies

- Students should also be able to explore this learning objective in the context of angles within circles.
- Angle measurement should be introduced with non- standard tools such as pattern blocks, unit angles, and/or wedges prior to introducing protractors. 360-degree protractors would make an explicit connection to the degrees of a circle and builds conceptual understanding of angles.

Progressions

- As with length, area, and volume, students need to understand equal partitioning and unit iteration to understand angle and turn measure. Whether defined as more statically as the measure of the figure formed by the intersection of two rays or as turning, having a given angle measure involves a relationship between components of plane figures and therefore is a property. (Please reference page 23 in the Progression document).

Examples

- The student can place four squares around the center of a circle. Since there are 360 degrees in a circle, 360 ÷ 4 = 90, so each square has 90-degree angles.
- Student Achievement Partners:
  - Angle Measure

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.GM.C.8

Standards Statement (2021):

Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.GM.C.7	4.GM.C.9	N/A	4.MD.C.6 4.GM.C Crosswalk

Standards Guidance:

Clarifications

- To understand measurement, students should measure in non-standard units, such as unit angles or wedges, before being introduced to tools with abstract units such as degrees.

Teaching Strategies

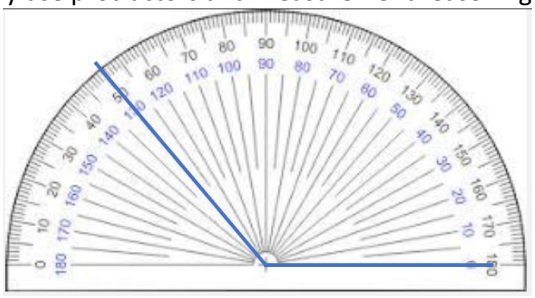
- Use angle measurement tools that help students connect non-standard units (wedges, unit angles, etc.) to standard units of angle measurement (degrees).

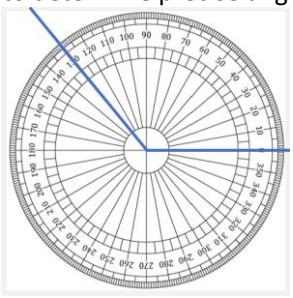
Progressions

- If examples and tasks are not varied, students can develop incomplete and inaccurate notions. For example, some come to associate all slanted lines with 45 degree measures and horizontal and vertical lines with measures of 90 degrees. Others believe angles can be "read off" a protractor in "standard" position, that is, a base is horizontal, even if neither arm of the angle is horizontal. Measuring and then sketching many angles with no horizontal or vertical arms, perhaps initially using circular 360 protractors, can help students avoid such limited conceptions. (Please reference page 23 in the Progression document).

Examples

- Students may be given angles to find precise measurements of angles. Here are some examples of how students may use protractors and measurement reasoning to determine precise angle measurements.





    - Sample student response: The angle is an obtuse angle because it is open more than a square corner angle. It measures 130 degrees.
- Illustrative Mathematics:
    - Measuring Angles

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 4.GM.C.9

Standards Statement (2021):

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.GM.C.7, 4.GM.C.8	7.GM.B.4	N/A	4.MD.C.7 4.GM.C Crosswalk

Standards Guidance:

Boundaries

- Expectation includes solving addition and subtraction problems to find unknown angles on a diagram in authentic contexts and mathematical problems such as by using an equation with a symbol for the unknown angle measure.

Progressions

- Students with an accurate conception of angle can recognize that angle measure is additive. As with length, area, and volume, when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Students can then solve interesting and challenging addition and subtraction problems to find the measurements of unknown angles on a diagram in real world and mathematical problems. (Please reference page 24 in the Progression document).

Examples

- Using an equation with a symbol for the unknown angle measure.
- Illustrative Mathematics:
  - Measuring Angles
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 4.MD.C.7

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.DR.A - Pose investigative questions and collect/consider data.

STANDARD: 4.DR.A.1

Standards Statement (2021):

Generate questions to investigate situations within the classroom, school or community. Determine strategies for collecting or considering data involving addition and subtraction of fractions that can naturally answer questions by using information presented in line plots.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.DR.A.1	5.DR.A.1	N/A	[new content] 4.DR Crosswalk

Standards Guidance:

Clarification

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.
- Students should be given opportunities to generate questions about things they notice and wonder from a real-life situation.

Terminology

- A statistical investigative question is one that requires data that will vary.

Teaching Strategies

- Students should be able to use rulers to measure to the nearest 1/8.
- By measuring repeatedly students learn that measurements can vary.
- Based on the posed question, create a plan that determines the appropriate population to survey and how to collect that data.

Progressions

- Students should be able to measure objects found in everyday life to collect data.
- Developing strategies for collecting data include students collaborating to determine ways to collect data.
- Data can be gathered from a variety of sources to answer the statistical investigative question posed.

Examples

- “How tall are the tomato plants in the class garden?” is a statistical investigative question because it anticipates variability in the lengths of the tomato plants.
- “How tall is the tomato plant right here?” is a question used to collect data to answer the investigative question.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 4.DR.B - Analyze, represent, and interpret data.

STANDARD: 4.DR.B.2

Standards Statement (2021):

Analyze line plots to display a distribution of numerical measurement data, which include displays of data sets of fractional measurements with the same denominator. Interpret information presented to answer investigative questions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.DR.B.2	5.DR.B.2	N/A	4.MD.B.4 4.DR Crosswalk

Standards Guidance:

Clarifications

- Students should be able to determine the appropriate representation for the type of data to be collected based on the statistical investigative question.
- Students should have opportunities to determine the difference between representations for categorical data and numerical data presented.
- Representations for data should include bar graphs, pictographs, and dot plots (line plots).

Terminology

- Dot plots and line plots can be used interchangeably.
- Numerical data: A data type expressed in numbers rather than natural language descriptions. This is sometimes called quantitative data.

Boundaries

- Fractional measurements can include 1/2, 1/4, 1/8 units.
- Students should record observations they notice about the shape of the distribution using informal language such as spread out and/or grouped.

Progressions

- Grade 4 students learn elements of fraction equivalence and arithmetic, including multiplying a fraction by a whole number and adding and subtracting fractions with like denominators. Students can use these skills to solve problems, including problems that arise from analyzing line plots. For example, with reference to the line plot above, students might find the difference between the greatest and least values in the data. (In solving such problems, students may need to label the measurement scale in eighths so as to produce like denominators. Decimal data can also be used in this grade.) (Please reference page 11 in the Progression document).

Examples

- Based on a class survey, the students determined each student's favorite flavor of ice cream. The student is able to determine that the best representation for the data would be a bar graph since the data are categorical.
- How long are the specimens in an insect collection? From a dot plot, find and interpret the difference in length between the longest and shortest specimens in an insect collection.
- Illustrative Mathematics: [Button Diameters]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4F: Grade 5 Math Standards and Guidance

Critical Areas of Focus

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Domains and Clusters

5.OA - Algebraic Reasoning: Operations

- 5.OA.A Write and interpret numerical expressions.
- 5.OA.B Analyze patterns and relationships.

5.NBT - Numeric Reasoning: Base Ten Arithmetic

- 5.NBT.A Understand the place value system.
- 5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NF - Numeric Reasoning: Fractions

- 5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.
- 5.NF.B Apply and extend previous understandings of multiplication and division.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5.GM - Geometric Reasoning and Measurement

- 5.GM.A Graph points on the coordinate plane to solve real-world and mathematical problems.
- 5.GM.B Classify two-dimensional figures into categories based on their properties.
- 5.GM.C Convert like measurement units within a given measurement system.
- 5.GM.D Geometric measurement: understand concepts of volume.

5.DR - Data Reasoning

- 5.DR.A Pose investigative questions and collect/consider data.
- 5.DR.B Analyze, represent, and interpret data.

Students should spend the large majority[1] of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

[1]At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see the K-8 major work of the grade developed by Student Achievement Partners.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.OA.A - Write and interpret numerical expressions.

STANDARD: 5.OA.A.1

Standards Statement (2021):

Write and evaluate numerical expressions that include parentheses.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	5.OA.A.2, 6.AEE.A.1	N/A	5.OA.A.1 5.OA.A Crosswalk

Standards Guidance:

Boundaries

- The expressions should be no more complex than the expressions one finds in a simple application of the associative and distributive properties.
- Simple expressions should only include two operations.
- Grouping symbols used in expressions may include parentheses, brackets, or braces.

Teaching Strategies

- Use of nested parentheses should be used in favor of brackets or braces in numerical expressions.
- Students should begin with concrete models. Concrete models may include color tiles or base ten blocks for constructing area models and rods for representing numerical values.

Examples

- Express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7).
- If this expression were tripled, then it could represented as 3 x (2 × (8 + 7)), where the number of open parentheses is the same as the number of closed parentheses.
- Karl brought 3 ten-packs of juice boxes to the class party. Joshua brought 4 six-packs of soda to the party. How many drinks did they bring altogether?
  - (3 × 10) + (4 × 6)
- Illustrative Mathematics:
  - Watch Out for Parentheses 1
  - Using Operations and Parentheses
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 5.OA.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.OA.A.2

Standards Statement (2021):

Write expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.OA.A.1	6.AEE.A.2, 6.AEE.A.3, 7.AEE.A.1	6.NS.B.4	5.OA.A.2 5.OA.A Crosswalk

Standards Guidance:

Boundaries

- The expressions should be no more complex than the expressions one finds in a simple application of the associative and distributive properties.
- Simple expressions should only include two operations.
- Grouping symbols used in expressions may include parentheses, brackets, or braces.
- Nested grouping symbols (more than one grouping symbol used within another grouping symbol in an expression) could also be used within expressions at this grade level.

Teaching Strategies

- Expressions included should contain numbers, operations, and grouping symbols.
- Students should begin with concrete models. Concrete models may include color tiles or base ten blocks for constructing area models and rods for representing numerical values.

Examples

- Recognize that 3 × (18,932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.
- Karl brought 3 ten-packs of juice boxes to the class party. Joshua brought 4 six-packs of soda to the party. How many drinks did they bring altogether?
    - (3 × 10) + (4 × 6)
- Express the calculation “Add 8 and 7, then multiply by 2” as 2×(8+7). Recognize that 12×(7+91) is twelve times as large as 7+91, without having to calculate the indicated sum or product.
- Illustrative Mathematics:
    - Video Game Scores
    - Comparing Products
    - Seeing is Believing
    - Words to Expressions 1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.OA.B - Analyze patterns and relationships.

STANDARD: 5.OA.B.3

Standards Statement (2021):

Generate two numerical patterns using two given rules. Identify and analyze relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph them on a coordinate plane.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.OA.C.5	6.AEE.A.2, 6.AEE.C.8	6.RP.A.1	5.OA.B.3 5.OA.B Crosswalk

Standards Guidance:

Clarifications

- This standard extends the work from fourth grade, where students generate numerical patterns when they are given one rule. In Fifth Grade, students are given two rules and generate two numerical patterns.

Boundaries

- Generating numerical patterns is a fourth grade standard, therefore is also an expectation for 5th grade.
- This learning objective is limited to patterns involving whole numbers.

Examples

- Given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences.
    - Identify and explain why the terms in one sequence are twice the value of the terms in the corresponding sequence.
- Sam and Terri live by a lake and enjoy going fishing together every day for five days. Sam catches 2 fish every day, and Terri catches 4 fish every day. Make a chart (table) to represent the number of fish that Sam and Terri catch.
- Illustrative Mathematics:
    - Sidewalk Patterns
- Student Achievement Partners:
    - Assessment Item Illustrating 5.OA.B.3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.NBT.A - Understand the place value system.

STANDARD: 5.NBT.A.1

Standards Statement (2021):

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NBT.A.1, 4.NBT.A.2, 4.NF.B.3	5.NBT.A.2, 5.NBT.A.3, 5.NBT.A.4, 5.NBT.B.5, 5.NBT.B.6, 5.NBT.B.7	4.NF.C.5, 4.NF.C.6, 4.NF.C.7	5.NBT.A.1 5.NBT.A Crosswalk

Standards Guidance:

Clarifications

- Students should identify the value of a digit up 100 times greater or 11000 of the value of a digit.
- Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Examples

- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.
    - For example, 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
    - 700 is 10 times as much as 70, and 70 is 1/10 of 700.
- Mara has a digital scale. He placed one playing card on the scale and it read 1.3 grams. How much would you expect 10 playing cards to weigh?
- Chris took the cards off the scale and then placed 10 pennies on the scale and the scale read 24 grams. How much would you expect one penny to weigh.
- Illustrative Mathematics:
    - Tenths and Hundredths
    - Kipton's Scale
    - Which number is it?
- Student Achievement Partners:
    - Decimal Place Value
    - Assessment Item Illustrating 5.NBT.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.NBT.A.2

Standards Statement (2021):

Use whole number exponents to denote powers of 10 and explain the patterns in placement of digits that occur when multiplying and/or dividing whole numbers and decimals by powers of 10.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.NBT.A.1	6.AEE.A.1, 8.AEE.A.3	N/A	5.NBT.A.2 5.NBT.A Crosswalk

Standards Guidance:

Clarifications

- Students should explain what happens to the value of a digit as it shifts to the left or right and discover the decimal point remains between the ones and tenths place as the digits shift.
- Use whole-number exponents to denote powers of 10, up to 10^3.

Boundaries

- Work with exponents at this grade is limited to powers of 10.

Progressions

- New at Grade 5 is the use of whole number exponents to denote powers of 10. Students understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left.
- For example, multiplying by 104 is multiplying by 10 four times. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right. So multiplying by 10 four times shifts every digit 4 places to the left. Patterns in the number of 0s in products of a whole number and a power of 10 can be explained in terms of place value. (Please reference page 18 in the Progression document).

Examples

- Observe and explain the patterns in the number of zeros of a product when multiplying a whole number by a power of 10, and the placement of the decimal point when a decimal is multiplied or divided by a power of 10.
- Illustrative Mathematics:
  - Marta's Multiplication Error

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.NBT.A.3

Standards Statement (2021):

Read, write, and compare decimals to thousandths.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NBT.A.2, 5.NBT.A.1, 4.NF.C.7	5.NBT.A.4	6.AEE.B.7	5.NBT.A.3 5.NBT.A Crosswalk

Standards Guidance:

Clarification

- Read and write decimals to thousandths using standard form, expanded form, and word from.
- Compare two decimals to thousandths based on meanings of the digits in each place, and record the results of the comparisons using >,=, and <.

Boundaries

- Students should be provided opportunities to simultaneously compare decimals and fractions, including equivalent fractions and decimals, on both single and double number lines.
- Base-ten numerals should range between millions and thousandths.
- Students are not expected to write decimal numbers in word form.
- Exponents and decimal numbers should not be included in expanded form notation.
- The decimal fractions used in Grade 5 should be limited to those for which the equivalent fraction can be written as a fraction where the denominator is a power of ten.

Teaching Strategies

- Students should be presented with decimal number comparisons from contextual, mathematical situations.
- Students should have opportunities to determine and explain comparisons using a variety of tools such as concrete materials, drawings, number lines, other visual representations, and strategies.

Examples

- Use >, =, and < symbols to record comparisons of two decimals. For example:
  - 347.392 =
  - = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
  - =three hundred forty-seven and three hundred ninety-two thousandths
- 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (110) + 9 × (1100) + 2 × (11000)
- Which is greater 0.13 or 0.031? Explain. Use a visual representation to illustrate your explanation.
  - I think 0.13 is greater because it fills up more of the whole square than 0.031 does.
- Illustrative Mathematics:
  - Are these equivalent to 9.52?
  - Placing Thousandths on the Number Line
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 5.NBT.A.3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.NBT.A.4

Standards Statement (2021):

Use place value understanding to round decimals to any place.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NBT.A.3, 5.NBT.A.1, 5.NBT.A.3	N/A	8.AEE.A.3	5.NBT.A.4 5.NBT.A Crosswalk

Standards Guidance:

Boundaries

- Work with decimals at this grade is limited to decimals up to the thousandths.

Teaching Strategies

- Students should round decimal numbers to the hundredths place in contextual, mathematical problems using visual aids, such as a number line.

Examples

- Illustrative Mathematics:
  - Rounding to Tenths and Hundredths

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.NBT.B - Perform operations with multi-digit whole numbers and with decimals to hundredths.

STANDARD: 5.NBT.B.5

Standards Statement (2021):

Fluently multiply multi-digit whole numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NBT.B.4, 4.NBT.B.5, 5.NBT.A.1	6.NS.B.3	N/A	5.NBT.B.5 5.NBT.B Crosswalk

Standards Guidance:

Terminology

- The National Council of Teachers of Mathematics provides the following definition of procedural fluency:
  - "Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another.

Boundaries

- Students may use but are not limited to partial products (area model).
- Students may also use a standard algorithm by making connections from previous part-whole strategies.
- Students should choose a strategy that makes sense to them based on the context of the problem. The focus should always be on efficiency.

Teaching Strategies

- Students should be presented with contextual, real-life situations involving multiplication of multi-digit whole numbers.
- Students should fluently (flexibly, accurately, and efficiently) multiply to solve contextual, mathematical problems using efficient strategies that are based on knowledge of place value and properties of operations.

Examples

- Student Achievement Partners:
  - Multi-Digit Multiplication Using the Standard Algorithm Mini-Assessment
  - Assessment Item Illustrating 5.NBT.B.5

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.NBT.B.6

Standards Statement (2021):

Use a variety of representations and strategies to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NBT.B.4, 4.NBT.B.6, 5.NBT.A.1	6.NS.B.2, 6.NS.B.3	N/A	5.NBT.B.6 5.NBT.B Crosswalk

Standards Guidance:

Clarification

- Use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
- Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models

Boundaries

- Students should divide multi-digit whole numbers up to 4- digit dividends and 2-digit divisors no greater than 25.
- Students may use but are not limited to partial quotients (area model).
- Students should choose a strategy that makes sense to them based on the context of the problem. The focus should always be on efficiency.

Teaching Strategies

- Students should be presented with contextual, real-life situations involving the division of multi-digit whole numbers.
- Students should fluently (flexibly, accurately, and efficiently) divide, to solve contextual, mathematical problems using an efficient algorithm and flexible strategies, based on knowledge of place value and properties of operations.
- Examples of different strategies and representations can be found within the Computational Strategies for Whole Numbers document found in the appendices.

Progressions

- Division in Grade 5 extends Grade 4 methods to two-digit divisors. Students continue to decompose the dividend into base-ten units and find the quotient place by place, starting from the highest place.
- Estimating the quotients is a new aspect of dividing by a two-digit number. Even if students round the dividend appropriately, the resulting estimate may need to be adjusted up or down. (Please reference page 18 in the Progression document).

Examples

- Student Achievement Partners:
  - Division Accuracy

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.NBT.B.7

Standards Statement (2021):

Use a variety of representations and strategies to add, subtract, multiply, and divide decimals to hundredths. Relate the strategy to a written method and explain the reasoning used.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NBT.B.4, 5.NBT.A.1, 5.NF.A.1	6.NS.B.3	5.NF.B.4, 5.GM.A.1, 5.GM.C.4	5.NBT.B.7 5.NBT.B Crosswalk

Standards Guidance:

Clarification

- As part of this standard, students must be able to use concrete models, visual drawings and strategies based on place value, properties of operations, and the relationship between addition and subtraction.

Boundaries

- Fluency with operations with decimals is part of the 6th grade standards.
- Students should be given the choice of which strategy they can use.

Teaching Strategies

- Students should be presented with a variety of contextual, real-life situations involving addition and subtraction of decimal numbers to the hundredths place.
- Students should add and subtract decimal numbers to hundredths, using concrete models, drawings, strategies based on place value, properties of operations, and the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Progressions

- Because of the uniformity of the structure of the base-ten system, students use the same place value understanding for adding and subtracting decimals that they used for adding and subtracting whole numbers. Like base-ten units must be added and subtracted, so students need to attend to aligning the corresponding places correctly (this also aligns the decimal points).
- General methods used for computing products of whole numbers extend to products of decimals. Because the expectations for decimals are limited to thousandths and expectations for factors are limited to hundredths at this grade level, students will multiply tenths with tenths and tenths with hundredths, but they need not multiply hundredths with hundredths.
- General methods used for computing quotients of whole numbers extend to decimals with the additional issue of placing the decimal point in the quotient. (Please reference page 19 in the Progression document)

Examples

- Illustrative Mathematics: [The Value of Education]
- Student Achievement Partners: [Assessment Item Illustrating 5.NBT.B.7]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.NF.A - Use equivalent fractions as a strategy to add and subtract fractions.

STANDARD: 5.NF.A.1

Standards Statement (2021):

Add and subtract fractions with unlike denominators, including common fractions larger than one and mixed numbers.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NF.A.1, 4.NF.B.3	5.NF.A.2, 5.NBT.B.7, 7.NS.A.1	6.AEE.B.6	5.NF.A.1 5.NF.A Crosswalk

Standards Guidance:

Terminology

- A common fraction is a fraction in which numerator and denominator are both integers, as opposed to fractions. Fractions such as 4/3, or 14/5 should be thought of as common fractions greater than one, which could also be written using mixed numbers as 1-1/3 and 2-4/5 respectively.
- Use of the term "improper fraction" should be avoided.

Boundaries

- Work with fractions at grade 5 should focus on fractions with denominators 2-10, 12, 16, 20, 25, 50, 100 and 1000.

Progressions

- In Grade 4, students have some experience calculating sums of fractions with different denominators...where one denominator is a divisor of the other, so that only one fraction has to be changed.
- Grade 5 students extend this reasoning to situations where it is necessary to re-express both fractions in terms of a new denominator. For example, in calculating 2/3 + 5/4 they reason that if each third in 2/3 is subdivided into fourths, and if each fourth in 5/4 is subdivided into thirds, then each fraction will be a sum of unit fractions with denominator 3 x 4 = 4 x 3 = 12:
  - 2/3 + 5/4 = 2x4 / 3x4 + 5x3 / 4x3 = 8/12 + 15/12 = 23/12.
- Please reference page 11 in the Progression document

Examples

- Include replacing given fractions with equivalent fractions to produce an equivalent sum or difference.
  - 2/3 + 5/4 = 8/12 + 15/12 = 23/12 or 1 11/12.
- Illustrative Mathematics:
  - Finding Common Denominators to Add
  - Finding Common Denominators to Subtract
- Student Achievement Partners:
  - Assessment Item Illustrating 5.NF.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.NF.A.2

Standards Statement (2021):

Solve problems in authentic contexts involving addition and subtraction of fractions with unlike denominators, including common fractions larger than one and mixed numbers.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NF.A.2, 4.NF.C.5, 5.NF.A.1	6.NS.B.2, 6.NS.B.3	5.DR.B.2, 5.GM.A.2	5.NF.A.2 5.NF.A Crosswalk

Standards Guidance:

Clarifications

- Use visual fraction models or equations to represent the problem.
- Use benchmark fractions and number sense of fractions to estimate and assess the reasonableness of answers.
- Students should use benchmark fractions and number sense of fractions to estimate and assess the reasonableness of answers as an introduction to addition and subtraction.

Boundaries

- Work with fractions at grade 5 should focus on fractions with denominators 2-10, 12, 16, 25, 100 and 1000.

Teaching Strategies

- Students should use numerical reasoning to add and subtract fractions and mixed numbers with unlike denominators in contextual, mathematical problems by finding a common denominator and equivalent fractions to produce like denominators using a variety of tools and strategies.

Progressions

- Students make sense of fractional quantities when solving word problems, estimating answers mentally to see if they make sense. For example in the problem:
    - Ludmilla and Lazarus each have a lemon. They need a cup of lemon juice to make hummus for a party. Ludmilla squeezes 1/2 a cup from hers and Lazarus squeezes 2/5 of a cup from his. How much lemon juice do they have? Is it enough?
- Students estimate that there is almost but not quite one cup of lemon juice, because 2/5 < 1/2. They calculate 1/2 + 2/5 = 9/10, and see this as 1/10 less than 1, which is probably a small enough shortfall that it will not ruin a recipe. (Please reference page 11 in the Progression document).

Examples

- Tom is baking a cake. He added 12 teaspoon of vanilla extract to the cake mix. He tasted the batter and determined he needed more, so he added another 34 teaspoon of vanilla extract. How much total vanilla extract did he add to the cake mix?
- Illustrative Mathematics: [Do These Add Up?]
- Student Achievement Partners: [Smarter Balanced Assessment Item Illustrating 5.NF.A.2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.NF.B - Apply and extend previous understandings of multiplication and division.

STANDARD: 5.NF.B.3

Standards Statement (2021):

Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve problems in authentic contexts involving division of whole numbers that result in answers that are common fractions or mixed numbers.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NF.B.3, 4.NF.B.4	6.RP.A.1, 6.RP.A.2, 6.RP.A.3	3.OA.A.2, 3.OA.B.6, 4.OA.A.1, 4.OA.A.2, 4.GM.B.5, 6.AEE.B.6	5.NF.B.3 5.NF.B Crosswalk

Standards Guidance:

Boundaries

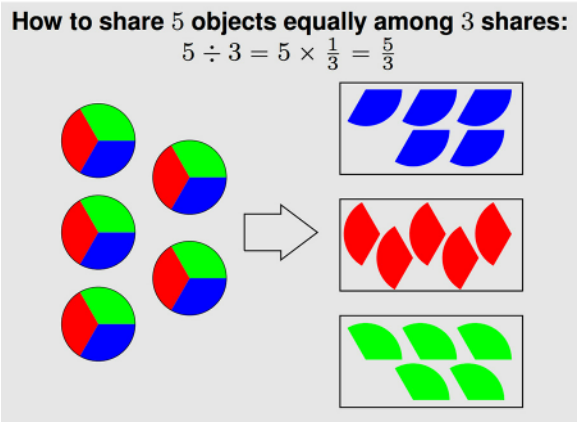
- As part of this standard, students should have opportunities to use visual models or equations to represent and solve problems.

Progressions

- In Grade 5, [students] connect fractions with division, understanding that
  - 5 div 3 = 5/3,
- or, more generally, a/b = a div b for whole numbers a and b, with b not equal to zero. (Please reference page 17 in the Progression document).

Examples

- If 5 cookies are shared equally among 3 people each person receives $\frac{5}{3}$ of a cookie.
  - If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute 1/3 of itself to each share.
  - Thus, each share consists of 5 pieces, each of which is 1/3 of an object, and so each share is $5 \times \frac{1}{3} = \frac{5}{3}$ of an object



- Illustrative Mathematics:
  - How Much Pie?
  - What is 23 ÷ 5?
- Student Achievement Partners:
  - Sharing Chocolate
  - Multiplication and Division of Fractions Mini-Assessment
  - Smarter Balanced Assessment Item Illustrating 5.NF.B.3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.NF.B.4

Standards Statement (2021):

Apply and extend previous understanding and strategies of multiplication to multiply a fraction or whole number by a fraction. Multiply fractional side lengths to find areas of rectangles, and represent fractional products as rectangular areas.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.OA.A.1, 3.GM.C.7, 4.OA.A.1, 4.NF.B.4	6.RP.A.1, 6.RP.A.2, 6.RP.A.3	5.NBT.B.7, 6.NS.B.2, 6.AEE.B.6, 6.GM.A.1, 7.NS.A.2	5.NF.B.4 5.NF.B Crosswalk

Standards Guidance:

Boundaries

- Students should explain the meaning of a fraction a/b as a multiple of $1/b$.
- Students should be exposed to fractions less than 1, equal to 1, and greater than 1.

Teaching Strategies

- Interpret the product of the fraction a/b and a whole number (q) as
  - partitioning the whole number into b parts and counting a parts
  - Repeating the fraction a/b q number of times.
- Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths
- Students should be presented with a variety of real-life, mathematical problems involving multiplication of a fraction and a whole number.
- Students should use their understanding of equivalency to flexibly reason with equivalent fractions based on the context of the problem. Simplifying fractions is not an expectation of this grade level.

Progressions

- Students should use a variety of models to conecptualize multiplicaiton of fractions, including use of a number line, fraction strip, and area models. Please reference page 17 in the Progression document for additional information.

Examples

- Understand that ⅔ x 4 can be seen as partitioning 4 into 3 equal parts as well as counting 2 of the 3 (4/3 x 2) parts or as iterating ⅔ four times [(2 x 4)/3]. In general, a/b x q = q/b x a = (a x q)/b.
- Use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15.
- Each cupcake takes 1/4 cup of frosting. If Betty wants to make 20 cupcakes for a party, how much frosting will she need?
- Illustrative Mathematics: [Connor and Makayla Discuss Mult.] [Cross Country Training]
- Student Achievement Partners: [Multiplication and Division of Fractions Mini-Assessment] [Smarter Balanced Assessment Item Illustrating 5.NF.B.4]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.NF.B.5

Standards Statement (2021):

Apply and extend previous understandings of multiplication and division to represent and calculate multiplication and division of fractions. Interpret multiplication as scaling (resizing) by comparing the size of products of two factors.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.NF.A.1, 4.NF.C.5	6.RP.A.1	3.OA.A.1, 3.OA.A.2, 4.OA.A.1, 4.OA.A.2, 4.GM.B.5	5.NF.B.5 5.NF.B Crosswalk

Standards Guidance:

Boundaries

- As part of this standard, students must be able to
  - Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
  - Explain that multiplying a given number by a fraction greater than 1 results in a product greater than the given number.
  - Explain that multiplying a given number by a fraction equivalent to 1 (such as 4/4) results in the same product as multiplying by 1.
  - Explain that multiplying a given number by a fraction less than 1 results in a product smaller than the given number.

Teaching Strategies

- Students should be presented with a variety of real-life, mathematical situations involving multiplication as scaling (resizing) that include fractions and whole numbers.

Progressions

- In preparation for Grade 6 work in ratios and proportional relationships, students learn to see products such as 5 x 3 or 1/2 x 3 as expressions that can be interpreted in terms of a quantity, 3, and a scaling factor, 5 or 1/2. Thus, in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is 5 times as big as 3, without evaluating the product. Likewise, the see 1/2 x 3 as half the size of 3.
- Grade 5 work with multiplying by unit fractions, and interpreting fractions in terms of division, enables students to see that multiplying a quantity by a number smaller than 1 produces a smaller quantity, as when the budget of a large state university is multiplied by 1/2, for example. (Please reference page 19 in the Progression document).

Examples

- Mrs. Cole needs to make lunch for 12 children at a day care. Each child gets 1/2 of a sandwich. How many whole sandwiches does Mrs. Cole need to make?
- Illustrative Mathematics: [Comparing a Number and a Product] [Calculator Trouble] [Reasoning about Multiplication]
- Student Achievement Partners: [Multiplication and Division of Fractions Mini-Assessment] [Smarter Balanced Assessment Item Illustrating 5.NF.B.5]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.NF.B.6

Standards Statement (2021):

Solve problems in authentic contexts involving multiplication of common fractions and mixed numbers.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.OA.A.2	6.RP.A.1, 6.NS.A.1	3.OA.A.1, 3.OA.A.2, 4.OA.A.1, 4.GM.B.5, 5.DR.B.2	5.NF.B.6 5.NF.B Crosswalk

Standards Guidance:

Teaching Strategies

- Students should be given opportunities to use both visual fraction models and equations to represent and solve problems.
- Students should be given opportunities to use both visual fraction models and equations to represent and solve problems.

Progressions

- Solve applied problems involving multiplication of fractions and mixed numbers by using visual fraction models and/or equations to represent the problem.

Examples

- Illustrative Mathematics:
  - Making Cookies
  - Comparing Heights of Buildings
  - Drinking Juice
  - Half of a Recipe
  - New Park
  - Running to School
- Student Achievement Partners:
  - Forest Road
  - Multi-Domain Application Mini-Assessment
  - Smarter Balanced Assessment Item Illustrating 5.NF.B.6

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.NF.B.7

Standards Statement (2021):

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions, including solving problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NF.A.1, 4.NF.B.4	6.RP.A.2, 6.NS.A.1	3.OA.B.6, 5.DR.B.2, 6.DR.A.1	5.NF.B.7 5.NF.B Crosswalk

Standards Guidance:

Boundaries

- Division of a fraction by a fraction is not a requirement at this grade. However, students who are able to multiply fractions can develop strategies to divide a fraction by a fraction by reasoning about the relationship between multiplication and division.

Teaching Strategies

- Students should be given opportunities to use both visual fraction models and equations to represent and solve problems.
- Students should begin with modeling for deeper understanding.
- Students should be presented with a variety of contextual, real-life problems involving division of a whole number by a unit fraction and division of a unit fraction by a whole number.

Examples

- Create a story context for (1/3) ÷ 4 and use a visual fraction model to show the quotient.
- Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.
- Create a story context for 4 ÷ (1/5) and use a visual fraction model to show the quotient.
- Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.
- How much chocolate will each person get if 3 people share ½ lb of chocolate equally?
- How many ⅓-cup servings are in 2 cups of raisins?
- Knowing the number of groups/shares and finding how many/much in each group/share Four students sitting at a table were given 1/3 of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally? The diagram shows the 1/3 pan divided into 4 equal shares with each share equaling 1/12 of the pan.
- Illustrative Mathematics:
  - Banana Pudding
  - How many servings of oatmeal?
- Student Achievement Partners:
  - Multiplication and Division of Fractions Mini-Assessment
  - Smarter Balanced Assessment Item Illustrating 5.NF.B.7

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.GM.A - Graph points on the coordinate plane to solve real-world and mathematical problems.

STANDARD: 5.GM.A.1

Standards Statement (2021):

Graph and name coordinate points in the first quadrant using the standard (x, y) notation. Understand the coordinate points values represent the distance traveled along the horizontal x-axis and vertical y-axis.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NF.A.2	5.GM.A.2	5.NBT.B.7, 6.NS.C.6	5.G.A.1 5.GM.A Crosswalk

Standards Guidance:

Clarifications

- This is students first formalized introduction to the conventions of coordinate graphing:
  - The first number indicates the distance from the origin on the x-axis.
  - The second number indicates the distance from the origin on the y-axis.
  - The names of the two axes and coordinates (or ordered pairs) correspond (x-axis and x-coordinate, y-axis and y-coordinate).
- In addition to whole numbers, ordered pairs should include the decimal and fractional values of halves and fourths.

Boundaries

- Graphing beyond the first quadrant is not a requirement at this grade.
- All four quadrants of the coordinate plane can be displayed, but students will only plot and label within the first quadrant.

Teaching Strategies

- Students should be provided with a variety of real-life, mathematical problems involving graphing points in the first quadrant.
- Students should interpret coordinate values of points in the context of the problem or situation.

Progressions

- Although students can often “locate a point,” these understandings are beyond simple skills. For example, initially, students often fail to distinguish between two different ways of viewing the point (2, 3), say, as instructions: “right 2, up 3”; and as the point defined by being a distance 2 from the y -axis and a distance 3 from the x -axis. In these two descriptions the 2 is first associated with the x -axis, then with the y -axis. (Please reference page 17 in the Progression document).

Examples

- Illustrative Mathematics:
  - Battle Ship Using Grid Paper
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 5.GM.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.GM.A.2

Standards Statement (2021):

Represent authentic contexts and mathematical problems by graphing points in the first quadrant of the coordinate plane. Interpret the meaning of the coordinate values based on the context of a given situation.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.NF.A.2, 5.GM.A.1	6.GM.A.3, 6.NS.C.8	5.NF.A.2, 6.RP.A.3	5.G.A.2 5.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students should be given ample experience with organizing, representing, and analyzing data from real-life contexts.
- Data should not be limited to numerical data collected from linear measurements.
- Students should continue to create dot plots (line plots) with measurements in fractions of a unit.

Terminology

- Distribution refers to how the data is spread across the graph.
- Dot plots and line plots can be used interchangeably.
- Dot plots should be used for numerical data representation on a number line.
- Numerical data is data that expressed in numbers rather than natural language. An example of numerical data that could be collected is the number of people who attended the movie theater over the course of a month.
- Categorical data is a type of data that is used to group information with similar characteristics. Examples of categorical data that could be collected might be marital status, favorite sport, or favorite type of movie.

Progressions

- Students connect ordered pairs of (whole number) coordinates to points on the grid, so that these coordinate pairs constitute numerical objects and ultimately can be operated upon as single mathematical entities. Students solve mathematical and real-world problems using coordinates.
- For example, they plan to draw a symmetric figure using computer software in which students' input coordinates that are then connected by line segments. (Please reference pages 17-18 in the Progression document).

Examples

- The coordinate (1,1.5) or (1,1½) means that in the first year, a person grew 1.5 or 1 ½ inches.
- Numerical variable(s): number of pets; categorical variable(s): type of pets, (e.g., cats, dogs, hamsters)"
- Illustrative Mathematics: [Meerkat Coordinate Plane Task]
- Student Achievement Partners: [Lighthouse] [Multi-Domain Application Mini-Assessment] [Smarter Balanced Assessment Item Illustrating 5.GM.A.2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.GM.B - Classify two-dimensional figures into categories based on their properties.

STANDARD: 5.GM.B.3

Standards Statement (2021):

Classify two-dimensional figures within a hierarchy based on their geometrical properties, and explain the relationship across and within different categories of these figures.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.GM.A.1, 4.GM.A.2	6.GM.A.3	N/A	5.G.B.4 5.GM.B Crosswalk

Standards Guidance:

Clarifications

- Students should explore, compare, and contrast polygons based on properties.
- Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

Boundaries

- This objective does not require students to create a hierarchy.
- Standards expecations inclue the inclusive definitions for the classification of shapes are used.
- Polygons should include triangles, quadrilaterals including kites and trapezoids (rectangles, squares, rhombuses, and other parallelograms), pentagons, hexagons, and octagons.
- Properties may include angles, side lengths, symmetry, congruence, and the presence or absence of parallel or perpendicular lines

Progressions

- Based on analysis of properties, students classify two-dimensional figures in hierarchies. For example, they conclude that all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite, parallel, equal-length sides (MP3). In this way, they relate certain categories of shapes as subclasses of other categories.
- This leads to understanding propagation of properties; for example, students understand that squares possess all properties of rhombuses and of rectangles. Therefore, if they then show that rhombuses' diagonals are perpendicular bisectors of one another, they infer that squares' diagonals are perpendicular bisectors of one another as well. (Please reference page 18 in the Progression document).

Examples

- Explain that since all rectangles have four right angles, and squares are rectangles, then all squares have four right angles.
- Explain that parallelograms and trapezoids are both quadrilaterals, and both have at least one set of parallel sides, but that they differ in that trapezoids have exactly one set and parallelograms have exactly two sets.
- Illustrative Mathematics: [Always, Sometimes, Never] [What do these shapes have in Common?]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.GM.C - Convert like measurement units within a given measurement system.

STANDARD: 5.GM.C.4

Standards Statement (2021):

Convert between different-sized standard measurement units within a given measurement system. Use these conversions in solving multi-step problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.GM.B.4, 4.GM.B.5	HS.NQ.B.3	5.NBT.B.7	5.MD.A.1 5.GM.C Crosswalk

Standards Guidance:

Boundaries

- Fifth grade is the first time students are expected to convert between different units within the same measurement system.
- Conversion chart should be provided.
- Students should be presented with contextual problems involving distance, weight, volume, and time that are practical and relevant to their everyday lives.
- Students should have opportunities to solve problems involving conversions within both metric and customary systems
    - Customary measurement units include weight (oz., lbs., tons) capacity (fl. oz, cups, pints, quarts, gallons), distance (in., ft., yds., miles).
    - Common metric units include weight (grams), capacity (liters), distance (meters)
        - Common metric conversions include Kilo- (1000), centi- (1/100), & milli- (1/1000)

Teaching Strategies

- Instruction could include grade appropriate use of different measurement systems and the conversion between similar units (e.g. kilometer (km) and miles (mi), kilograms (kg) and pounds (lb), liters (l) to gallons (g), etc).

Progressions

- In Grade 5, students extend their abilities from Grade 4 (4.MD.A.1) to express measurements in larger or smaller units within a measurement system. This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and make connections between fractions and decimals (e.g., 2 1/2 meters can be expressed as 2.5 meters or 250 centimeters). (Please reference page 26 in the Progression document).

Examples

- Convert 5 cm to 0.05 m
- Convert 1 gallon = 4 quarts = 8 pints = 16 cups.
- Illustrative Mathematics: [Minutes and Days] [Converting Fractions of a Unit into a Smaller Unit]
- Student Achievement Partners:
    - Multi-Domain Application Mini-Assessment
    - Smarter Balanced Assessment Item Illustrating 5.GM.C.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.GM.D - Geometric measurement: understand concepts of volume.

STANDARD: 5.GM.D.5

Standards Statement (2021):

Recognize that volume is a measurable attribute of solid figures.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
3.GM.C.5	5.GM.D.6, 5.GM.D.7	N/A	5.MD.C.3 5.GM.C Crosswalk

Standards Guidance:

Teaching Strategies

- A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
- A solid figure that can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

Progressions

- "Packing" volume is more difficult than iterating a unit to measure length and measuring area by tiling. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube (5.GM.D.5). They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build (5.GM.D.6). (Please reference page 27 in the Progression document).

Examples

- Student Achievement Partners:
  - Volume Units
  - Box of Clay
  - Assessment Item Illustrating 5.GM.D.5

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.GM.D.6

Standards Statement (2021):

Measure the volume of a rectangular prism by counting unit cubes using standard and nonstandard units.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.GM.A.2, 5.GM.D.5	5.GM.D.7	N/A	5.MD.C.4 5.GM.D Crosswalk

Standards Guidance:

Teaching Strategies

- Students should have opportunities to use metric, customary and improvised units

Progressions

- "Packing" volume is more difficult than iterating a unit to measure length and measuring area by tiling. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube (5.GM.D.5). They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build (5.GM.D.6). (Please reference page 27 in the Progression document).

Examples

- Student Achievement Partners:
  - Box of Clay
  - Smarter Balanced Assessment Item Illustrating 5.GM.D.6

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 5.GM.D.7

Standards Statement (2021):

Relate volume of rectangular prisms to the operations of multiplication and addition. Solve problems in authentic contexts involving volume using a variety of strategies.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.GM.D.5, 5.GM.D.6	6.GM.A.2	N/A	5.MD.C.5 5.GM.D Crosswalk

Standards Guidance:

Terminology

- Total volume is defined as the total number of units that fill the space.
  - A solid figure packed with n unit cubes is said to have a volume of n cubic units.
- The dimensions of a rectangular prism can be referred to as length, width, and height.
- A cube with side length 1 unit, called "a unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume (e.g., cubic cm, cubic m, cubic in, cubic ft).

Boundaries

- Work with volume at fifth grade is limited to whole number edge lengths.
- If students are provided with an image of a right rectangular prism, the unit cubes should be visible.

Teaching Strategies

- Students should be provided opportunities to use a variety of strategies including counting cubes, addition and multiplication, and applying the formula.
- Students should explore the volume of solid figures from real-life contexts by packing them with unit cubes with no gaps or overlaps.

Progressions

- Students understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes. They also learn that the height of the prism tells how many layers would fit in the prism. (Please reference page 27 in the Progression document).

Examples

- Find the volume of a rectangular prism with whole-number side lengths by packing it with unit cubes.
- Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping rectangular prisms by adding the volumes of the non-overlapping parts.
- Given the volume and 2 side lengths, determine the missing side length.
- Illustrative Mathematics: [Cari's Aquarium] [Breaking Apart Composite Solids]
- Student Achievement Partners: [Box of Clay] [Multi-Domain Application Mini-Assessment] [Smarter Balanced Assessment Item Illustrating 5.MD.C.5]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.DR.A - Pose investigative questions and collect/consider data.

STANDARD: 5.DR.A.1

Standards Statement (2021):

Generate questions to investigate situations within the classroom, school or community. Determine strategies for collecting or considering data involving operations with fractions for this grade that can naturally answer questions by using information presented in line plots.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.DR.A.1	6.DR.A.1, 6.DR.B.2	N/A	[new content] 5.DR Crosswalk

Standards Guidance:

Clarifications

- Students can generate questions about things they notice and wonder from a real-life situation.
- Based on the posed question, create a plan that determines the appropriate population to survey and how to collect that data.

Terminology

- A statistical investigative question is one that requires data that will vary.

Boundaries

- Expectations in this domain should be taught throughout the year and applied contextually to the current expectation and real-life events.

Teaching Strategies

- Students should be provided with learning experiences to collect and analyze both numerical data and categorical data.
- Developing strategies for collecting data include students collaborating to determine ways to collect data.
- Data can be gathered from a variety of sources to answer the statistical investigative question posed.

Examples

- Survey question: “How many pets do you have at home?” and “What grade are you in?” to make sure that the sample included only 5th grade students.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 5.DR.B - Analyze, represent, and interpret data.

STANDARD: 5.DR.B.2

Standards Statement (2021):

Analyze graphical representations and describe the distribution of the numerical data through line plots or categorical data through bar graphs. Interpret information presented to answer investigative questions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.DR.B.2	6.DR.C.3, 6.DR.D.4	5.NF.B.6, 5.NF.B.7	5.MD.B.2 5.DR Crosswalk

Standards Guidance:

Clarifications

- Students should be given ample experience with organizing, representing, and analyzing data from real-life contexts.
- Data should not be limited to numerical data collected from linear measurements.
- Students should continue to create dot plots (line plots) with measurements in fractions of a unit (1/2, 1/4, 1/8).

Terminology

- Distribution refers to how the data is spread across the graph.
- Dot plots and line plots can be used interchangeably.
- Dot plots should be used for numerical data representation on a number line.
- Numerical data is data that expressed in numbers rather than natural language. An example of numerical data that could be collected is the number of people who attended the movie theater over the course of a month.
- Categorical data is a type of data that is used to group information with similar characteristics. Examples of categorical data that could be collected might be favorite sport, or favorite type of movie.

Boundaries

- The mean formula is not an expectation in 5th grade. This concept should be explored visually and conceptually.
- This is the beginning of the progression of the concept of measures of center and will continue to be developed in 6th grade.

Progressions

- Grade 5 students grow in their skill and understanding of fraction arithmetic, including multiplying a fraction by a fraction, dividing a unit fraction by a whole number or a whole number by a unit fraction, and adding and subtracting fractions with unlike denominators. Students can use these skills to solve problems, including problems that arise from analyzing line plots. (Please reference page 11 in the Progression document).

Examples

- Numerical variable(s): number of pets; categorical variable(s): type of pets, (e.g., cats, dogs, hamsters)
- Illustrative Mathematics: [Fractions on a Line Plot]
- Student Achievement Partners: [Smarter Balanced Assessment Item Illustrating 5.DR.B.2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4G: Grade 6 Math Standards and Guidance

Critical Areas of Focus

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Domains and Clusters

6.AEE - Algebraic Reasoning: Expressions and Equations

- 6.AEE.A Apply and extend previous understandings of arithmetic to algebraic expressions.
- 6.AEE.B Reason about and solve one-variable equations and inequalities.
- 6.AEE.C Represent and analyze quantitative relationships between dependent and independent variables.

6.RP - Proportional Reasoning: Ratios and Proportions

- 6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.

6.NS - Numeric Reasoning: Number Systems

- 6.NS.A Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- 6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.
- 6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

6.GM - Geometric Reasoning and Measurement

- 6.GM.A Solve real-world and mathematical problems involving area, surface area, and volume.

6.DR - Data Reasoning

- 6.DR.A Formulate Statistical Investigative Questions.
- 6.DR.B Collect and Consider Data.
- 6.DR.C Analyze, summarize, and describe data.
- 6.DR.D Interpret data and answer investigative questions.

Students should spend the large majority[1] of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

[1]At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see the K-8 major work of the grade developed by Student Achievement Partners.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.AEE.A - Apply and extend previous understandings of arithmetic to algebraic expressions.

STANDARD: 6.AEE.A.1

Standards Statement (2021):

Write and evaluate numerical expressions involving whole-number bases and exponents.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.OA.B.4, 5.OA.A.1	6.AEE.A.2, 7.AEE.B.3, 8.AEE.A.1	5.NBT.A.2	6.EE.A.1 6.AEE.A Crosswalk

Standards Guidance:

Teaching Strategies

- Extend previous understanding by using brackets and parentheses and order of operations and exponents.
- Students should interpret real-life, mathematical situations to write and evaluate numerical expressions.

Progressions

- In Grade 6 [students] start to incorporate whole number exponents into numerical expressions, for example when they describe a square with side length 50 feet as having an area of 50 ft^2 (square feet). (Please reference page 4 in the Progression document)

Examples

- Illustrative Mathematics:
  - Seven to the What?!?
  - Sierpinski's Carpet
- Student Achievement Partners:
  - Extending Previous Understandings of Properties Mini-Assessment

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.AEE.A.2

Standards Statement (2021):

Write, read, and evaluate expressions in which letters stand for numbers. Apply knowledge of common mathematical terms to move between the verbal and mathematical forms of an expression including expressions that arise from authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.OA.A.2, 5.OA.B.3, 6.AEE.A.1	6.AEE.A.3, 6.AEE.B.4, 6.AEE.B.5, 8.AEE.A.2	HS.AEE.A.1	6.EE.A.2 6.AEE.A Crosswalk

Standards Guidance:

Clarifications

- Students should write expressions that record operations with numbers and with letters standing for numbers.
- Students should evaluate algebraic expressions for a given value of a variable, using the order of operations.

Boundaries

- Evaluate expressions at specific values of their variables. Numeric values should align with grade level expecations of positive rational numbers.
- Includes identificaiton of the parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
- Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Teaching Strategies

- Include expressions that arise from formulas used in authentic problems.
- Students should understand letters called variables represent unknown numbers and the same rules apply in operations with numbers also apply in operations with variables.

Progressions

- Abstracting the pattern they write 10 - *p* for a book costing *p* dollars, thus summarizing a calculation that can be carried out repeatedly with different numbers. Such work also helps students interpret expressions. For example, if there are 3 and with letters standing for numbers. loose apples and 2 bags of A apples each, students relate quantities in the situation to the terms in the expression 3 + 2A. (Please reference page 4 in the Progression document)

Example

- Express the calculation subtract y from 5 as 5 – y.
- Describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
- Illustrative Mathematics: [Rectangle Perimeter 1]
- Student Achievement Partners: [Expression] [Extending Previous Understandings of Properties Mini-Assessment] [Smarter Balanced Assessment Item Illustrating 6.AEE.A.2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.AEE.A.3

Standards Statement (2021):

Apply the properties of operations to generate equivalent expressions and to determine when two expressions are equivalent.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.OA.A.2, 6.AEE.A.2	7.AEE.A.1	6.NS.B.4	6.EE.A.3, 6.EE.A.4 6.AEE.A Crosswalk

Standards Guidance:

Clarification

- Identify when two expressions are equivalent such as when the two expressions name the same number regardless of which value is substituted into them.

Boundaries

- This standard includes distributive property and combining like terms.

Progressions

- A firm grasp on variables as numbers helps students extend their work with the properties of operations from arithmetic to algebra. For example, students who are accustomed to mentally calculating 5 x 37 as 5 x (30 + 7) = 150 + 35 can now see that 5(3a + 7) = 15a + 35 for all numbers a. (Please reference page 5 in the Progression document).

Examples

- Apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x.
- Apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y)
- Apply properties of operation to the expression y + y + y to produce the equivalent expression 3y and know they are equivalent because they name the same number regardless of which number y stands for.
- Illustrative Mathematics:
  - Rectangle Perimeter 2
  - Equivalent Expressions
- Student Achievement Partners:
  - Extending Previous Understandings of Properties Mini-Assessment
  - Smarter Balanced Assessment Item Illustrating 6.AEE.A.3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.AEE.B - Reason about and solve one-variable equations and inequalities.

STANDARD: 6.AEE.B.4

Standards Statement (2021):

Understand solving an equation or inequality as a process of answering which values from a specified set, if any, make the equation or inequality true. Use substitution to determine which number(s) in a given set make an equation or inequality true.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.A.2	8.AEE.A.2, 8.AEE.C.8, HS.AEE.C.8, HS.AEE.D.9	N/A	6.EE.B.5 6.AEE.B Crosswalk

Standards Guidance:

Teaching Strategies

- Students should be able to use algebraic reasoning to solve an equation as a process of answering a contextual question and explain their reasoning.
- When solving an equation or inequality as a process of answering a question, students should be able to explain why specific values from a specified set, if any, make the equation or inequality true.
- Students should use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Progressions

- Solving is a process of reasoning to find the numbers which make an equation true, which can include checking if a given number is a solution. Although the process of reasoning will eventually lead to standard methods for solving equations, students should study examples where looking for structure pays off, such as in $4x + 3x = 3x + 20$, where they can see that $4x$ must be 20 to make the two sides equal. (Please reference page 6 in the Progression document)

Examples

- Use an inequality of the form $x > c$ or $x < c$.
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 6.AEE.B.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.AEE.B.5

Standards Statement (2021):

Use variables to represent numbers and write expressions when solving problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.A.2	7.AEE.B.4	N/A	6.EE.B.6 6.AEE.B Crosswalk

Standards Guidance:

Clarifications

- Understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Progressions

- As with all their work with variables, it is important for students to state precisely the meaning of variables they use when setting up equations (MP6). This includes specifying whether the variable refers to a specific number, or to all numbers in some range. For example, in the equation 0.44n = 11 the variable n refers to a specific number (the number of stamps you can buy for $11); however, if the expression 0.44n is presented as a general formula for calculating the price in dollars of n stamps, then n refers to all numbers in some domain. That domain might be specified by inequalities, such as n > 0. (Please reference page 7 in the Progression document).

Examples

- Illustrative Mathematics:
  - Firefighter Allocation
- Student Achievement Partners:
  - Assessment Item Illustrating 6.AEE.B.5

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.AEE.B.6

Standards Statement (2021):

Write and solve equations of the form x + p = q and px = q in problems that arise from authentic contexts for cases in which p, q and x are all nonnegative rational numbers.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.C.8	7.AEE.B.4	6.NS.A.1, 5.NF.A.1, 5.NF.B.3, 5.NF.B.4, 5.NF.B.5, 5.NF.B.6	6.EE.B.7 6.AEE.B Crosswalk

Standards Guidance:

Teaching Strategies

- p, x, and q include non-whole numbers. Students should be able to solve equations of this form using strategies such as related equations, fact families, inverse operations, and visual models.
- Students should have opportunities to use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction and multiplication and division when solving one-step equations.
- Students should be able to solve equations presented in contextual, mathematical problems involving positive rational numbers using number sense, properties of arithmetic and the idea of maintaining equality on both sides of the equation.
- Students should be able to interpret a solution in the original context and assess the reasonableness of results.

Progressions

- For example, how many 44-cent stamps can you buy with $11? Students are accustomed to solving such problems by division; now they see the parallel with representing the problem algebraically as 0.44n = 11, from which they use the same reasoning as in the numerical solution to conclude that n = 11 0.44. (Please reference page 7 in the Progression document).

Examples

- Illustrative Mathematics:
  - Morning Walk
  - Firefighter Allocation
  - Fruit Salad
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 6.AEE.B.6

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.AEE.B.7

Standards Statement (2021):

Write inequalities of the form x > c and x < c to represent constraints or conditions to solve problems in authentic contexts. Describe and graph on a number line solutions of inequalities of the form x > c and x < c.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.NS.C.7, 6.NS.C.6	7.AEE.B.4	5.NBT.A.3	6.EE.B.8 6.AEE.B Crosswalk

Standards Guidance:

Clarification

- Recognize that inequalities of the form $x>c$ or $x<c$ have infinitely many solutions.
- Represent solutions of such inequalities on number line diagrams.

Teaching Strategies

- Students should represent contextual, mathematical situations using inequalities involving variables.
- Students should be able to create contextual, mathematical situations corresponding to specific inequalities.
- This objective includes the use of the symbols, < , > , = , ≤ , ≥.

Examples

- Illustrative Mathematics:
  - Height Requirements

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.AEE.C - Represent and analyze quantitative relationships between dependent and independent variables.

STANDARD: 6.AEE.C.8

Standards Statement (2021):

Use variables to represent and analyze two quantities to solve problems in authentic contexts. Including those that change in relationship to one another; write an equation to express one quantity in terms of the other quantity.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.OA.B.3	6.AEE.B.6, 7.AEE.B.4	6.RP.A.3	6.EE.C.9 6.AEE.C Crosswalk

Standards Guidance:

Boundaries

- Students should be able to represent equations involving positive variables and rational numbers.
- Students should have opportunities to solve contextual, mathematical problems.

Teaching Strategies

- Students should have an opportunity to solve problem situations with variables in all positions.
- Students should be able to explain that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.

Progressions

- As [students] work with such equations [they] begin to develop a dynamic understanding of variables, an appreciation that they can stand for any number from some domain.
- This use of variables arises when students study expressions such as 0.60n, [presented as a general formula for calculating the price in dollars of n stamps that cost $0.60],
- or equations in two variables such as d = 5 + 5t describing [the] relationship between distance in miles, d, and time in hours, t, for a person starting 5 miles from home and walking away at 5 miles per hour. Students can use tabular and graphical representations to develop an appreciation of varying quantities. (Please reference page 7 in the Progression document).

Examples

- Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example:
  - In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.
- Illustrative Mathematics:
  - Chocolate Bar Sales
- Student Achievement Partners:
  - Assessment Item Illustrating 6.AEE.C.8

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.RP.A - Understand ratio concepts and use ratio reasoning to solve problems.

STANDARD: 6.RP.A.1

Standards Statement (2021):

Understand the concept of a ratio in authentic contexts, and use ratio language to describe a ratio relationship between two quantities.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.NF.B.3, 5.NF.B.4, 5.NF.B.5, 5.NF.B.6	6.RP.A.2, 6.RP.A.3	4.OA.A.2, 5.OA.B.3, 4.GM.B.4	6.RP.A.1 6.RP.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to explain the concept of a ratio, such as using part-to-part or part-to-whole.
- Students should be able to fluently use ratio language to describe a ratio relationship between two quantities.
- Students should be able to identify standard fractional notation to compare.

Teaching Strategies

- Students should be able to solve problems involving ratios found in real-life situations.
- Students should be given the opportunity to represent and explain the concept of a ratio and the relationship between two quantities using concrete materials, drawings, tape diagrams (bar models), double number line diagrams, equations, and standard fractional notation

Progressions

- It is important for students to focus on the meaning of the terms “for every,” “for each,” “for each 1,” and “per” because these equivalent ways of stating ratios and rates are at the heart of understanding the structure in these tables, providing a foundation for learning about proportional relationships in Grade 7. (Please reference page 5 in the Progression document).

Examples

- The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak
- For every vote candidate A received, candidate C received nearly three votes.
- Describe a ratio as a multiplicative relationship between two quantities.
- Model a ratio relationship using a variety of representations.
- Illustrative Mathematics:
  - The Escalator, Assessment Variation
  - Bag of Marbles
  - Games at Recess
- Student Achievement Partners:
  - Ratios and Rates Mini-Assessment
  - Smarter Balanced Assessment Item Illustrating 6.RP.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.RP.A.2

Standards Statement (2021):

Understand the concept of a unit rate in authentic contexts and use rate language in the context of a ratio relationship.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.NF.B.3, 5.NF.B.4, 5.NF.B.7, 6.RP.A.1	7.RP.A.1, 7.RP.A.2	4.OA.A.2	6.RP.A.2 6.RP.A Crosswalk

Standards Guidance:

Clarifications

- When asked contextual, mathematical questions, should demonstrate an understanding of simple multiplicative relationships involving unit rates.
- Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

Terminology

- Students should understand a unit rate as a relationship of a:b where b = 1 (ab associated with a ratio a: b with b ≠ 0 (b not equal to zero), and use rate language).

Teaching Strategies

- Students should create a table of values displaying the ratio relationships to graph ordered pairs of distances and times.
- Students should write equations to represent the relationship between distance and time where the unit rate is the simple multiplicative relationship.
- Students should be able to determine the independent and dependent relationship of rate relationships within contextual, mathematical situations.

Examples

- This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.
- We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.
- In a problem involving motion at a constant speed, list and graph ordered pairs of distances and times, and write an equation such as d = 65t to represent the relationship between distance and time. In this example, 65 is the unit rate or simple multiplicative relationship.
- Illustrative Mathematics:
  - Price per pound and pounds per dollar
  - Riding at a Constant Speed, Assessment Variation
  - The Escalator, Assessment Variation
  - Ticket Booth
- Student Achievement Partners:
  - Ratios and Rates Mini-Assessment

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.RP.A.3

Standards Statement (2021):

Use ratio and rate reasoning to solve problems in authentic contexts that use equivalent ratios, unit rates, percents, and/or measurement units.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.NF.B.3, 5.NF.B.4, 6.RP.A.1	7.RP.A.2, 7.RP.A.3	5.GM.A.2, 6.AEE.C.8, HS.NQ.B.3	6.RP.A.3 6.RP.A Crosswalk

Standards Guidance:

Clarifications

- Students should have opportunities to explore the concept of percents and recognize the connection between fractions, decimal numbers, and percents, such as, 25% of a quantity means 25/100 or .25 times the quantity.
- Students should be able to convert fractions with denominators of 2, 4, 5 and 10 to the decimal notation.
- Students should be able to calculate the percent of a number using proportional reasoning developed through working with ratios and rates.
- Students should be able to solve contextual problems involving finding the whole given a part and the part given the whole.
- Students should determine what percent one number is of another number to solve contextual, mathematical problems.

Teaching Strategies

- Students should be able to use flexible, strategic thinking to manipulate and transform units appropriately when multiplying or dividing quantities to solve contextual, mathematical problems.
- Instruction could include grade appropriate use of different measurement systems (e.g. feet/inches/yards and meters/centimeters/millimeters) which includes conversion of measurement units when given a conversion factor within one system, or across two systems (e.g. customary or metric), using proportional reasoning developed through working with ratios and rates.

Progressions

- Students should be given the opportunity to use concrete materials, drawings, tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and equations when solving problems. (Please reference pages 3-5 and 7 in the Progression document).

Examples

- Use unit rates to solve problems, including problems involving unit pricing and constant speed.
- If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
- Given 1 in. = 2.54 cm, how many centimeters are in 6 inches?
- Illustrative Mathematics: [Painting a Barn] [Riding at a Constant Speed, Assessment Variation] [Fizzy Juice] [Converting Square Units]
- Student Achievement Partners: [Planting Corn]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.NS.A - Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

STANDARD: 6.NS.A.1

Standards Statement (2021):

Represent, interpret, and compute quotients of fractions to solve problems in authentic contexts involving division of fractions by fractions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.NF.B.6, 5.NF.B.7, 6.NS.B.4	7.NS.A.2	3.OA.B.6, 6.AEE.B.6	6.NS.A.1 6.NS.A Crosswalk

Standards Guidance:

Clarifications

- Students should use their understanding of equivalency to flexibly reason with equivalent fractions based on the context of the problem. Simplifying fractions is not an expectation of this grade level.
- Students should be able to use the meanings of fractions, multiplication, division and the inverse relationship between multiplication and division to make sense of multiplying and dividing fractions.

Terminology

- Fraction quotients can be represented visually by fraction diagram, concretely with manipulatives, or symbolically with equations to represent the problem.

Teaching Strategies

- Students should be able to utilize fractions with denominators including 2, 3, 4, 5, 6, 8, 10, and 12.
- Students should be able to use numerical reasoning to interpret contextual, mathematical situations involving fractions.
- Students can use a variety of strategies, including but not limited to concrete models, visual fraction models, student-generated strategies, a standard algorithm, or other strategies based on numerical reasoning to represent and solve problems.

Progressions

- Students should use flexible, accurate, and efficient written methods to express computational thinking based on numerical reasoning and sense-making developed from learning experiences that focus on the numbers as quantities. (Please reference pages 5 and 6 in the Progression document).

Examples

- Reason and solve problems with quotients of fractions using both the measurement and partition models of division (based on what is most appropriate for the fractions in the quotient).
- For example, (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.)
- How many 3/4-cup servings are in 2/3 of a cup of yogurt?
- How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?
- Illustrative Mathematics:
  - [Traffic Jam] [Cup of Rice] [Video Game Credits]
  - How Many Containers in One Cup / Cups in One Container?
  - Making Hot Cocoa [Variation 1] [Variation 2]
- Student Achievement Partners: [Shoelace] [Smarter Balanced Assessment Item Illustrating 6.NS.A.1]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.NS.B - Compute fluently with multi-digit numbers and find common factors and multiples.

STANDARD: 6.NS.B.2

Standards Statement (2021):

Fluently divide multi-digit numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.NBT.B.6, 5.NF.A.2	6.NS.B.3	5.NF.B.4	6.NS.B.2 6.NS.B Crosswalk

Standards Guidance:

Clarifications

- Fluently/Fluency – Students choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Terminology

- Decimal number – a number whose whole number part and fractional part are separated by a decimal point.

Teaching Strategies

- Students should be able to use a variety of part- whole strategies to compute efficiently (area model, partial product, partial quotient).
- The part-whole strategies used should be flexible and extend from previous computation strategies and future work with computation.
- Students should use models and student-selected strategies as an efficient written method of demonstrating place value understanding for each operation (addition, subtraction, multiplication, and division).

Examples

- Student Achievement Partners:
  - Number System
  - Assessment Item Illustrating 6.NS.B.2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.NS.B.3

Standards Statement (2021):

Fluently add, subtract, multiply, and divide positive rational numbers using accurate, efficient, and flexible strategies and algorithms.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.NBT.B.5, 5.NBT.B.6, 5.NBT.B.7, 5.NF.A.2, 6.NS.B.2	7.NS.A.3	6.AEE.A.3	6.NS.B.3 6.NS.B Crosswalk

Standards Guidance:

Terminology

- Positive rational numbers includes numbers that can be represented by a ratio a/b where a is positive whole number greater than or equal to zero, and b is a non-zero whole number. Such numbers include whole numbers, fractions, and decimals greater than or equal to zero.
- Fluently/Fluency – Students choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.

Boundaries

- Students should be allowed to choose an appropriate strategy to demonstrate fluency.

Teaching Strategies

- Students should be able to use numerical reasoning to interpret contextual, mathematical situations involving fractions.
- Students should be given the opportunity to apply reasoning strategies while solving problems.

Examples

- Student Achievement Partners:
  - Number System
  - Smarter Balanced Assessment Item Illustrating 6.NS.B.3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.NS.B.4

Standards Statement (2021):

Determine greatest common factors and least common multiples using a variety of strategies. Apply the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.OA.B.4	6.NS.A.1, 6.AEE.A.3	5.OA.A.2, 7.AEE.A.1	6.NS.B.4 6.NS.B Crosswalk

Standards Guidance:

Clarification

- Students should also be able to apply the least common multiple of two whole numbers less than or equal to 12 to solve contextual, mathematical problems.
- Students should be able to determine the greatest common factor of 2 whole numbers (from 1-100) and use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factors (GCF).

Boundaries

- Find the greatest common factor of two whole numbers less than or equal to 100
- Find the least common multiple of two whole numbers less than or equal to 12.

Teaching Strategies

- Investigate the distributive property using sums and its use in adding numbers 1-100 with a common factor.
- Students should apply these strategies to solve real- life, mathematical problems.
- Note GCF & LCM support use of distributive property.

Examples

- Express 36 + 8 as 4 (9 + 2).
- Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.
- Hotdogs come in a package of 8 and buns in a package of 12. How many packages of hot dogs and packages of buns would you need to purchase to have an equal number of hot dogs and buns?
- Student Achievement Partners:
  - Assessment Item Illustrating 6.NS.B.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.NS.C - Apply and extend previous understandings of numbers to the system of rational numbers.

STANDARD: 6.NS.C.5

Standards Statement (2021):

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in authentic contexts, explaining the meaning of zero in each situation.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	6.NS.C.6, 7.NS.A.1	N/A	6.NS.C.5 6.NS.C Crosswalk

Standards Guidance:

Clarifications

- Students should be able to explain that zero is its own opposite.
- Students should be able to explain that the sign of an integer represents its position relative to zero on a number line.
- Students should be able to show and explain why –(–a) = a. Which is read as, "The opposite of the opposite of a is the same as a."

Terminology

- Rational numbers are numbers that can be written as a fraction where the numerator and denominator are integers.

Teaching Strategies

- Students should be able to use numerical reasoning to interpret and explain the meaning of numerical statements of inequality as the relative position of two integers positioned on a number line.
- Students are introduced to rational numbers. Students should connect their understanding of fractions and integers to comprehend rational numbers as numbers that can be written as a fraction where the numerator and denominator are integers.

Progressions

- The Standards do not introduce integers separately from the entire system of rational numbers, and examples of negative fractions or decimals can be included from the beginning. (Please reference page 7 in the Progression document).

Examples

- Example contexts: Temperature above/below zero; Elevation above/below sea level; Debits/credit; Positive/negative electric charge.
- Write –3 degrees Celsius > –7 degrees Celsius to express the fact that –3 degree Celsius is warmer than –7 degrees Celsius.
- Illustrative Mathematics:
  - Mile High
  - It's Warmer in Miami

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.NS.C.6

Standards Statement (2021):

Represent a rational number as a point on the number line. Extend number line diagrams and coordinate axes to represent points on the line and in the coordinate plane with negative number coordinates.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.NS.C.5	6.NS.C.7, 6.NS.C.8, 7.NS.A.1	6.AEE.B.7, 3.NF.A.2, 5.GM.A.1	6.NS.C.6 6.NS.C Crosswalk

Standards Guidance:

Clarifications

- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line.
- Recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane.
- Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- Students should use numerical and graphical reasoning to plot points in all four quadrants on the coordinate plane.

Teaching Strategies

- Students should have opportunities to explore this concept using visual models to develop a deeper understanding.
- Number lines should be indicated both vertically and horizontally.
- Students should use numerical and graphical reasoning to show and explain the relationship between ordered pairs and location in quadrants of the coordinate plane.
- Students should extend understanding of number lines and coordinate axes from previous grades to represent points on the line and in the plane with negative number coordinates.

Progressions

- Students come to see p as the opposite of p, located an equal distance from 0 in the opposite direction. In order to avoid the common misconception later in algebra that any symbol with a negative sign in front of it should be a negative number, it is useful for students to see examples where p is a positive number, for example if p = -3 then -p = -(-3) = 3. (Please reference page 7 in the Progression document).

Examples

- Find and position integers and rational numbers on a horizontal or vertical number line diagram.
- Find and position pairs of integers and other rational numbers on a coordinate plane.
- Students should be able to recognize that -a is the same distance from zero as a, and therefore, are opposites of each other.
- Student Achievement Partners: Smarter Balanced Assessment Item Illustrating 6.NS.C.6

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.NS.C.7

Standards Statement (2021):

Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. Write, interpret, and explain statements of order for rational numbers and absolute value in authentic applications.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.NS.C.6	6.NS.C.8, 7.NS.A.1	6.AEE.B.7	6.NS.C.7 6.NS.C Crosswalk

Standards Guidance:

Clarifications

- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation
- Distinguish comparisons of absolute value from statements about order.
- Students should be introduced to the absolute value symbol with this learning objective, i.e., |−3/4|.
- Students should conclude through exploration that absolute value and distance are always expressed as a positive value.

Terminology

- Absolute value is a number's distance from zero (0) on a number line.

Progressions

- Comparing negative numbers requires closer attention to the relative positions of the numbers on the number line rather than their magnitudes.
- Comparisons such as -7 < -5 can initially be confusing to students, because -7 is further away from 0 than -5, and is therefore larger in magnitude. (Please reference page 8 in the Progression document).

Examples

- Interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.
- Write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.
- For an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.
- Recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.
- For an account balance of –51.25 dollars, write |– 51.25| = 51.25 to describe the size of the debt in dollars.
- Illustrative Mathematics:
  - Above and below sea level
- Student Achievement Partners:
  - Assessment Item Illustrating 6.NS.C.7

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.NS.C.8

Standards Statement (2021):

Graph points in all four quadrants of the coordinate plane to solve problems in authentic contexts. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.GM.A.2, 6.NS.C.6, 6.NS.C.7	7.NS.A.1, 8.GM.B.7, 8.GM.B.8	6.GM.A.3	6.NS.C.8 6.NS.C Crosswalk

Standards Guidance:

Teaching Strategies

- Students should be expected to solve problems within the context of a graph only.

Progressions

- Students should be able to solve contextual, mathematical problems when graphing points.

Examples

- Rectangle RSTU has vertices at (−4,3), SS(−4, −2), TT(5, −2) and UU(5,3).
- Plot the rectangle on a coordinate plane and find the perimeter of the figure.
- Illustrative Mathematics:
  - Distances between Points
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 6.NS.C.8

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.GM.A - Solve real-world and mathematical problems involving area, surface area, and volume.

STANDARD: 6.GM.A.1

Standards Statement (2021):

Find the area of triangles, quadrilaterals, and other polygons by composing into rectangles or decomposing into triangles and other shapes. Apply these techniques to solve problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.GM.B.6	6.GM.A.4, 7.GM.A.1, 7.GM.B.3, 7.GM.B.5, HS.GM.C.8	5.NF.B.4	6.G.A.1 6.GM.A Crosswalk

Standards Guidance:

Terminology

- A polygon is a closed figure with at least three straight sides and angles;
- A regular polygon is when all sides are equal and all angles are equal
- An irregluar polygon is when all sides are not equal or all angles are not equal.

Teaching Strategies

- Apply these techniques in the context of solving authentic mathematical problems.
- Students should be able to use knowledge of area of a rectangle to determine the area of a triangle. Students should have opportunities to find the area of a triangle by decomposing the rectangle into two triangles. Thus, students should conclude the area of the triangle is half the area of the rectangle and the area of the rectangle is twice the area of the triangle.
- Students should be able to use geometric and spatial reasoning to calculate the area of a triangle, quadrilateral, and regular polygon by composing or decomposing into triangles, rectangles, and other shapes.
- Students should be able to decompose regular and irregular polygons into triangles and quadrilaterals in a way that makes sense from their perspective.

Progressions

- Problems involving areas and volumes ... provide a context for developing and using equations. The ability to view a triangle as part of a parallelogram composed of two copies of that triangle and the understanding that area is additive ... provides a justification (MP3) for halving the product of the base times the height, helping students guard against the common error of forgetting to take half. (Please reference page 19 in the Progression document).

Examples

- Illustrative Mathematics: [Wallpaper Decomposition] [Same Base and Height, Variation 2] [Finding Areas of Polygons]
- Student Achievement Partners: [Quilt Triangles] [Smarter Balanced Assessment Item Illustrating 6.G.A.1]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.GM.A.2

Standards Statement (2021):

Find the volume of a right rectangular prism with fractional edge lengths by filling it with unit cubes of appropriate unit fraction edge lengths. Connect and apply to the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths to solve problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.GM.D.7	7.GM.B.5	N/A	6.G.A.2 6.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students should make the connection between (length) x (width) and the area of the base to connect this formula to other three-dimensional volume formulas.
- Show that the volume is the same as would be found by multiplying the edge lengths of the prism.

Teaching Strategies

- Apply these techniques in the context of solving authentic mathematical problems.
- Students should be able to calculate the volume of a right rectangular prism with fractional edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism.
- Students should apply the formula for the volume of a right rectangular prism in the context of solving contextual, mathematical problems to meet this learning objective.

Examples

- Illustrative Mathematics:
  - Computing Volume Progression 1
  - Computing Volume Progression 3
  - Banana Bread
- Student Achievement Partners:
  - Assessment Item Illustrating 6.G.A.2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.GM.A.3

Standards Statement (2021):

Draw polygons in the 4-quadrant coordinate plane given coordinates for the vertices and find the length of a side. Apply these techniques to solve problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.GM.A.2, 5.GM.B.3	8.GM.A.3	6.NS.C.8	6.G.A.3 6.GM.A Crosswalk

Standards Guidance:

Boundaries

- Measurements of between vertices can be done using standardized measurement tools on graph paper. Formal use of the distance formula is not an expectation at this grade level.

Teaching Strategies

- Students should be able to solve problems with polygons when given coordinate pairs with or without a coordinate grid.

Progressions

- Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.
- Students should apply the techniques of graphing in the coordinate plane in the context of solving contextual problems involving the application of algebra through geometry.

Examples

- Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.
- Illustrative Mathematics:
  - Polygons in the Coordinate Plane
  - Walking the Block

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 6.GM.A.4

Standards Statement (2021):

Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures, including those from authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.GM.A.1	7.GM.B.5	N/A	6.G.A.4 6.GM.A Crosswalk

Standards Guidance:

Teaching Strategies

- Apply these techniques in the context of solving authentic mathematical problems.
- Students should use various tools and strategies including a picture or physical model of a net to measure the surface area of three-dimensional figures that are composed of rectangular and triangular faces when solving contextual, mathematical problems.
- Students should be provided the net of three- dimensional figures to ensure grade level appropriateness.

Progressions

- Students also analyze and compose and decompose polyhedral solids. They describe the shapes of the faces, as well as the number of faces, edges, and vertices. They measure the attributes of these shapes, allowing them to apply area formulas to solve surface area problems (MP7). They learn to plan the construction of complex three-dimensional compositions through the creation of corresponding two-dimensional nets (e.g., through a process of digital fabrication and/or graph paper).
- For example, they may design a living quarters (e.g., a space station) consistent with given specifications for surface area and volume (MP2, MP7). (Please reference pages 19-20 in the Progression document).

Examples

- Illustrative Mathematics:
  - Nets for Pyramids and Prisms
- Student Achievement Partners:
  - Assessment Item Illustrating 6.G.A.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.DR.A - Formulate Statistical Investigative Questions

STANDARD: 6.DR.A.1

Standards Statement (2021):

Formulate and recognize statistical investigative questions as those that anticipate changes in descriptive data related to the question and account for it in the answers.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.DR.A.1	7.DR.A.1	5.NF.B.7	6.SP.A.1 6.DR Crosswalk

Standards Guidance:

Clarifications

- Students can generate questions about things they notice and wonder from a real-life situation.
- Students should be able to generate their own statistical questions.

Terminology

- A statistical question is one that requires data that vary.
- A statistical investigative question is one that allows for exploration through statistical inquiry and reasoning.

Teaching Strategies

- Students should be able to use the statistical process to formulate questions. The statistical process involves asking a statistical investigative question, collecting the data, analyzing the data, and interpreting the results.

Progressions

- Statistical investigations begin with a question, and students now see that answers to such questions always involve variability in the data collected to answer them. (Please reference page 4 in the Progression document).

Examples

- "How old are the students in my school?" is a statistical question because it anticipates variability in students' ages.
- "How old am I?" is a question used to collect data to answer the investigative question.
- Student Achievement Partners:
    - Assessment Item Illustrating 6.DR.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.DR.B - Collect and Consider Data

STANDARD: 6.DR.B.2

Standards Statement (2021):

Collect and record data with technology to identify and describe the characteristics of numerical data sets using quantitative measures of center and variability.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.DR.A.1	7.DR.B.2	N/A	6.SP.A.2, 6.SP.B.4 6.DR Crosswalk

Standards Guidance:

Clarifications

- Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (6.SP.A.2)
- Students should understand the concept of outliers.
- Students should be able to describe the nature of the statistical attribute under investigation, including how it was measured and its units of measurement.

Boundaries

- A set of data collected to answer a statistical question has a distribution, which can be described by
    - Measures of center - mean, median, mode
    - Measures of variation (spread) - range, interquartile range, and/or mean absolute deviation
    - Descriptions of overall shape - symmetrical vs non- symmetrical
- The focus of mean absolute deviation (MAD) is visualizing deviations from the mean as a measure of variability as opposed to a focus on calculating MAD.
    - In sixth grade, students should explore the conceptual idea of MAD – not the formula.
    - Students should be able to apply their understanding of absolute value (rather than use operations on negative integers) in the context of MAD.

Progressions

- Students extend their knowledge of symmetric shapes, to describe data displayed in dot plots and histograms in terms of symmetry. They identify clusters, peaks, and gaps, recognizing common shapes and patterns in these displays of data distributions (MP7). (Please reference page 4 in the Progression document).

Examples

- Arthur and Aaron are on the same 6th grade basketball team. Both players have scored an average of ten points over the past ten games. Here are the students' number of points scored during each of the last ten games.
    - Arthur: 9, 10, 10, 11, 11, 9, 10, 10, 10, 10
    - Aaron: 16, 18, 4, 3, 5, 13, 18, 3, 13, 7
    - Which student is more consistent?
    - Possible Student Response/Solution: Arthur is more consistent because his MAD is smaller than Aaron's MAD; Arthur has less variability than Aaron.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.DR.C - Analyze, summarize, and describe data

STANDARD: 6.DR.C.3

Standards Statement (2021):

Analyze data representations and describe measures of center and variability of quantitative data using appropriate displays.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.DR.B.2	7.DR.C.3	N/A	6.SP.A.3 6.DR Crosswalk

Standards Guidance:

Clarification

- Recognize that a measure of center for a numerical data set is a single number that summarizes all of the values in the data set, while a measure of variation is a single number that describes how the values in the data set vary from one another. (6.SP.A.3)
- Display numerical data in plots on a number line, including dot plots and histograms. (6.SP.A.4)
- Students have experience with displaying categorical data using bar graphs from elementary grades. In sixth grade, students are extending their understanding of analyzing categorical data displayed on histograms.

Boundaries

- Sixth grade students should be able to create dot plots and box plots to analyze the results of a statistical investigation.
- Sixth grade students should focus on describing and interpreting data displayed.
- Students should be able to identify that each quartile presented in a box plot represents 25% of the data set.

Teaching Strategies

- Students should be able to analyze the shape of a data distribution and determine the impact single data points have on the data set represented visually.
- Describe the impact that inserting or deleting a data point has on the mean, median, and mode of a data set.
- As a result of an investigation, students should summarize categorical and quantitative (numerical) data sets in relation to the context.
- Students should be able to describe the nature of the statistical attribute under investigation, including how it was measured and its units of measurement.

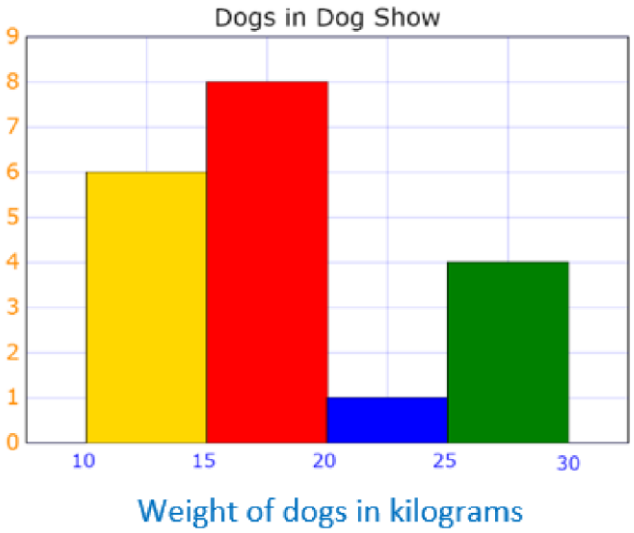
Progressions

- To be useful, center and spread must have well-defined numerical descriptions that are commonly understood by those using the results of a statistical investigation. (Please reference page 4 in the Progression document).

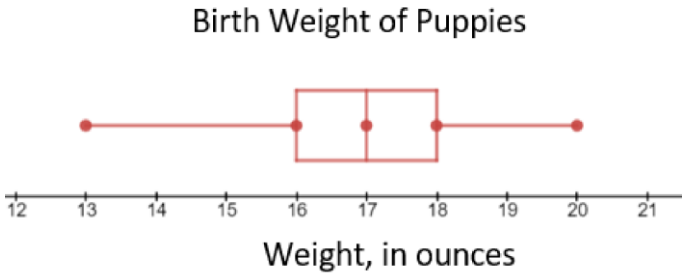
This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Examples

- Categorical Example:



  - What could be the weight of the smallest dog? The largest?
- Quantitative (numerical) Example:
  - Here are the birth weights, in ounces, of all the puppies born at a kennel in the past month.



- What do you notice and wonder about the distribution of the puppy weights?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 6.DR.D - Interpret data and answer investigative questions

STANDARD: 6.DR.D.4

Standards Statement (2021):

Interpret quantitative measures of center to describe differences between groups from data collected to answer investigative questions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.DR.B.2	7.DR.D.4	N/A	6.SP.B.5 6.DR Crosswalk

Standards Guidance:

Clarifications

- Identify, describe, and interpret the characteristics of numerical data sets using quantitative measures of center and variability.
- Additional descriptions include number of observations, measurement attributes, and shape of distribution. (6.SP.A.5)

Terminology

- In sixth grade, students should explore the conceptual idea of MAD – not the formula.
- Students should be able to apply their understanding of absolute value (rather than use operations on negative integers) in the context of MAD.

Boundaries

- Students should be able to determine the number of observations from a context or diagram.
- Students should be able to describe the distribution of a quantitative (numerical) variable collected to answer a statistical investigative question, including its center (median, mean), variability (interquartile range (IQR), mean absolute deviation (MAD), and range), and overall shape (symmetrical vs non-symmetrical).
- Students should be able to describe the nature of the statistical attribute under investigation, including how it was measured and its units of measurement.

Teaching Strategies

- Identification and description of data characteristics related to their context includes:
    - Reporting the number of observations.
    - Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    - Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
- Students should explore conceptually the measures of center (mean, median) and variability (interquartile range and range) for a set of numerical data gathered from contextual, mathematical situations and use these measures to describe the shape of the data presented in various forms.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4H: Grade 7 Math Standards and Guidance

Critical Areas of Focus

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Domains and Clusters

7.AEE - Algebraic Reasoning: Expressions and Equations

- 7.AEE.A Use properties of operations to generate equivalent expressions.
- 7.AEE.B Solve mathematical problems in authentic contexts using numerical and algebraic expressions and equations.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

7.RP - Proportional Reasoning: Ratios and Proportions

- 7.RP.A Analyze proportional relationships and use them to solve mathematical problems in authentic contexts.
- 7.RP.B Investigate chance processes and develop, use, and evaluate probability models.

7.NS - Numeric Reasoning: Number Systems

- 7.NS.A Apply and extend previous understandings of operations with fractions.

7.GM - Geometric Reasoning and Measurement

- 7.GM.A Draw, construct, and describe geometrical figures and describe the relationships between them.
- 7.GM.B Solve mathematical problems in authentic contexts involving angle measure, area, surface area, and volume.

7.DR - Data Reasoning

- 7.DR.A Formulate Statistical Investigative Questions.
- 7.DR.B Collect and Consider Data.
- 7.DR.C Analyze, summarize, and describe data.
- 7.DR.D Interpret data and answer investigative questions.

Students should spend the large majority[1] of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

[1]*At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see the K-8 major work of the grade developed by Student Achievement Partners.*

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.AEE.A - Use properties of operations to generate equivalent expressions.

STANDARD: 7.AEE.A.1

Standards Statement (2021):

Identify and write equivalent expressions with rational numbers by applying associative, commutative, and distributive properties.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.OA.A.2, 6.AEE.A.3	7.AEE.A.2, 8.AEE.C.7, HS.AEE.A.1, HS.AEE.A.2	6.NS.B.4	7.EE.A.1 7.AEE.A Crosswalk

Standards Guidance:

Teaching Strategies

- Identify like terms and combine like terms to create equivalent expressions.
- Apply the distributive property to factor and expand linear expressions.
- Use numerical substitution to identify equivalent expressions.

Progressions

- In Grade 7 students start to simplify general linear expressions with rational coefficients. Building on work in Grade 6, where students used conventions about the order of operations to parse, and properties of operations to transform, simple expressions such as 2(3 + 8x) or 10 - 2p, students now encounter linear expressions with more operations and whose transformation may require an understanding of the rules for multiplying negative numbers, such as 7 - 2(3 - 8x).
- In simplifying this expression students might come up with answers such as
    - 5(3 - 8x), mistakenly detaching the 2 from the indicated multiplication
    - 7 - 2(-5x), through a determination to perform the computation in parentheses first, even though no simplification is possible
    - 7 - 6 - 16x, through an imperfect understanding of the way the distributive law works or of the rules for multiplying negative numbers.
- Please reference page 8 in the Progression document for additional information.

Examples

- $4x+2=2(2x+1)$ and $-3(x-5/3)=-3x+5$
- If Massey and Brenda both get paid a wage of $11 per hour, but Massey was paid an additional $55 for overtime, the expression 11(M+B) + 55 may be more clearly interpreted as 11M+55+11B for purposes of understanding Brenda's pay separated from Massey's pay.
- Student Achievement Partners:
    - Smarter Balanced Assessment Item Illustrating 7.EE.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.AEE.A.2

Standards Statement (2021):

Understand that rewriting an expression in different forms in a contextual problem can show how quantities are related.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.AEE.A.1	HS.AEE.A.1, HS.AEE.A.2	N/A	7.EE.A.2 7.AEE.A Crosswalk

Standards Guidance:

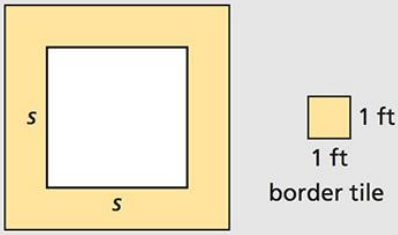
Clarifications

- Building on work in Grade 6, where students used conventions about the order of operations to rewrite simple expressions such as 2(3 + 8x) as 6 +16x and 10p-2 as 2(5p-1), students now encounter linear expressions with more operations that require an understanding of integers, such as 7 - 2(3 - 8x).

Progressions

- In the example [right], the connection between the expressions and the figure emphasize that they all represent the same number.
- The connection between the structure of each expression and a method of calculation emphasize the fact that expressions are built up from operations on numbers. (Please reference page 8 in the Progression document).

Writing expressions in different forms



In expressing the number of tiles needed to border a square pool with side length s feet (where s is a whole number), students might write $4(s + 1)$, $s + s + s + s + 4$, or $2s + 2(s + 2)$, each indicating a different way of breaking up the border in order to perform the calculation. They should see all these expressions as equivalent.

Examples

- For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."
- For example, 3 friends each buy a drink for x dollars and popcorn for y dollars. The total cost could be expressed by "x + x + x +y + y + y", "3x +3y" and "3(x+y)"
- A shirt at a clothing store is on sale for 20% off the regular price, p. The discount can be expressed as $0.2p$. The new price for the shirt can be expressed as $p-0.2p$ or $0.8p$.
- A rectangle is twice as long as it is wide. One way to write an expression to find the perimeter would be w + w + 2w + 2w. Write the expression in two other ways.
- Write an equivalent expression for 9 – 7(2x + 4).
- Illustrative Mathematics:
  - Ticket to Ride
- Student Achievement Partners:
  - Assessment Item Illustrating 7.EE.A.2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.AEE.B - Solve mathematical problems in authentic contexts using numerical and algebraic expressions and equations.

STANDARD: 7.AEE.B.3

Standards Statement (2021):

Write and solve problems in authentic contexts using expressions and equations with positive and negative rational numbers in any form. Contexts can be limited to those that can be solved with one or two-step linear equations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.A.1	8.AEE.A.4	7.NS.A.2, 7.NS.A.3	7.EE.B.3 7.AEE.B Crosswalk

Standards Guidance:

Clarifications

- Students should be able to fluently solve equations of the specified forms presented.
- Students should use the properties of equality to solve for the value of a variable.

Boundaries

- Continue to build on 6th grade objectives of writing and solving one-step equations from a problem situation to multi-step problem situations. This is also another context for students to practice using rational numbers including: integers, and positive and negative fractions and decimal numbers.

Teaching Strategies

- Students should be able to represent relationships in various contextual, mathematical situations with equations involving variables and positive and negative rational numbers and explain the meaning of the solution based on the context.
- Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Progressions

- As they build a systematic approach to solving equations in one variable, students continue to compare arithmetical and algebraic solutions to word problems. For example they solve the problem
    - The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
        - by subtracting 2 * 6 from 54 and dividing by 2, and also by setting up the equation
        - 2w + 2 * 6 = 54.
- Please reference page 9 in the Progression document for additional information.

Examples

- Vicky and Bob went to a store to buy school supplies. Vicky spent a total of $22 on school supplies. She spent $13 on a book and spent the rest of the money on notebooks. The store sells notebooks for $1.50 each. Without using a variable, determine the number of notebooks Vicky bought.
- Illustrative Mathematics: [Stained Glass] [Shrinking]
- Student Achievement Partners: [Micro-Models and Reasoned Estimates Mini-Assessment] [Assessment Item Illustrating 7.EE.B.3]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.AEE.B.4

Standards Statement (2021):

Use variables to represent quantities and construct one- and two-step linear inequalities with positive rational numbers to solve authentic problems by reasoning about the quantities.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.B.5, 6.AEE.B.6, 6.AEE.B.7, 6.AEE.C.8	8.AEE.C.7, 8.AEE.C.8, 8.AFN.A.2, HS.AEE.A.1, HS.AEE.D.11, HS.AEE.B.5	7.RP.A.2	7.EE.B.4 7.AEE.B Crosswalk

Standards Guidance:

Clarification

- Solve word problems leading to equations of the form $px+q=r$ and $(x+q)=r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- Solve word problems leading to inequalities of the form $px+q>r$ or $px+q<r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Teaching Strategies

- Students should be able to represent relationships in various contextual, mathematical situations with inequalities involving variables and positive and negative rational numbers.
- Students should be able to fluently solve inequalities of the specified forms. To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.
- Students should use the properties of inequality to solve for the value of a variable.
- When identifying a specific value for p, q, and r, any rational number can be used.
- Students should be able to graph and interpret the solution of an inequality used as a model to explain real-life phenomena.

Examples

- For example, as a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.
- As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions.
- Illustrative Mathematics:
  - Fishing Adventures 2
  - Gotham City Taxis
- Student Achievement Partners:
  - Piggy Bank
  - Smarter Balanced Assessment Item Illustrating 7.EE.B.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.RP.A - Analyze proportional relationships and use them to solve mathematical problems in authentic contexts.

STANDARD: 7.RP.A.1

Standards Statement (2021):

Solve problems in authentic contexts involving unit rates associated with ratios of fractions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.RP.A.2	8.AEE.B.5	HS.NQ.B.3, HS.GM.C.11	7.RP.A.1 7.RP.A Crosswalk

Standards Guidance:

Terminology

- Ratios of fractions refers to complex fractions where the numerator and/or denominator of a ratio includes a fraction, such as ¼ ÷ ½ is also the ratio of (¼)/(½)

Teaching Strategies

- This includes ratios of lengths, areas and other quantities measured in like or different units.
- Students should have opportunities to create visual representations to solve complex ratio problems.
- Students should build upon their understanding of fractions as a form of division.
- Students should build upon their fluency in division of fractions.
- Students should be able to solve problems involving unit rate presented in practical, real-life situations.

Examples

- For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour.
- Illustrative Mathematics:
  - Cooking with the Whole Cup
  - Molly's Run, Assessment Variation
  - Cider versus Juice - Variation 2
- Student Achievement Partners:
  - Proportional Relationships Mini-assessment
  - Assessment Item Illustrating 7.RP.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.RP.A.2

Standards Statement (2021):

Recognize and represent proportional relationships between quantities in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Identify the constant of proportionality (unit rate) within various representations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.RP.A.2, 6.RP.A.3	7.RP.A.3, 8.AEE.B.5, 8.AEE.B.6, 8.AFN.A.1, 8.AFN.A.2, 8.AFN.B.4	7.AEE.B.4, 7.GM.A.1	7.RP.A.2 7.RP.A Crosswalk

Standards Guidance:

Clarifications

- Students should demonstrate a conceptual understanding of slope.
- Students should recognize equations in the form y = mx are proportional.
- Students should know that a graph with a straight line through the origin is proportional.
- Explain what a point $(x,)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0,0) and (1,r) where r is the unit rate.
- This standard builds on students' understanding of unit rates from 6th grade.

Boundaries

- In seventh grade, students are expected to understand that unit rate and constant of proportionality are the same.

Teaching Strategies

- Have students represent proportional relationships using equations, and decide whether two quantities are in a proportional relationship.

Progressions

- Students identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Recognize the constant of proportionality as both the unit rate and as the multiplicative comparison between two quantities. (Please reference page 9 in the Progression document).

Examples

- If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
- Jennifer rides on a train for 6 hours and travels 360 miles. How many miles per hour does she travel?
- Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
- Illustrative Mathematics: [Art Class, Assessment Variation] [Buying Bananas, Assessment Version] [Robot Races, Assessment Variation]
- Student Achievement Partners: [Snow Vehicle] [Proportional Relationships Mini-assessment] [Smarter Balanced Assessment Item Illustrating 7.RP.A.2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.RP.A.3

Standards Statement (2021):

Use proportional relationships to solve ratio and percent problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.RP.A.3, 7.RP.A.2	7.RP.B.5, 7.RP.B.6, 7.RP.B.7	HS.AEE.A.2	7.RP.A.3 7.RP.A Crosswalk

Standards Guidance:

Terminology

- Simple interest – a quick and easy method of calculating the interest charge on a loan. Simple interest is determined by multiplying the daily interest rate by the principal by the number of days that elapse between payments. Simple Interest = (principal) * (rate) * (# of periods)
- Markups and markdowns - increase and decrease in the amount of a quantity

Boundaries

- This includes solving multi step problems involving simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.
- Students should be able to represent proportional relationships using equations.

Teaching Strategies

- Students should be able to analyze and make decisions about relationships using proportional reasoning strategies, which may include but not limited to graphing on a coordinate plane and/or observing whether a graph is a straight line passing through the origin
- Students may use flexible strategies such as a + 0.05a = 1.05a with the understanding that adding a 5% tax to a total is the same as multiplying the total by 1.05.

Progressions

- Student should be able to identify, represent, and use proportional relationships between quantities using verbal descriptions, tables of values, equations, and graphs to model contextual, mathematical problem and translate from one representation to another. (Please reference page 10 in the Progression document).

Examples

- If the total cost, t, is proportional to the number, n, of items purchased at a constant price, p, the relationship between the total cost and the number of items can be expressed as t = np.
- Jane runs 12 miles in 2.5 hours. Sarah runs 14 miles 3.5 hours. Are Jane and Sarah running at the same rate? Justify your answer.
- Illustrative Mathematics:
  - [Sand Under the Swing Set] [Lincoln's math problem] [Comparing Years] [Tax and Tip] [The Price of Bread]
- Student Achievement Partners:
  - Proportional Relationships Mini-assessment
  - Smarter Balanced Assessment Item Illustrating 7.RP.A.3 [Option 1] [Option 2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.RP.B - Investigate chance processes and develop, use, and evaluate probability models.

STANDARD: 7.RP.B.4

Standards Statement (2021):

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Represent probabilities as fractions, decimals, and percents.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.RP.A.3, 7.RP.A.2	HS.DR.E.14	N/A	7.SP.C.5 7.RP.B Crosswalk

Standards Guidance:

Terminology

- Descriptions may include impossible, unlikely, equally likely, likely, and certain.
- Know that a probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is equally likely, and a probability near 1 indicates a likely event.

Teaching Strategies

- Students should be able to represent the probability as a fraction, decimal numbers, or percent.

Progressions

- In Grade 7, students build their understanding of probability on a relative frequency view of the subject, examining the proportion of "successes" in a chance process—one involving repeated observations of random outcomes of a given event, such as a series of coin tosses.
- "What is my chance of getting the correct answer to the next multiple choice question?" is not a probability question in the relative frequency sense. "What is my chance of getting the correct answer to the next multiple choice question if I make a random guess among the four choices?" is a probability question because the student could set up an experiment of multiple trials to approximate the relative frequency of the outcome. And two students doing the same experiment will get nearly the same approximation. These important points are often overlooked in discussions of probability. (Please reference page 7 in the Progression document).

Examples

- Student Achievement Partners:
  - Prize Game

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.RP.B.5

Standards Statement (2021):

Use experimental data and theoretical probability to make predictions. Understand the probability predictions may not be exact.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.RP.A.3	HS.DR.E.14	N/A	7.SP.C.6 7.RP.B Crosswalk

Standards Guidance:

Terminology

- Approximate the (theoretical) probability of a chance event by collecting data and observing its long-run relative frequency (experimental probability). Predict the approximate relative frequency given the (theoretical) probability.

Teaching Strategies

- Students should draw upon understanding of proportional relationships to make predictions.
- Students should be able to predict the approximate, relative frequency given the theoretical probability.

Progressions

- It must be understood that the connection between relative frequency and probability goes two ways. If you know the structure of the generating mechanism (e.g., a bag with known numbers of red and white chips), you can anticipate the relative frequencies of a series of random selections (with replacement) from the bag.
- If you do not know the structure (e.g., the bag has unknown numbers of red and white chips), you can approximate it by making a series of random selections and recording the relative frequencies. This simple idea, obvious to the experienced, is essential and not obvious at all to the novice. The first type of situation, in which the structure is known, leads to "probability"; the second, in which the structure is unknown, leads to "statistics." (Please reference page 7 in the Progression document).

Examples

- When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
- When drawing chips out of a bag containing an unknown number of red and white chips, estimate the probability of selecting a particular chip color given 50 draws.
- Illustrative Mathematics:
  - Rolling Dice
  - Heads or Tails
  - Tossing Cylinders
- Student Achievement Partners:
  - Assessment Item Illustrating 7.RP.B.5

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.RP.B.6

Standards Statement (2021):

Develop a probability model and use it to find probabilities of events. Compare theoretical and experimental probabilities and explain possible sources of discrepancy if any exists.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.RP.A.3	HS.DR.E.14, HS.DR.E.15	N/A	7.SP.C.7 7.RP.B Crosswalk

Standards Guidance:

Clarification

- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Teaching Strategies

- Probability models may include various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips.
- Students should have multiple opportunities to collect data using physical objects, graphing calculators, or web-based simulations.

Progressions

- A probability model provides a probability for each possible nonoverlapping outcome for a chance process so that the total probability over all such outcomes is unity. The collection of all possible individual outcomes is known as the sample space for the model. For example, the sample space for the toss of two coins (fair or not) is often written as {TT, HT, TH, HH}.
- The probabilities of the model can be either theoretical (based on the structure of the process and its outcomes) or empirical (based on observed data generated by the process). In the toss of two balanced coins, the four outcomes of the sample space are given equal theoretical probabilities of 1/4 because of the symmetry of the process—because the coins are balanced, an outcome of heads is just as likely as an outcome of tails. (Please reference page 7 in the Progression document).

Examples

- Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
- Kim calculates the probability of landing on heads when tossing a coin to be 50%. She uses this to predict that when Tiffany tosses a coin 20 times, the coin will land on heads 10 times. When Tiffany performed the experiment, the coin landed on heads 7 times. Explain possible reasons why Kim's prediction and Tiffany's results do not match.
- Illustrative Mathematics:
  - Rolling Dice

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.RP.B.7

Standards Statement (2021):

Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.RP.A.3	HS.DR.E.14, HS.DR.E.15	N/A	7.SP.C.8 7.RP.B Crosswalk

Standards Guidance:

Clarifications

- Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Teaching Strategies

- Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
- Design and use simulations to generate experimental frequency data for compound events.

Progressions

- The product rule for counting outcomes for chance events should be used in finite situations like tossing two or three coins or rolling two number cubes. There is no need to go to more formal rules for permutations and combinations at this level.
- Students should gain experience in the use of diagrams, especially trees and tables, as the basis for organized counting of possible outcomes from chance processes. For example, the 36 equally likely (theoretical probability) outcomes from the toss of a pair of number cubes are most easily listed on a two-way table. (Please reference page 8 in the Progression document).

Examples

- Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?
- Determine the probability of "rolling double sixes"
- Illustrative Mathematics:
  - Red, Green, or Blue?
  - Rolling Twice
  - Waiting Times
- Student Achievement Partners:
  - Assessment Item Illustrating 7.RP.B.7

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.NS.A - Apply and extend previous understandings of operations with fractions.

STANDARD: 7.NS.A.1

Standards Statement (2021):

Apply and extend previous understandings of addition, subtraction and absolute value to add and subtract rational numbers in authentic contexts. Understand subtraction as adding the additive inverse, p – q = p + (–q).

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.NS.C.5, 6.NS.C.6, 6.NS.C.7, 6.NS.C.8	7.NS.A.2, 7.NS.A.3	5.NF.A.1	7.NS.A.1 7.NS.A Crosswalk

Standards Guidance:

Terminology

- Part-whole reasoning refers to how numbers can be split into parts to add and subtract numbers more efficiently.
- In the equation 3 + −3 = 0, 3 and −3 are additive inverses of each other.
- Students should represent a variety of types of rational numbers on a number line diagram presented both horizontally and vertically.

Teaching Strategies

- Represent operations with rational numbers both visually and numerically, including number line diagrams.
- Students should be allowed to explore the signs of integers and what they really mean to discover integer rules.
- It is common to use colored chips to represent integers, with one color representing positive integers and another representing negative integers, subject to the rule that chips of different colors cancel each other out; thus, a number is not changed if you take away or add such a pair. Also implicit in the use of chips is that the commutative and associative properties extend to addition of integers, since combining chips can be done in any order.

Progressions

- A fundamental fact about addition of rational numbers is that p + (-p) = 0 for any rational number p; in fact, this is a new property of operations tha comes into play when negative numbers are introduced. (Please reference pages 9 and 10 in the Progression document).

Examples

- (–8) + 5 + (–2) may be solved as (–8) +(–2) + 5 to first make –10 by using the Commutative Property.
- Your bank account balance is − $25.00. You deposit $25.00 into your account. The net balance is $0.00.
- 6 + (–4) is 4 units to the left of 6 on a horizontal number line or 4 units down from 6 on a vertical number line.
- Illustrative Mathematics: [Bookstore Account] [Operations on the number line] [Rounding and Subtracting]
- Student Achievement Partners: [Signed Numbers] [Operations on Rational Numbers Mini-Assessment] Smarter Balanced Assessment Item Illustrating 7.NS.A.1 [Option 1] [Option 2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.NS.A.2

Standards Statement (2021):

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. Interpret operations of rational numbers solving problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.NS.A.1, 7.NS.A.1	8.NS.A.1	5.NF.B.4, 7.AEE.B.3	7.NS.A.2 7.NS.A Crosswalk

Standards Guidance:

Clarifications

- Students should be allowed to explore the signs of integers and what they really mean to discover integer rules.
- If p and q are integers ($q \neq 0$), then $-(-p/q)=(-p)/q= p/(-q)$
- Students should be able to reason about direction on a number line when representing multiplication and division using the tool.

Boundaries

- Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number.
- Apply properties of operations as strategies to multiply and divide rational numbers.
- Convert a rational number to a decimal using division; know that the decimal form of a rational number terminates or eventually repeats.

Teaching Strategies

- Represent operations with rational numbers both visually and numerically,
- Apply properties of operations such as identity, inverse, distributive, associative and commutative properties.
- Student should have opportunities to use concepts of repeated addition and the meaning of a negative sign as the "opposite of," with both models and representations, leading to deriving the rules for multiplying signed numbers.

Progressions

- Just as the relationship between addition and subtraction helps students understand subtraction of rational numbers, so the relationship between multiplication and division helps them understand division.To calculate -8 divided by 4, students recall that (-2) x 4 = -8, and so -8 divided by 4 = -2. By the same reasoning, -8 divided by 5 = -8/5 because -8/5 x 5 = -8. (Please reference page 11 in the Progression document).

Examples

- $-(20/5) = -4$ is the same as $(-20)/5= -4$ and $20/(-5) = -4$
- Illustrative Mathematics:
  - Why is a Negative Times a Negative Always Positive?
- Student Achievement Partners:
  - [Operations on Rational Numbers Mini-Assessment] [SBAC Item Illustrating 7.NS.A.2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.NS.A.3

Standards Statement (2021):

Understand that equivalent rational numbers can be written as fractions, decimals and percents.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.NS.B.3, 7.NS.A.1	8.NS.A.1	7.AEE.B.3, 8.AEE.A.2	7.NS.A.3 7.NS.A Crosswalk

Standards Guidance:

Clarifications

- Students should build upon their understanding of percents as a ratio comparison to 100.
- This is an extension of previous understanding from 6th grade of writing common fractions as decimal numbers and percents.

Boundaries

- Use long division to convert fractions to decimals.
- Students should know that every rational number can be written as the ratio of two integers, terminating decimal numbers, or repeating decimal numbers.

Examples

- A water well drilling rig has dug to a height of −60 feet after one full day of continuous use. If the rig has been running constantly and is currently at a height of −143.6 feet, for how long has the rig been running? (Modified from Illustrative Mathematics)
- Identify whether the decimal form of a rational number is a terminating or repeating decimal.
- Convert terminating decimals to fractions.
- If Sara makes $25 an hour gets a 10% raise, she will make an additional 110 of her salary an hour, or $2.50, for a new salary of $27.50.
- Illustrative Mathematics:
  - Sharing Prize Money
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 7.NS.A.3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.GM.A - Draw construct, and describe geometrical figures and describe the relationships between them.

STANDARD: 7.GM.A.1

Standards Statement (2021):

Solve problems involving scale drawings of geometric figures. Reproduce a scale drawing at a different scale and compute actual lengths and areas from a scale drawing.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.GM.A.1	HS.GM.C.10	7.RP.A.2	7.G.A.1 7.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students should understand scale factor as a rate comparison between similar figures and scale drawings.
- Students should build upon their understanding of proportional relationships.

Teaching Strategies

- Students should be given opportunities to use technology and tools to reproduce scale drawings.
- Students should have opportunities to use proportional reasoning to compute unknown lengths by setting up proportions in tables or equations, or they can reason about how the lengths compare multiplicatively.
- Students should be able to determine the dimensions of figures when given a scale and identify the impact of a scale on actual length (one-dimension) and area (two–dimensions). Students should be able to identify the scale factor given two figures.
- Opportunity to connect to proportional reasoning to explain why the slope, m, is the same between any two distinct points (7.NRP.A.2).

Progressions

- Three-dimensional objects can be represented without distortion by scale models such as doll houses, model trains, architectural models, and souvenirs. Students compute or estimate lengths in the real object by computing or measuring lengths in the drawing and multiplying by the scale factor. Angles in a scale drawing are the same as the corresponding angles in the real object. Lengths are not the same, but differ by a constant scale factor. (Please reference pages 6-7 in the Progression document).

Examples

- Mariko has an 1/4 inch scale-drawing (1/4 inch=1 foot) of the floor plan of her house. On the floor plan, the scaled dimensions of her rectangular living room are 4-1/2 inches by 8-3/4 inches. What is the area of her living room in square feet?
- Illustrative Mathematics:
  - [Circumference of a Circle] [Scaling angles and polygons] [Floor Plan] [Rescaling Washington Park]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.GM.A.2

Standards Statement (2021):

Draw triangles from three measures of angles or sides. Understand the possible side lengths and angle measures that determine a unique triangle, more than one triangle, or no triangle.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	8.GM.A.1, HS.GM.B.5, HS.GM.B.7	N/A	7.G.A.2 7.GM.A Crosswalk

Standards Guidance:

Clarifications

- Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine unique triangles, more than one triangle, or no triangle.

Boundaries

- Know when 3 side lengths will form a triangle.
- Know that the angle measures in a triangle have a sum of 180 degrees.

Teaching Strategies

- Students should be provided opportunities to draw triangles with ruler and protractor, and with technology.

Progressions

- By sketching geometric shapes that obey given conditions, students lay the foundation for the concepts of congruence and similarity in Grade 8, and for the practice of geometric deduction that will grow in importance throughout the rest of their school careers.
- For example, given three side lengths, perhaps in the form of physical or virtual rods, students try to construct a triangle. Two important possibilities arise: there is no triangle or there is exactly one triangle. By examining many situations where there is no triangle, students can identify the culprit: one side that is longer than the other two put together. From this they can reason that in a triangle the sum of any two sides must be greater than the third. (Please reference page 6 in the Progression document).

Examples

- A triangle with side lengths 3 cm, 4 cm, and 5 cm exists. Use a compass and ruler to draw a triangle with these side lengths. (Modified from Engage NY M6L9)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.GM.B - Solve mathematical problems in authentic contexts involving angle measure, area, surface area, and volume.

STANDARD: 7.GM.B.3

Standards Statement (2021):

Understand the relationship between area and circumference of circles. Choose and use the appropriate formula to solve problems with radius, diameter, circumference and area of circles.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.GM.A.1	8.GM.C.9, HS.GM.C.8, HS.GM.C.10	N/A	7.G.B.4 7.GM.B Crosswalk

Standards Guidance:

Clarifications

- Know that a circle is a two-dimensional shape created by connecting all of the points equidistant from a fixed point called the center of the circle.
- Informally derive and know the formulas for the area and circumference of a circle and use them to solve problems.

Terminology

- Students should know how to write responses in terms of pi.
- Special Note: The terms pi, radius, diameter, and circumference are new academic vocabulary for students.

Boundaries

- Square roots are an 8th grade expectation.

Teaching Strategies

- Students should use proportional reasoning to explain the relationship between the diameter and circumference of a circle and that the unit rate (constant of proportionality) is π in order to derive the formulas for the circumference and area of a circle.

Progressions

- Students have been long familiar with circles and now they undertake a calculation of their perimeters and areas. This is a step forward from their previous methods of calculating area by decomposing figures into rectangles and triangles. Students must now grapple with the meaning of the area of a figure with curved boundary. The area can be estimated by superimposing a square grid and counting squares inside the figure, with the estimate becoming more and more accurate as the grid is made finer and finer. (Please reference page 8 in the Progression document).

Examples

- Illustrative Mathematics:
  - [Circumference of a Circle] [Designs] [Eight Circles] [Wedges of a Circle]
- Student Achievement Partners:
  - Lake Pingualuit
  - Smarter Balanced Assessment Item Illustrating 7.GM.B.3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.GM.B.4

Standards Statement (2021):

Apply facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to determine an unknown angle in a figure.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.GM.C.7, 4.GM.C.9	8.GM.A.1, HS.GM.B.6	N/A	7.G.B.5 7.GM.B Crosswalk

Standards Guidance:

Clarifications

- In previous grades, students have studied angles by type according to size: acute, obtuse, and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical, and adjacent angles.

Terminology

- Supplementary angles – two angles add up to 180 degrees
- Complementary angles – two angles add up to 90 degrees
- Vertical angles – angles opposite each other when two lines intersect
- Adjacent angles – Two angles that have a common side and a common vertex (corner point), and do not overlap.

Boundaries

- This includes writing and solving simple equations for an unknown angle in a figure.

Progressions

- In Grade 7, students build on earlier experiences with angle measurement (see the Grade 4 section of the Geometric Measurement Progression) to solve problems that involve supplementary angles, complementary angles, vertical angles, and adjacent angles.
- Vertical angles have the same number of degrees because they are both supplementary to the same angle. Keeping in mind that two geometric figures are "the same" in Grade 7 if one can be superimposed on the other, it follows that angles that are the same have the same number of degrees. Conversely, if two angles have the same measurement, then one can be superimposed on the other, so having the same number of degrees is a criterion for two angles to be the same.
- An angle is called a right angle if, after extending the rays of the angle to lines, it is the case that all the angles at the vertex are the same. In particular, the measurement of a right angle is 90°. In this situation, the intersecting lines are said to be perpendicular.
- Knowledge of angle measurements allows students to use algebra to determine missing information about particular geometric figures, using algebra in the service of geometry, rather than the other way around. (Please reference page 8 in the Progression document).

Examples

- The ratio of the measurement of an angle to its complement is 1:2. Create and solve an equation to find the measurement of the angle and its complement. (From Engage NY M5L1)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 7.GM.B.5

Standards Statement (2021):

Solve problems in authentic contexts involving two- and three-dimensional figures. Given formulas, calculate area, volume and surface area.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.GM.A.1, 6.GM.A.2, 6.GM.A.4	8.GM.B.8, HS.GM.C.8, HS.GM.C.9, HS.GM.C.10	N/A	7.G.B.6 7.GM.B Crosswalk

Standards Guidance:

Clarifications

- Students should understand the formulas for prisms as the general statement of the area of the base times the height. Students may build upon this generalization for volumes of figures in 8th grade.
- Students should relate the formulas for parallelograms, triangles and trapezoids to the formula for a rectangle.

Terminology

- Cylinder – any three-dimensional figure with two congruent, opposite faces called bases connected by adjacent curved or flat faces (bases can include circles, triangles, rectangles, or other shapes).
- Right prism – any three-dimensional figure with two polygons for bases that are opposite, congruent, and perpendicular to the adjacent faces

Boundaries

- This includes two- and three-dimensional objects composed of polygons.
- Cylinders explored in Grade 7 should be limited to right circular cylinders. Right circular cylinders are three-dimensional solid figures with two congruent, parallel, circular bases that are connected by a curved face that is perpendicular to each base.

Teaching Strategies

- Students should apply knowledge of cross sections as a strategy for revealing a base of cylinders including right prisms.
- Students should apply reasoning about the volume of rectangular prisms to explore the volume of cylinders and other three-dimensional objects composed of cubes and right prisms.
- Students should have opportunities to discover the surface area of a cylinder by decomposing the figure into circles and rectangles.

Progressions

- In Grade 7, students extend the use of geometric terms and definitions with which they have become familiar: polygons, perimeter, area, volume and surface area of two-dimensional and three-dimensional objects, etc. In Grade 6, students found the area of a polygon by decomposing it into triangles and rectangles whose areas they could calculate, making use of structure (MP.7) to make collections of simpler problems (MP.1). Now they apply the same sort of reasoning to three-dimensional figures, dissecting them in order to calculate their volumes. (Please reference page 7 in the Progression document).

Examples

- Illustrative Mathematics: [Sand Under the Swing Set]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.DR.A - Formulate Statistical Investigative Questions

STANDARD: 7.DR.A.1

Standards Statement (2021):

Formulate summary, comparative investigative questions to gain information about a population and that a sample is valid only if the sample is representative of that population.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.A.1	8.DR.A.1	N/A	7.SP.A.1 7.DR.A Crosswalk

Standards Guidance:

Clarification

- Students can generate questions about things they notice and wonder from an authentic situation.
- Understand that statistics can be used to gain information about a population and that a sample is valid only if the sample is representative of that population. (7.SP.A.1)
- Understand that random sampling tends to produce representative samples and support valid inferences.

Terminology

- A statistical investigative question is one that requires data that will vary.
- Understand that random sampling tends to produce representative samples and support valid inferences.
- Potential limitations may include how the sample was selected and/or how the questions were asked.

Teaching Strategies

- Students should have opportunities to answer statistical investigative questions about a population by collecting data from a representative sample, using random sampling techniques to collect the data.

Progressions

- A statistic computed from a random sample, such as the mean of the sample, can be used as an estimate of that same characteristic of the population from which the sample was selected. This estimate must be viewed with some degree of caution because of the variability in both the population and sample data. A basic tenet of statistical reasoning, then, is that random sampling allows results from a sample to be generalized to a much larger body of data, namely, the population from which the sample was selected. (Please reference page 8 in the Progression document).

Examples

- "How old are the students in my class?" is a statistical investigative question because it anticipates variability in students' ages. "How old am I?" is a question used to collect data to answer the investigative question.
- Illustrative Mathematics: Mr. Briggs's Class Likes
- Student Achievement Partners: Smarter Balanced Assessment Item Illustrating 7.DR.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.DR.B - Collect and Consider Data

STANDARD: 7.DR.B.2

Standards Statement (2021):

Collect or consider data from a random sample to compare and draw inferences about a population with an unknown characteristic of interest.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.B.2	8.DR.B.2	N/A	7.SP.A.2 7.DR.B Crosswalk

Standards Guidance:

Clarifications

- Use data from a random sample to gauge how far off the estimate or prediction might be.
- Students should use sample data collected to draw inferences.

Terminology

- A statistical investigative question is one that requires data that will vary.
- Potential limitations may include how the sample was selected and/or how the questions were asked.

Teaching Strategies

- Students should have opportunities to answer statistical investigative questions about a population by collecting data from a representative sample, using random sampling techniques to collect the data.
- Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
- Students should have opportunities to critique examples of sampling techniques.
- Students should conclude when conditions of sampling methods may be biased, random, and not representative of the population.

Progressions

- In short, students realize that conclusions drawn from random samples generalize beyond the sample to the population from which the sample was selected, but a sample statistic is only an estimate of a corresponding population parameter and there will be some discrepancy between the two. Understanding the variability in sampling allows the investigator to gauge the expected size of that discrepancy. (Please reference page 9 in the Progression document).

Examples

- Estimate the mean word length in a book by randomly sampling words from the book. Gauge how far off the estimate is from the actual mean.
- Predict the winner of a school election based on randomly sampled survey data. Gauge how far off the prediction might be.
- Illustrative Mathematics:
  - Valentine Marbles

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.DR.C - Analyze, summarize, and describe data

STANDARD: 7.DR.C.3

Standards Statement (2021):

Analyze two data distributions visually to compare multiple measures of center and variability.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.C.3	8.DR.C.3	N/A	7.SP.B.3 7.DR.C Crosswalk

Standards Guidance:

Teaching Strategies

- Given visual representations of data from dot plots, line graphs, histograms and box-plots, create statements that compare the measures of center and variability between two data sets.
- Students should use side by side bar graphs or segmented bar graphs to compare categorical data distributions of samples from two populations.
- Students should compare data of two samples or populations displayed in box plots and dot plots to make inferences.
- Students should be able to draw inferences using measures of central tendency (mean, median, mode) and/or variability (range, mean absolute deviation and interquartile range) from random samples.
- Students should be given multiple opportunities to compare quantitative data distributions of samples from two populations.

Progressions

- If all measurements in a population are known, no sampling is necessary and data comparisons involve the calculated measures of center. Even then, students should consider variability.
- Conclusions should be made related to a population, using a random sample, by describing a distribution using measures of central tendency (mean, median, mode) and/or variability (range, mean absolute deviation, and interquartile range). (Please reference page 10 in the Progression document).

Examples

- By comparing distributions, investigate whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.
- The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
- Illustrative Mathematics:
  - Offensive Linemen
  - College Athletes

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 7.DR.D - Interpret data and answer investigative questions

STANDARD: 7.DR.D.4

Standards Statement (2021):

Interpret measures of center and measures of variability for numerical data from random samples to compare between two populations, and to answer investigative questions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.D.4	8.DR.D.4	N/A	7.SP.B.4 7.DR.D Crosswalk

Standards Guidance:

Clarification

- Students should use sample data collected to draw inferences.

Teaching Strategies

- Students should have opportunities to critique examples of sampling techniques.
- Students should conclude when conditions of sampling methods may be biased, random, and not representative of the population.

Progressions

- For random samples, students should understand that medians and means computed from samples will vary from sample to sample and that making informed decisions based on such sample statistics requires some knowledge of the amount of variation to expect. Just as for proportions, a good way to gain this knowledge is through simulation, beginning with a population of known structure. (Please reference page 10 in the Progression document).

Examples

- Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.
- Estimate the mean word length in a book by randomly sampling words from the book. Gauge how far off the estimate is from the actual mean.
- Predict the winner of a school election based on randomly sampled survey data. Gauge how far off the prediction might be.
- Illustrative Mathematics:
  - College Athletes
  - Offensive Linemen
- Student Achievement Partners:
  - Assessment Item Illustrating 7.SP.B.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4I: Grade 8 Math Standards and Guidance

Critical Areas of Focus

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Domains and Clusters

8.AEE - Algebraic Reasoning: Expressions and Equations

- 8.AEE.A Expressions and Equations Work with radicals and integer exponents.
- 8.AEE.B Understand the connections between proportional relationships, lines, and linear equations.
- 8.AEE.C Analyze and solve linear equations and pairs of simultaneous linear equations.

8.AFN - Algebraic Reasoning: Functions

- 8.AFN.A Define, evaluate, and compare functions.
- 8.AFN.B Use functions to model relationships between quantities.

8.NS - Numeric Reasoning: Number Systems

- 8.NS.A Know that there are numbers that are not rational, and approximate them by rational numbers.

8.GM - Geometric Reasoning and Measurement

- 8.GM.A Understand congruence and similarity using physical models, transparencies, or geometry software.
- 8.GM.B Understand and apply the Pythagorean Theorem.
- 8.GM.C Solve mathematical problems in authentic contexts involving volume of cylinders, cones, and spheres.

8.DR - Data Reasoning

- 8.DR.A Formulate Statistical Investigative Questions.
- 8.DR.B Collect and Consider Data.
- 8.DR.C Analyze, summarize, and describe data.
- 8.DR.D Interpret data and answer investigative questions.

Students should spend the large majority[1] of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

[1]*At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see the K-8 major work of the grade developed by Student Achievement Partners.*

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.AEE.A - Expressions and Equations Work with radicals and integer exponents.

STANDARD: 8.AEE.A.1

Standards Statement (2021):

Apply the properties of integer exponents using powers of 10 to generate equivalent numerical expressions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.A.1	8.AEE.A.3, 8.AEE.A.4	N/A	8.EE.A.1 8.AEE.A Crosswalk

Standards Guidance:

Boundaries

- Exploration of integer exponents can be limited to base 10 for this standard. Additional bases could be explored as a possible extension.

Teaching Strategies

- Students should use numerical reasoning to identify patterns associated with properties of integer exponents.
- The following properties should be addressed: product rule, quotient rule, power rule, power of product rule, power of a quotient rule, zero exponent rule, and negative exponent rule.

Progressions

- Students have been denoting whole number powers of 10 with exponential notation since Grade 5, and they have seen the pattern in the number of zeros when powers of 10 are multiplied. They express this as $10^a \cdot 10^b = 10^{a+b}$ for whole numbers a and b. Requiring this rule to hold when a and b are integers leads to the definition of the meaning of pwers with 0 and negative exponents. For example, we define 10^0=1 because we want $10^a \cdot 10^0 = 10^{a+0} = 10^a$, so 10^0 must equal 1. (Please reference page 11 in the Progression document)

Examples

- Generate equivalent numerical expressions. For example, $10^2 * 10^{-5} = 10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$
- Illustrative Mathematics:
  - Raising to the zero and negative powers
  - Extending the Definitions of Exponents, Variation 1
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 8.AEE.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.AEE.A.2

Standards Statement (2021):

Represent solutions to equations using square root and cube root symbols.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.B.4	HS.AEE.B.5, 8.GM.B.7	7.NS.A.3, 8.NS.A.2, 8.GM.B.6, 8.GM.C.9, HS.NQ.A.1	8.EE.A.2 8.AEE.A Crosswalk

Standards Guidance:

Clarifications

- Equations should include those with irrational number solutions, such as the solution for $x^2 = 14$ would include $x = \pm\sqrt{14}$

Boundaries

- Use square root and cube root symbols to represent solutions to equations of the form $x^2=p$ and $x^3=p$, where p is a positive rational number.
- Evaluate square roots of small perfect squares up to 225 and cube roots of small perfect cubes up to 1000.
- Know irrational numbers include square roots of non-perfect squares, such as $\sqrt{2}$, and cube roots of non-perfect cubes.

Teaching Strategies

- Students should be able to find patterns within the list of square numbers and then with cube numbers.
- Students should be able to recognize that squaring a number and taking the square root of a number are inverse operations; likewise, cubing a number and taking the cube root are inverse operations.

Progressions

- Notice that students do not learn the properties of rational exponents until high school. However, they prepare in Grade 8 by starting to work systematically with the square root and cube root symbols, writing, for example, $\sqrt{64} = \sqrt{8^2}$and $\sqrt[3]{5^3} = 5$.
- Since $\sqrt{p}$ is defined to mean the positive solution to the equation $x^2 = p$ (when it exists). It is not mathematically correct to say $\sqrt{64} = \pm 8$ (as is a common misconception). In describing the solutions to $x^2 = 64$, students should write $x = \pm\sqrt{64} = \pm 8$ (Please reference page 11 in the Progression document).

Examples

- $\sqrt{64} = \left(\sqrt{8}\right)^2 = 8$ and $\sqrt[3]{125} = 5$
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 8.AEE.A.2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.AEE.A.3

Standards Statement (2021):

Estimate very large or very small quantities using scientific notation with a single digit times an integer power of ten.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.OA.A.2, 5.NBT.A.2, 8.AEE.A.1	8.AEE.A.4	5.NBT.A.4	8.EE.A.3 8.AEE.A Crosswalk

Standards Guidance:

Clarifications

- Students should use place value reasoning which supports the understanding of digits shifting to the left or right when multiplied by a power of 10.
- Product and quotient rules for powers is relevant at 8th grade, and only for powers of 10

Teaching Strategies

- Students should use the magnitude of quantities to compare numbers written in scientific notation to determine how many times larger (or smaller) one number written in scientific notation is than another.
- Students should have opportunities to compare numbers written in scientific notation in contextual problems.

Examples

- Compare two quantities written in this format. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.
- Illustrative Mathematics:
  - Orders of Magnitude
  - Ant and Elephant

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.AEE.A.4

Standards Statement (2021):

Perform operations with numbers expressed in scientific notation.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.AEE.B.3, 8.AEE.A.1, 8.AEE.A.3	N/A	HS.NQ.B.3	8.EE.A.4 8.AEE.A Crosswalk

Standards Guidance:

Clarifications

- Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.
- Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.
- Interpret scientific notation that has been generated by technology.

Teaching Strategies

- Students should use place value reasoning, which supports the understanding of digits shifting to the left or right when multiplied by a power of 10.
- Students combine knowledge of integer exponent rules and scientific notation to perform operations with numbers expressed in scientific notation.
- Students should solve problems involving real-life contexts.

Progressions

- Include authentic contexts where both standard and scientific notation are used. Use scientific notation to choose units of appropriate size for measurements of very large or very small quantities. (Please reference page 11 in the Progression document).

Examples

- Use millimeters per year for seafloor spreading.
- Interpret scientific notation that has been generated by technology such as 1.2×10^6 may be displayed in a calculator as “1.2E6”.
- Illustrative Mathematics:
  - Choosing appropriate units
  - Ants versus humans
  - Giantburgers
- Student Achievement Partners:
  - Assessment Item Illustrating 8.AEE.A.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.AEE.B - Understand the connections between proportional relationships, lines, and linear equations.

STANDARD: 8.AEE.B.5

Standards Statement (2021):

Graph proportional relationships in authentic contexts. Interpret the unit rate as the slope of the graph, and compare two different proportional relationships represented in different ways.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.RP.A.1, 7.RP.A.2	8.AFN.A.2, HS.AEE.D.9	HS.GM.A.3	8.EE.B.5 8.AEE.B Crosswalk

Standards Guidance:

Terminology

- Various forms of linear functions include standard and slope-intercept forms.
- Key features include rate of change (slope), intercepts, strictly increasing or strictly decreasing, positive, negative, and end behavior.

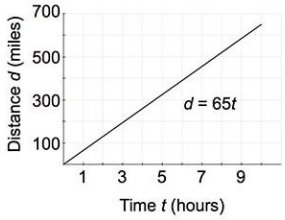
Teaching Strategies

- Use verbal descriptions, tables and graphs created by hand and/or using technology.

Progressions

- As students start to build a unified notion of the concept of function they are able to compare proportional relationships presented in different ways.
- For example, the table on the right shows 300 miles in 5 hours, whereas the graph shows more than 300 miles in the same time. (Please reference page 12 in the Progression document).

t (hours)	1	2	3	4	5	6
$60t$ (miles)	60	120	180	240	300	360



Examples

- Interpret the unit rate as the slope of the graph. Compare one or more proportional relationships represented in different ways.
- For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
- Illustrative Mathematics:
  - Peaches and Plums
  - Equations of Lines
  - Coffee by the Pound
  - Comparing Speeds in Graphs and Equations
  - Stuffing Envelopes
  - Who Has the Best Job?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.AEE.B.6

Standards Statement (2021):

Write the equation for a line in slope intercept form y = mx + b, where m and b are rational numbers, and explain in context why the slope m is the same between any two distinct points.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.RP.A.2	8.AEE.C.8, 8.AFN.A.2, 8.AFN.A.3, HS.AEE.D.9	HS.GM.A.3	8.EE.B.6 8.AEE.B Crosswalk

Standards Guidance:

Clarification

- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane
- Derive the equation $y=mx$ for a line through the origin and the equation $y=mx+b$ for a line intercepting the vertical axis at b.

Boundaries

- Content expecation would include students using both standard and slope-intercept forms for a linear equation. Students should be able to rewrite linear equations written in different forms depending on the given context.
- This work could also include generating equations using the point-slope form to generate an equation for a line that passes through a point with a given slope.

Terminology

- Forms of linear equations:
  - Standard Form: $Ax + By = C$
  - Slope-Intercept Form: $y = mx + b$
  - Point-Slope Form: $(y - y_1) = m(x - x_1)$ for a line with slope m, that passes through the point (x_1,y_1)

Progressions

- Content expecation would be for a student to calcluate the slope between two points (x_1,y_1) and (x_2,y_2) as the difference between change in y (e.g. "rise") over the change in x (eg. "run") as $m = \Delta y/\Delta x$. A fact that a line has a well-defined slope where the ratio between the rise and run for any two points on the line is always the same—depends on similar triangles. (Please reference page 12 in the Progression document).

Examples

- Know that the slope *m* is the same between any two distinct points on a non-vertical line and be able to explain or demonstrate why.
- Derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at b.
- Illustrative Mathematics: [Slopes Between Points on a Line] [Equations of Lines] [Coupon versus discount]
- Student Achievement Partners: [Assessment Item Illustrating 8.AEE.B]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.AEE.C - Analyze and solve linear equations and pairs of simultaneous linear equations.

STANDARD: 8.AEE.C.7

Standards Statement (2021):

Solve linear equations with one variable including equations with rational number coefficients, with the variable on both sides, or whose solutions require using the distributive property and/or combining like terms.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.AEE.A.1, 7.AEE.B.4	HS.AEE.A.3, HS.AEE.A.2, HS.AEE.C.8, HS.AEE.D.11	N/A	8.EE.C.7 8.AEE.C Crosswalk

Standards Guidance:

Clarifications

- To achieve fluency, students should be able to choose flexibly among methods and strategies to solve mathematical problems accurately and efficiently.
- Students should rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Interpret and explain the results.

Terminology

- Parts of an expression include terms, factors, coefficients, and operations.

Boundaries

- This standard also includes solving or giving examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.
- Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Teaching Strategies

- Students should use algebraic reasoning in their descriptions of the solutions to linear equations.
- Problems should be practical and contextual providing a purpose for analyzing equivalent forms of an expression.

Progressions

- Building upon skills from grade 7, students combine like terms on the same side of the equal sign and use the distributive property to simplify the equation when solving. Emphasis in this standard is also on using rational coefficients. Solutions of certain equations may elicit infinitely many or no solutions. Include linear equations and inequalities with rational number coefficients and whose solutions require expanding expressions using the distributive property and collecting like terms.

Examples

- Given ax + 3 = 7, solve for x.
- Illustrative Mathematics: [The Sign of Solutions] [Sammy's Chipmunk and Squirrel Observations]
- Student Achievement Partners: [Movie Flop] [Equations Procedural Skill and Fluency Mini-Assessment]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.AEE.C.8

Standards Statement (2021):

Find, analyze, and interpret solutions to pairs of simultaneous linear equations using graphs or tables.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.B.4, 7.AEE.B.4, 8.AEE.B.6	HS.AEE.B.4, HS.AEE.B.6, HS.AEE.D.9, HS.AEE.C.7, HS.AEE.D.11	HS.GM.A.3	8.EE.C.8 8.AEE.C Crosswalk

Standards Guidance:

Clarification

- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs.
- Estimate solutions by graphing the equations; solve simple cases by inspection, or by using tables.

Teaching Strategies

- Include mathematical problems in authentic contexts leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
- Students should be able to analyze and find solutions to systems of equations presented numerically in tables, or graphically.

Progressions

- Students should be provided with opportunities to explore systems of equations represented using technology such as interactive graphs to analyze and interpret the solutions to the systems.
- Students should have the opportunity to explore visual graphs of equations that are parallel, perpendicular or neither parallel nor perpendicular to develop a deep, conceptual understanding. (Please reference page 13 in the Progression document).

Examples

- Given coordinates for two pairs of points, a student can determine whether the line through the first pair of points intersects the line through the second pair.
- A student can graph two linear equations that represent a culturally relevant problem using digital graphing tools (e.g., Desmos, graphing calculators, or other) and visually make sense of the graphed lines in context. A student can provide a verbal or written explanation of their reasoning.
- A student can recognize that there is no solution to the system of equations formed by $3x + 2y = 5$ and $3x + 2y = 6$ because the lines are parallel and $3x + 2y$ cannot simultaneously be 5 and 6.
- Illustrative Mathematics:
  - Fixing the Furnace
  - How Many Solutions?
- Student Achievement Partners:
  - Simultaneous Equations
  - Simultaneous Linear Equations Mini-Assessment
  - Smarter Balanced Assessment Item Illustrating 8.AEE.C.8

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.AFN.A - Define, evaluate, and compare functions.

STANDARD: 8.AFN.A.1

Standards Statement (2021):

Understand in authentic contexts, that the graph of a function is the set of ordered pairs consisting of an input and a corresponding output.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.RP.A.2	8.AFN.A.2, 8.AFN.A.3, 8.AFN.B.5, HS.AFN.A.1	HS.GM.A.1	8.F.A.1 8.AFN.A Crosswalk

Standards Guidance:

Clarification

- Understanding that a function is a rule that assigns exactly one output to each input.

Boundaries

- Use of function notation is not required in Grade 8.

Teaching Strategies

- Students should be able to use algebraic reasoning when formulating an explanation or justification regarding whether or not a relationship is a function or not a function.

Communication

- Describe the graph of a function as the set of ordered pairs consisting of an input and the corresponding output. Formal language, such as domain and range, and function notation may be postponed until high school. (Please reference pages 4 and 5 in the Progression document).

Examples

- If a function gives the number of hours it takes a person to assemble n engines in a factory, then the set of positive integers would be an appropriate domain for the function.
- Illustrative Mathematics:
  - Garbage, Version 1
  - Function Rules
  - Introducing Functions
  - The Customers
- Student Achievement Partners:
  - Functions Mini-Assessment
  - Assessment Item Illustrating 8.AFN.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.AFN.A.2

Standards Statement (2021):

Compare the properties of two functions represented algebraically, graphically, numerically in tables, or verbally by description.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.AEE.B.4, 7.RP.A.2, 8.AFN.A.1, 8.AEE.B.5, 8.AEE.B.6	8.AFN.B.5, HS.AFN.B.4	N/A	8.F.A.2 8.AFN.A Crosswalk

Standards Guidance:

Teaching Strategies

- Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

Examples

- Given a linear function represented by a table of values and a linear function represented by an algebraic equation, determine which function has the greater rate of change.
- Illustrative Mathematics:
  - Battery Charging
- Student Achievement Partners:
  - Functions Mini-Assessment
  - Assessment Item Illustrating 8.AFN.A.2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.AFN.A.3

Standards Statement (2021):

Understand and identify linear functions, whose graph is a straight line, and identify examples of functions that are not linear.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AFN.A.1, 8.AFN.A.2, 8.AEE.B.6	8.AFN.B.4, 8.AFN.B.5, HS.AFN.D.10	N/A	8.F.A.3 8.AFN.A Crosswalk

Standards Guidance:

Clarifications

- Students should be given opportunities to explore how an equation in the form y = mx + b is a translation of the equation y = mx.
- In Grade 7, students had multiple opportunities to build a conceptual understanding of slope as they made connections to unit rate and analyzed the constant of proportionality for proportional relationships.
- Students should be given opportunities to explore and generalize that two lines with the same slope but different intercepts, are also translations of each other.
- Students should be encouraged to attend to precision when discussing and defining b (i.e., b is not the intercept; rather, b is the y-coordinate of the y- intercept). Students must understand that the x-coordinate of the y-intercept is always 0.

Teaching Strategies

- Students should be given the opportunity to explore and discover the effects on a graph as the value of the slope and y-intercept changes using technology.
- Students should be able to model contextual situations using graphs and interpret graphs based the contextual situations.
- Students should analyze a graph by determining whether the function is increasing or decreasing, linear or non-linear.
- Students should have the opportunity to explore a variety of graphs including time/distance graphs and time/velocity graphs.

Progressions

- Students learn to recognize linearity in a table: when constant differences between input values produce constant differences between output values. The proof that y = mx + b is also the equation of a line, and hence that the graph of a linear function is a line, is an important piece of reasoning connecting algebra with geometry in Grade 8. (Please reference page 5 in the Progression document).

Examples

- For example, A) determine if an equation represents a linear function and give examples of both linear and non-linear functions and B) show that the function A = s^2 is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.
- Illustrative Mathematics: [Introduction to Linear Functions]
- Student Achievement Partners: [Domino Effect] [Functions Mini-Assessment] [Assessment Item Illustrating 8.AFN.A.3]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.AFN.B - Use functions to model relationships between quantities.

STANDARD: 8.AFN.B.4

Standards Statement (2021):

Construct a function to model a linear relationship in authentic contexts between two quantities.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.RP.A.2, 8.AFN.A.3	8.AFN.B.5, HS.AFN.A.3, HS.AFN.D.8, HS.AFN.B.4, HS.AEE.B.4, HS.AFN.D.10	8.DR.D.4, HS.DR.D.11	8.F.B.4 8.AFN.B Crosswalk

Standards Guidance:

Clarification

- Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Teaching Strategies

- This learning objective also includes verbal descriptions and scenarios of equations, tables, and graphs.

Progressions

- Graphs are ubiquitous in the study of functions, but it is important to distinguish a function from its graph. For example, a linear function does not have a slope but the graph of a non-vertical line has a slope.
- The slope of a vertical line is undefined and the slope of a horizontal line is 0. Either of these cases might be considered “no slope.” Thus, the phrase “no slope” should be avoided because it is ambiguous and “non-existent slope” and “slope of 0” should be distinguished from each other. (Please reference page 6 in the Progression document).

Examples

- Illustrative Mathematics:
  - Delivering the Mail, Assessment Variation
  - High School Graduation
  - Downhill
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 8.AFN.B.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.AFN.B.5

Standards Statement (2021):

Describe qualitatively the functional relationship between two quantities in authentic contexts by analyzing a graph.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AFN.A.1, 8.AFN.A.2, 8.AFN.A.3, 8.AFN.B.4	HS.AFN.D.10, HS.AFN.C.6	N/A	8.F.B.5 8.AFN.B Crosswalk

Standards Guidance:

Clarification

- Identify where the function is increasing or decreasing, linear or nonlinear.
- Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Teaching Strategies

- Students should use algebraic reasoning to show and explain that the graph of an equation represents the set of all its solutions.
- Students continue to build upon their understanding of proportional relationships, using the idea that one variable is conditioned on another.
- Students should relate graphical representations to contextual situations.
- Students should use tables to relate solution sets to graphical representations on the coordinate plane.

Examples

- Illustrative Mathematics:
  - Bike Race
  - Distance
  - Riding by the Library
- Student Achievement Partners:
  - Domino Effect
  - Functions Mini-Assessment
  - Assessment Item Illustrating 8.AFN.B.5

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.NS.A - Know that there are numbers that are not rational, and approximate them by rational numbers.

STANDARD: 8.NS.A.1

Standards Statement (2021):

Know that real numbers that are not rational are called irrational.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.NS.A.2, 7.NS.A.3	8.NS.A.2, HS.NQ.A.1, HS.NQ.A.2	N/A	8.NS.A.1 8.NS.A Crosswalk

Standards Guidance:

Terminology

- Rational numbers are numbers that can be represented by a ratio $\frac{a}{b}$ where "a" is an integer, and "b" is a non-zero whole number (e.g. natural number set).
- Rational numbers have decimal expansions that terminate in zeros or eventually repeat.
- Irrational numbers cannot be represented by a ratio $\frac{a}{b}$ and would include non-terminating, non-repeating decimals.

Teaching Strategies

- Students should be provided with experiences to use numerical reasoning when describing decimal expansions.
- Students should be able to classify real numbers as rational or irrational.
- Students should know that when a square root of a positive integer is not an integer, then it is irrational.
- Students should use prior knowledge about converting fractions to decimals learned in 6th and 7th grade to connect changing decimal expansion of a repeating decimal into a fraction and a fraction into a repeating decimal.
- Emphasis is placed on how all rational numbers can be written as an equivalent decimal. The end behavior of the decimal determines the classification of the number.

Examples

- Understand that every number has a decimal expansion.
- For rational numbers show that the decimal expansion terminates or repeats eventually.
- Convert a decimal expansion which terminates or repeats eventually into a rational number expressed as a fraction.
- Illustrative Mathematics:
  - Identifying Rational
  - Converting Decimal Representations of Rational Numbers to Fraction Representations
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 8.NS.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.NS.A.2

Standards Statement (2021):

Use rational approximations of irrational numbers to compare size and locate on a number line.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.NS.A.1	HS.NQ.A.2	8.AEE.A.2	8.NS.A.2 8.NS.A Crosswalk

Standards Guidance:

Teaching Strategies

- Students should use visual models and numerical reasoning to approximate irrational numbers.

Boundaries

- Locate the approximate location of irrational numbers on a number line and estimate the value of expressions.
- For decimal approximations, the concept for this grade level extends to comparing irrational numbers to at least the hundredths place on a number line.

Examples

- Compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of square roots. For example,
  - Start with locating the nearest perfect squares and obtain closer and closer successive decimal approximations.
- Using successive approximations, estimate the decimal expansion of $\sqrt{17}$, such as by showing that $\sqrt{17}$ is between 4 and 5, then closer to 4 (between 4.0 and 4.5) on a number line.
- Estimate the value of $\sqrt{2}$.
- By truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.
- Illustrative Mathematics:
  - Irrational Numbers on the Number Line
  - Comparing Rational and Irrational Numbers
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 8.NS.A.2 [Option 1] [Option 2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.GM.A - Understand congruence and similarity using physical models, transparencies, or geometry software.

STANDARD: 8.GM.A.1

Standards Statement (2021):

Verify experimentally the properties of rotations, reflections, and translations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.GM.A.2, 7.GM.B.4	8.GM.A.2, 8.GM.A.3	N/A	8.G.A.1 8.GM.A Crosswalk

Standards Guidance:

Clarifications

- Understand that:
  - Lines are taken to lines, and line segments to line segments of the same length.
  - Angles are taken to angles of the same measure.
  - Parallel lines are taken to parallel lines.

Boundaries

- Rotations can be limited to 90, 180, 270 and 360 degrees around the origin
- Reflections can be limited to reflection over horizontal and vertical lines

Progressions

- Students should get a sense that rigid motions are special transformations. They should encounter and experience transformations which do not preserve lengths, do not preserve angles, or do not preserve either. (Please reference page 9 in the Progression document).

Examples

- Show these properties using physical models, transparencies, and/or geometry software.
- Illustrative Mathematics:
  - Origami Silver
- Student Achievement Partners:
  - Rigid Rotation
  - Assessment Item Illustrating 8.GM.A.1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.GM.A.2

Standards Statement (2021):

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.GM.A.1	8.GM.A.4, 8.GM.A.5, HS.GM.A.1	N/A	8.G.A.2 8.GM.A Crosswalk

Standards Guidance:

Clarification

- Students describe a series of rigid transformations that map a two dimensional figure onto its image.

Terminology

- Rigid transformations include translations (slides), reflections (flips), rotations (turns), or glide reflections.

Progressions

- Two figures in the plane are said to be congruent if there is a sequence of rigid motions that takes one figure onto the other. It should be noted that if we find a sequence of rigid motions taking figure A to figure B, then we can also find a sequence taking figure B to figure A. In high school mathematics the topic of congruence will be developed in a coherent, logical way, giving students the tools to investigate many geometric questions. In Grade 8, the treatment is informal, and students discover what they can about congruence through experimentation with actual motions. (Please reference page 9 in the Progression document).

Examples

- Given two congruent figures, describe a sequence of transformations that demonstrates the congruence between them.
- Illustrative Mathematics:
  - Congruent Rectangles
  - Triangle congruence with coordinates
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 8.GM.A.2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.GM.A.3

Standards Statement (2021):

Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.GM.A.3, 8.GM.A.1	8.GM.A.4, HS.GM.A.1, HS.GM.A.2	6.NS.C.8	8.G.A.3 8.GM.A Crosswalk

Standards Guidance:

Progressions

- In Grade 7, students study scale drawings to as a prelude to the transition from "same shape" to similarity in Grade 8. In Grade 8, change in scale becomes understood in terms of transformations that expand or contract the plane and the previous work with scale drawings flows naturally into describing dilations in terms of coordinates. (Please reference pages 9 & 10 in the Progression document).

Examples

- Given a triangle with given coordinates, give the new coordinates after a prescribed transformation.
- The image of Triangle ABC with A=(−3,0), B=(−3,−2) and C=(4,−2) would have coordinates A'=(−3−3,0+2)=(−6,2), B'=(−3−3,−2+2)=(−6,0), and C'=(4−3,−2+2)=(1,0) following a translation 3 units to the left and 2 units up.
- The center of dilation should be limited to a) the origin on the coordinate plane or b) one vertex of a figure such as a triangle.
- Illustrative Mathematics:
  - Point Reflection
  - Triangle congruence with coordinates
  - Effects of Dilations on Length, Area, and Angles

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.GM.A.4

Standards Statement (2021):

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and/or dilations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.GM.A.2, 8.GM.A.3	8.GM.A.5, HS.GM.A.2	N/A	8.G.A.4 8.GM.A Crosswalk

Standards Guidance:

Progressions

- Students observe the properties of dilations by experimenting with them, just as they did with rigid motions. They notice that shape is preserved under dilations, but that size is not preserved unless r =1. This observation suggests that the idea of "same shape" can be made precise as similarity: Two figures are similar if there is a sequence of rigid motions and dilations that places one figure directly on top of the other. (Please reference page 10 in the Progression document).

Examples

- Given two similar two-dimensional figures, describe a sequence of transformations that demonstrates the similarity between them.
- Illustrative Mathematics:
  - Creating Similar Triangles
- Student Achievement Partners:
  - Assessment Item Illustrating 8.GM.A.4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.GM.A.5

Standards Statement (2021):

Use informal arguments to establish facts about interior and exterior angles of triangles and angles formed by parallel lines cut with a transversal.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.GM.A.2, 8.GM.A.4	HS.GM.A.1, HS.GM.A.2, HS.GM.B.6	N/A	8.G.A.5 8.GM.A Crosswalk

Standards Guidance:

Terminology

- Including identify alternate exterior angles, alternate interior angles, linear pairs, same side interior angles, same side exterior angles, and corresponding angles.

Boundaries

- This standard includes using the properties of the angle sum of the interior angles of a triangle, exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles to find missing angle measures.

Progressions

- Use informal (visual) construction with tools (patty paper, protractor, etc.) to discover the angle relationships between angles formed when two lines are cut by a transversal.
- When using more than one transversal, tie into similar triangles and to set up problems using triangle sum relationships (angle sum). (Please reference page 10 in the Progression document).

Examples

- Arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so.
- Illustrative Mathematics:
  - Find the Missing Angle
  - A Triangle's Interior Angles
  - Find the Angle
  - Rigid motions and congruent angles
  - Tile Patterns I: octagons and squares

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.GM.B - Understand and apply the Pythagorean Theorem.

STANDARD: 8.GM.B.6

Standards Statement (2021):

Distinguish between applications of the Pythagorean Theorem and its Converse in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.GM.B.7	HS.GM.D.14, HS.GM.D.12	8.AEE.A.2	8.G.B.6 8.GM.B Crosswalk

Standards Guidance:

Clarification

- Analysis and justification can be done using a variety of representation including use of pictures, diagrams, narratives, or models.

Terminology

- The Pythagorean Theorem states, that in any right triangle, the square length hypotenuse is equals the sum of the square length of the other two sides (e.g. $a^2+b^2=c^2$).
- The converse of the Pythagorean Theorem states that if a triangle has sides of length a, b, and c and if $a^2+b^2=c^2$ then the angle opposite the side of length c is a right angle.

Teaching Strategies

- Students should have the opportunity to explore one or more proofs of the Pythagorean Theorem, but are not required to prove the Theorem.
- Geometric and spatial reasoning should be used when explaining the Pythagorean Theorem.

Progressions

- Students learn that there are lengths that cannot be represented by a rational number. For example, by looking at areas of figures in the coordinate plane, students discover that the hypotenuse of a triangle with legs of length 1 is an irrational number. Students can continue this line of reasoning to explain a dissection proof of the Pythagorean Theorem. (Please reference page 11 in the Progression document).

Examples

- Many ancient cultures used simple Pythagorean triples such as (3,4,5) in order to accurately construct right angles: if a triangle has sides of lengths 3, 4, and 5 units, respectively, then the angle opposite the side of length 5 units is a right angle.
- The Pythagorean Theorem tells us that a certain relation holds amongst the side lengths of a right triangle. These ancient architects, however, do not have a right triangle but rather want to *produce* a right triangle. The converse of the Pythagorean Theorem enables them to do just this: they can conclude that an angle is a right angle provided a certain relationship holds between side lengths of a triangle.
- Illustrative Mathematics:
  - Converse of the Pythagorean
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 8.GM.B.6

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.GM.B.7

Standards Statement (2021):

Apply the Pythagorean Theorem in authentic contexts to determine unknown side lengths in right triangles.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.NS.C.8, 8.AEE.A.2	8.GM.B.6, 8.GM.B.8, HS.GM.D.12	N/A	8.G.B.7 8.GM.B Crosswalk

Standards Guidance:

Teaching Strategies

- Geometric and spatial reasoning should be used to solve problems involving the Pythagorean theorem.
- Models and drawings may be useful as students solve contextual problems in two- and three-dimensions.

Boundaries

- Include authentic mathematical problems in two and three dimensions.

Examples

- How tall is the Great Pyramid of Giza below?



- Illustrative Mathematics:
  - Area of a Trapezoid
  - Running on the Football Field
  - Circle Sandwich
  - Spiderbox
- Student Achievement Partners:
  - Blueprint

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: 8.GM.B.8

Standards Statement (2021):

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.NS.C.8, 7.GM.B.5, 8.GM.B.7	HS.GM.D.14, HS.GM.D.13	N/A	8.G.B.8 8.GM.B Crosswalk

Standards Guidance:

Boundaries

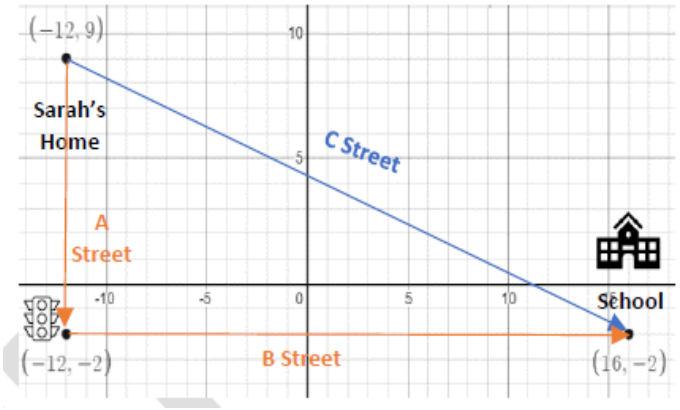
- The Distance Formula is NOT included in the 8th grade standard.
- Students should apply their understanding of the Pythagorean Theorem to find the distance. Use of the distance formula is not an expectation for this grade level.

Progressions

- In Grade 6, students calculate distances in the coordinate plane between points lying on the same horizontal or vertical line. In particular, they calculate the lengths of the vertical and horizontal legs of a triangle corresponding to two points in the coordinate plane. In Grade 7, they can use the Pythagorean Theorem to calculate the length of its hypotenuse, which is the distance between the two points.
- Calculating this distance as an application of the Pythagorean Theorem before doing so in high school as an application of the distance formula provides students an opportunity to look for and make use of structure in the coordinate plane (MP.7), and provides an opportunity for students to connect the distance formula to previous learning. (Please reference pages 11 & 12 in the Progression document).

Examples

- There are two paths that Sarah can take when walking to school. One path is to take A Street from home to the traffic light and then walk on B street from the traffic light to the school, and the other way is for her to take C street directly to the school. How much shorter is the direct path along C Street?



- Illustrative Mathematics:
  - Finding isosceles triangles
  - Finding the distance between points

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.GM.C - Solve mathematical problems in authentic contexts involving volume of cylinders, cones, and spheres.

STANDARD: 8.GM.C.9

Standards Statement (2021):

Choose and use the appropriate formula for the volume of cones, cylinders, and spheres to solve problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.GM.B.3, 8.AEE.A.2	HS.GM.C.8, HS.GM.C.9, HS.GM.C.10, HS.GM.C.11	N/A	8.G.C.9 8.GM.C Crosswalk

Standards Guidance:

Boundaries

- Memorizing the formulas is NOT included in this standard.

Teaching Strategies

- Given the volume, solve for an unknown dimension of the figure. Students will need to be able to express the answer in terms of pi and as a decimal approximation.

Progressions

- Students learn and use formulas for the volumes of cylinders, cones, and spheres. Explanations for these formulas do not occur until high school. However, Grade 8 students can look for structure in these formulas (MP.7). They know that the volume of a cube with sides of length s is s^3.
- A cube can be decomposed into three congruent pyramids, each of which has a square base, where the height is equal to the side length of the square. Each of these pyramids must have the volume $(1/3)s^3$, suggesting that the volume of a pyramid whose base has area B and whose height is h might be (1/3)Bh. The volume formulas for cylinders and cones have an analogous relationship.
  - cylinder $B \cdot h = \pi r^2 h$
  - cone $\frac{1}{3}B \cdot h = \frac{1}{3}\pi r^2 h$
- Please reference page 12 in the Progression document.

Examples

- Illustrative Mathematics:
  - Flower Vases
  - Glasses
- Student Achievement Partners:
  - Smarter Balanced Assessment Item Illustrating 8.GM.C.9 [Example 1] [Example 2]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.DR.A - Formulate Statistical Investigative Questions

STANDARD: 8.DR.A.1

Standards Statement (2021):

Formulate statistical investigative questions to articulate research topics and uncover patterns of association seen in bivariate categorical data.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.DR.A.1	HS.DR.A.1, HS.DR.A.2, HS.DR.A.3, HS.DR.A.4	N/A	8.SP.A.4 8.DR.A Crosswalk

Standards Guidance:

Clarifications

- Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.
- Students can generate questions about things they notice and wonder from a real-life situation.

Terminology

- A statistical investigative question is one that requires data that will vary.

Teaching Strategies

- Students should be able to use statistical reasoning to anticipate patterns of association as they formulate questions, such as and anticipated positive or negative association between the categorial variables of interest.
- Provide opportunities to engage in an analysis of sources of bias within assumptions as students formulate questions.

Progressions

- Building on experience with decimals and percent, and the ideas of association between measurement variables, students now take a more careful look at possible association between categorical variables. “Is there a difference between sixth graders and eighth graders with regard to their preference for rock, rap, or country music?”
- Data from a random sample of sixth graders and another random sample of eighth graders are summarized by frequency counts in each cell in a two-way table of preferred music type by grade. The proportions of favored music type for the sixth graders are then compared to the proportions for eighth graders. If the two proportions for each music type are about the same, there is little or no association between the grade and music preference because both grades have about the same preferences. If the two proportions differ, there is some evidence of association because grade level seems to make a difference in music preferences. The nature of the association should then be described in more detail. (Please reference pages 11 & 12 in the Progression document).

Examples

- Illustrative Mathematics:
  - Music and Sports
  - What's Your Favorite Subject?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.DR.B - Collect and Consider Data

STANDARD: 8.DR.B.2

Standards Statement (2021):

Collect or consider data using surveys and measurements to capture patterns of association, and critically analyze data collection methods.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.DR.B.2	HS.DR.B.5, HS.DR.B.6, HS.DR.B.7	N/A	8.SP.A.2 8.DR.B Crosswalk

Standards Guidance:

Clarification

- Know that straight lines are widely used to model relationships between two quantitative variables.

Terminology

- The line of best fit shows the linear relationship between two variables in a data set.
- It is important to indicate 'predicted' to indicate this is a probabilistic interpretation in context, and not deterministic.

Teaching Strategies

- Students should be able to use statistical reasoning to consider patterns of association, such as clustering, outliers, positive or negative association, linear association, and nonlinear association through the analysis of data presented in multiple ways.
- Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
- Use relative frequencies calculated for rows or columns to describe possible association between the two variables.
- Students should discover the line of best fit as the one that comes closest to most of the data points.
- Provide opportunities to engage in an analysis of sources of bias within collection methods used by students.

Progressions

- For a data showing a linear pattern, students sketch a line through the "center" of the cloud of points that captures the essential nature of the trend, at first by use of an informal fitting procedure, perhaps as informal as laying a stick of spaghetti on the plot. How well the line "fits" the cloud of points is judged by how closely the points are packed around the line, considering that one or more outliers might have tremendous influence on the positioning of the line. (Please reference page 11 in the Progression document).

Examples

- Illustrative Mathematics:
  - Laptop Battery Charge
  - Animal Brains

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.DR.C - Analyze, summarize, and describe data

STANDARD: 8.DR.C.3

Standards Statement (2021):

Analyze patterns of association between two quantitative or categorical variables and reason about distributions to compare groups.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.DR.C.3	HS.DR.C.8, HS.DR.C.9, HS.DR.C.10	N/A	8.SP.A.1 8.DR.C Crosswalk

Standards Guidance:

Clarification

- Construct and interpret scatter plots for bivariate data to investigate patterns of association between two quantities.
- Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- Students should be given opportunities to analyze the data distribution displayed graphically to answer the statistical investigative question generated from a real-life situation.

Terminology

- Bivariate data are data for two variables (usually two types of related data), such as height and weight.

Teaching Strategies

- Students should be able to use statistical reasoning to describe patterns of association, such as clustering, outliers, positive or negative association, linear association, and nonlinear association through the analysis of data presented in multiple ways.
- Create a scatter plot for bivariate data and, if appropriate, informally fit a straight line and use the line to predict values. Informally assess the model fit by judging the closeness of the data points to the line.

Progressions

- Working with paired measurement variables that might be associated linearly or in a more subtle fashion, students construct a scatter plot, describing the pattern in terms of clusters, gaps, and unusual data points (much as in the univariate situation). Then, they look for an overall positive or negative trend in the cloud of points, a linear or nonlinear (curved) pattern, and strong or weak association between the two variables, using these terms in describing the nature of the observed association between the variables. (Please reference page 11 in the Progression document).

Examples

- Illustrative Mathematics:
  - Animal Brains
  - Texting and Grades I

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: 8.DR.D - Interpret data and answer investigative questions

STANDARD: 8.DR.D.4

Standards Statement (2021):

Interpret scatter plots for bivariate quantitative data to investigate patterns of association between two quantities to answer investigative questions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.DR.D.4	HS.DR.D.11, HS.DR.D.12, HS.DR.D.13	8.AFN.B.4, HS.AEE.B.4,	8.SP.A.3 8.DR.D Crosswalk

Standards Guidance:

Clarification

- Interpret the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Terminology

- A linear model shows the relationship between two variables in a data set, such as lines of best fit.
- Bivariate data are data for two variables (usually two types of related data), such as height and weight.
- It is important to indicate 'predicted' to indicate this is a probabilistic interpretation in context, and not deterministic.

Teaching Strategies

- Students should interpret contextual linear problems involving situations using bivariate quantitative data.

Progressions

- After a line is fit through the data, the slope of the line is approximated and interpreted as a rate of change, in the context of the problem. The slope has important practical interpretations for most statistical investigations of this type (MP2). (Please reference page 11 in the Progression document).

Examples

- In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
- Illustrative Mathematics:
  - US Airports, Assessment Variation
- Student Achievement Partners:
  - Candle Burning

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4J: HS Algebra Standards and Guidance

Critical Areas of Focus

Expressions

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x2 – 2 = 0 are real numbers, not rational numbers; and the solutions of x2 + 2 = 0 are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, A = ((b1+b2)/2)h, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Connections to Functions and Modeling

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Numbers and Number Systems

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, and real numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (51/3)3 should be 5(1/3)3 = 51 = 5 and that 51/3 should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities

In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Domains and Clusters

HS.AEE - Algebraic Reasoning: Expressions and Equations

- HS.AEE.A Use algebraic reasoning to rewrite expressions in equivalent forms.
- HS.AEE.B Use algebraic reasoning to find solutions to an equation, inequality, and systems of equations or inequalities.
- HS.AEE.C Analyze the structure of an equation or inequality to determine an efficient strategy to find and justify a solution.
- HS.AEE.D Make predictions in different applications using expressions, equations, and inequalities to analyze authentic contexts.

HS.AFN - Algebraic Reasoning: Functions

- HS.AFN.A Describe functions by using both symbolic and graphical representations.
- HS.AFN.B Compare and relate functions using common attributes.
- HS.AFN.C Represent functions graphically and interpret key features in terms of the equivalent symbolic representation.
- HS.AFN.D Model a wide variety of authentic situations using functions through the process of making and changing assumptions, assigning variables, and finding solutions to contextual problems.

HS.NQ - Numeric Reasoning: Number and Quantity

- HS.NQ.A Understand and apply the real number system.
- HS.NQ.B Attend to units of measurement needed to solve problems through quantitative reasoning and mathematical modeling.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.AEE.A - Use algebraic reasoning to rewrite expressions in equivalent forms.

STANDARD: HS.AEE.A.1

Standards Statement (2021):

Interpret an expression which models a quantity by viewing one or more of its parts as a single entity. Reason about how changes in parts of the expression impact the whole, and vice versa.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.A.2, 7.AEE.A.1, 7.AEE.A.2, 7.AEE.B.4	HS.AEE.A.3, HS.AEE.B.4, HS.AEE.B.5, HS.AEE.C.7, HS.AEE.D.11	N/A	HSA.SSE.A.1 HS.AEE.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to interpret parts of an expression, such as terms, factors, leading coefficient, coefficients, constant and degree in context.
- Given contextual situations that utilize formulas or expressions with multiple terms and/or factors, students should be able to interpret the meaning in context of individual terms or factors.

Terminology

- Parts include terms, factors, coefficients, exponents, numerators and denominators.

Examples

- Illustrative Mathematics:
  - Animal Populations
  - Delivery Trucks
  - Exponential Parameters
  - Kitchen Floor Tiles
  - Mixing Candies
  - Mixing Fertilizer
  - Quadrupling Leads to Halving
  - Radius of a Cylinder
  - Seeing Dots
  - The Bank Account
  - The Physics Professor
  - Throwing Horseshoes

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AEE.A.2

Standards Statement (2021):

Create and recognize an equivalent form of an expression to understand the quantity represented in an authentic context.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.AEE.A.1, 7.AEE.A.2, 8.AEE.C.7, HS.AEE.A.3	HS.AEE.B.5	7.RP.A.3, HS.GM.D.13	HSA.SSE.B.3 HS.AEE.A Crosswalk

Standards Guidance:

Clarifications

- Equivalent forms are found through application of algebraic properties including properties of exponents, combining like terms, and distributive property.

Boundaries

- Algebraic manipulation for its own sake should be avoided.

Teaching Strategies

- Students should be able to use interactive graphing technologies to make sense of equivalent expressions in context.
- Students should be able to move fluently (flexibly, accurately, efficiently) between equivalent forms of an expression.

Progressions

- MP2, 7 & 8: quantitative & abstract reasoning, using structure & generalizing --Equivalent forms are found through application of algebraic properties including properties of exponents, combining like terms, and distributive property.
- Exponential equations are limited to those containing like bases, or exponential equations that could easily be transferred to like bases with linear operations.

Examples

- Illustrative Mathematics:
  - Taxes and Sales
  - Ice Cream
  - Increasing or Decreasing? Variation 2
  - Profit of a company
  - Forms of exponential expressions

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AEE.A.3

Standards Statement (2021):

Rearrange formulas and equations to highlight a specific quantity.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AEE.C.7, HS.AEE.A.1, HS.AEE.D.11	HS.AEE.A.2	N/A	HSA.CED.A.4, HSF.IF.C.8 HS.AEE.A Crosswalk

Standards Guidance:

Clarifications

- Represent constraints using systems of equations and/or inequalities and interpret solutions as viable or non- viable options in a modeling context

Boundaries

- Full proficiency in rearranging linear equations and developing proficiency with exponential (solved via roots, not logs) is expected.
- Opportunities with simple quadratic and rational situations when called for by context are also included.
- MP - MP2: quantitative & abstract reasoning

Examples

- Rearrange Ohm's law V = IR to highlight resistance R.
- Illustrative Mathematics:
  - Equations and Formulas
  - Rewriting equations
  - Springboard Dive
  - Which Function?
  - Carbon 14 dating in practice I

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.AEE.B - Use algebraic reasoning to find solutions to an equation, inequality, and systems of equations or inequalities.

STANDARD: HS.AEE.B.4

Standards Statement (2021):

Define variables and create equations with two or more variables to represent relationships between quantities in order to solve problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AFN.B.4, 8.AEE.C.8, HS.AEE.A.1, HS.AEE.D.9	HS.AEE.C.8	8.DR.D.4	HSA.CED.A.2 HS.AEE.B Crosswalk

Standards Guidance:

Clarifications

- Students should be able to define variables to represent the quantities and write equations to show the relationship
- Students should have the opportunity to use graphs to show a visual representation of the relationship while adhering to appropriate labels and scales.
- Full proficiency in creating equations arising from linear situations and developing proficiency for exponential situations is expected

Boundaries

- Focus of the standard is creating equations to solve problems in authentic contexts.
- Representations include expressions, equations, and graphing equations on coordinate axes with labels and scales.
- Opportunities to explore simple quadratic and rational equations when called for by context are also included.

Teaching Strategies

- When necessary, students should be able to rewrite equiations in various forms, such as slope-intercept form, for graphing.
- Students should be given opportunities to solve contextual linear equations graphically and algebraically.

Examples

- Four people may be seated at one rectangular table. If two rectangular tables are placed together end-to-end, 6 people may be seated at the table. If 10 tables are placed together end-to-end, how many people can be seated? How many tables are needed for n people?
- The cost of parking in the parking garage is $2.00 for the first hour and $1.00 for every hour after that. Write an equation in terms of x and y that shows the total cost for parking, y, for x hours. Use the equation to calculate the cost for parking in the garage for 5 hours.

Examples

- Illustrative Mathematics: [Clea on an Escalator] [Global Positioning System I] [Silver Rectangle] [Throwing a Ball] [Uranium 238]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AEE.B.5

Standards Statement (2021):

Define variables and create inequalities with one or more variables and use them to solve problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.AEE.B.4, 8.AEE.A.2, HS.AEE.A.1, HS.AEE.A.2, HS.AEE.D.11	HS.AEE.C.8	N/A	HSA.CED.A.1 HS.AEE.B Crosswalk

Standards Guidance:

Clarification

- Students should be given the opportunity to explore the difference between solid lines and dashed lines through exploration on an interactive graph.
- Students should have had opportunities to create and solve linear equations and inequalities throughout middle school mathematics.
- Students should recognize that the graph of a linear inequality in two variables is a half- plane.
- Full proficiency in creating inequalities arising from linear situations and developing proficiency for exponential situations is expected.

Boundaries

- Focus of the standard is creating inequalities to solve problems in authentic contexts.
- Representations include expressions, inequalities, and graphing equations on coordinate axes with labels and scales.

Examples

- Illustrative Mathematics:
  - Basketball
  - Buying a Car
  - Paper Folding
  - Introduction to Polynomials - College Fund
  - Paying the rent
  - Planes and wheat
  - Sum of angles in a polygon

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AEE.B.6

Standards Statement (2021):

Solve systems of linear equations and systems of linear inequalities in authentic contexts through reasoning, algebraic means, or strategically using technology.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AEE.C.8, HS.AEE.B.4, HS.AEE.C.7	HS.AEE.D.11	N/A	HSA.REI.C.6 HS.AEE.B Crosswalk

Standards Guidance:

Clarifications

- Ensure constraints are represented.
- Students in Grade 8 mathematics modeled with and solved systems of linear equations to solve real-life problems graphcially and with tables.
    - High school expectations would also include algebraic methods as well as strategic use of technololgy to solve linear equations, inequalities, and systems of equations and inequalities.

Terminology

- Simple system of linear equations would include solving on pairs of linear equations in two variables represented by the intersection of two lines graphically.

Boundaries

- Simple systems of equations refer to those that could easily be solved algebraically by hand, including whole number coefficients and/or rational number solutions. Technology should be used to find solutions for more complex systems of equations or inequalties.
- Full proficiency with pairs of linear equations in two variables is expected. Opportunities with non-linear systems when called for by context are also included.
- MP5: Using graphing technology

Teaching Strategies

- Students should be provided opportunities to use technology tools to solve systems of linear inequalities graphically.

Examples

- A school club is selling hats and t-shirts for a fundraiser. The group expects to sell a total of 50 items. They make a profit of 15 dollars for each t-shirt sold and 5 dollars for each hat sold. How many hats and t-shirts will the school club need to sell to make a profit of $300?
- Illustrative Mathematics:
    - [Accurately weighing pennies I] [Accurately weighing pennies II] [Cash Box] [Find A System] [Estimating a Solution via Graphs] [Pairs of Whole Numbers] [Quinoa Pasta 2] [Quinoa Pasta 3]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.AEE.C - Analyze the structure of an equation or inequality to determine an efficient strategy to find and justify a solution.

STANDARD: HS.AEE.C.7

Standards Statement (2021):

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities; interpret solutions as viable or nonviable options in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AEE.C.8, HS.AEE.A.1, HS.AEE.B.4, HS.AEE.D.11	HS.AEE.B.6	N/A	HSA.CED.A.3 HS.AEE.C Crosswalk

Standards Guidance:

Terminology

- Possible data points are solutions to the inequality or inequalities; data points that are not possible are non-solutions to the inequality or inequalities.

Boundaries

- Full proficiency in creating and interpreting equations or inequalities arising from linear situations is expected.
- Opportunities to explore exponentials, simple quadratic and rational situations when called for by context are also included.
- MP4: Mathematical Modeling

Examples

- Represent inequalities describing nutritional and cost constraints on combinations of different foods.
- Illustrative Mathematics:
  - Bernardo and Sylvia Play a Game
  - Dimes and Quarters
  - Growing coffee
  - How Much Folate?
  - Writing constraints

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AEE.C.8

Standards Statement (2021):

Construct a viable argument to justify a method for solving equations or inequalities.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.B.4, 8.AEE.A.1, 8.AEE.C.7, HS.AEE.B.4, HS.AEE.B.5	N/A	N/A	HSA.REI.A.1 HS.AEE.C Crosswalk

Standards Guidance:

Clarifications

- Students should be able to interpret parts of an expression, such as terms, factors, leading coefficient, coefficients, constant and degree in context.
- Given contextual situations which utilize formulas or expressions with multiple terms and/or factors, students should be able to interpret the meaning in context of individual terms or factors.

Boundaries

- Limit to real number solutions
- Viable arguments must give reasoning for important steps. Supporting this work may also involve justifying algebraic rules through models.
- Full proficiency in solving linear equations, quadratics which are solvable without factoring, completing the square or quadratic formula, exponentials solvable without logarithms and simple rational equations that can be solved without factoring.
- MP3: Construct arguments & critique the reasoning of others

Teaching Strategies

- Students should be able to move fluently (flexibly, accurately, efficiently) between different equivalent forms of an expression. Students should be able to analyze and explain what the zeros describe in context.

Examples

- Illustrative Mathematics:
  - Products and Reciprocals
  - Zero Product Property 1
  - Zero Product Property 2
  - Zero Product Property 3
  - Zero Product Property 4

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.AEE.D - Make predictions in different applications using expressions, equations, and inequalities to analyze authentic contexts.

STANDARD: HS.AEE.D.9

Standards Statement (2021):

Understand that the solution to an equation in two variables is a set of points in the coordinate plane that form a curve, which could be a line.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.AEE.B.4, 8.AEE.B.5, 8.AEE.B.6, 8.AEE.C.8	HS.AEE.B.4, HS.AEE.D.10, HS.AEE.D.11	HS.GM.D.13	HSA.REI.D.10 HS.AEE.D Crosswalk

Standards Guidance:

Boundaries

- Common graphs include lines, parabolas, circles, and exponential curves.
- Students can show that any point on the graph of an equation in two variables is a solution to the equation.

Examples

- Illustrative Mathematics:
  - Collinear points
  - Taxi!
  - A Linear and Quadratic System

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AEE.D.10

Standards Statement (2021):

Recognize and explain why the point(s) of intersection of the graphs of f(x) and g(x) are solutions to the equation f(x)=g(x). Interpret the meaning of the coordinates of these points in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
HS.AEE.D.9	N/A	N/A	HSA.REI.D.11 HS.AEE.D Crosswalk

Standards Guidance:

Teaching Strategies

- Use technology to graph the functions, make tables of values, or find successive approximations.
- Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- *MP4: mathematical modeling
- MP5: using graphing technology

Examples

- Illustrative Mathematics:
  - Ideal Gas Law
  - Population and Food Supply
  - Two Squares are Equal

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AEE.D.11

Standards Statement (2021):

Graph and explain why the points in a half plane are solutions to a linear inequality and the solutions to a system of inequalities are the points in the intersection of corresponding half planes. Interpret the meaning of the coordinates of these points in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
HS.AEE.A.1, HS.AEE.B.6, HS.AEE.D.9	HS.AEE.A.3, HS.AEE.B.5, HS.AEE.C.7	N/A	HSA.REI.D.12, HSA.REI.B.3 HS.AEE.D Crosswalk

Standards Guidance:

Boundaries

- Graphs can be created by hand in simple cases but in general with technology to allow the emphasis on the interpretations of solutions.
- MP4: mathematical modeling
- MP5: using graphing technology

Examples

- Illustrative Mathematics:
  - Fishing Adventures 3
  - Solution Sets
  - Reasoning with linear inequalities
  - Integer Solutions to Inequality

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.AFN.A - Describe functions by using both symbolic and graphical representations.

STANDARD: HS.AFN.A.1

Standards Statement (2021):

Understand a function as a rule that assigns a unique output for every input and that functions model situations where one quantity determines another.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AFN.A.1	HS.AFN.A.2, HS.AFN.B.4, HS.AFN.B.5, HS.AFN.C.6, HS.AFN.C.7	N/A	HSF.IF.A.1 HS.AFN.A Crosswalk

Standards Guidance:

Clarification

- Functions are often represented by tables, expressions or graphs. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
- Modeling examples should include both contexts where only one quantity can be considered the independent variable as well as contexts where both quantities could.

Boundaries

- Standard included expectation students connect the concept of a function to use of notation where if f is a function and x is an element of its domain, then (x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y=(x).
- Concept of a function introduced in grade 8, but formal use of function notation is not an expectation until high school.

Examples

- Illustrative Mathematics:
  - Interpreting the graph
  - Domains
  - Do Two Points Always Determine a Linear Function?
  - Do Two Points Always Determine a Linear Function II?
  - Finding the domain
  - Parabolas and Inverse Functions
  - Points on a graph
  - The Parking Lot
  - Your Father

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AFN.A.2

Standards Statement (2021):

Use function notation and interpret statements that use function notation in terms of the context and the relationship it describes.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
HS.AFN.A.1	HS.AFN.D.9	HS.GM.A.1	HSF.IF.A.2 HSF.BF.A.1 HS.AFN.A Crosswalk

Standards Guidance:

Clarifications

- Student should develop a deep understanding of function notation to build, evaluate, and interpret linear functions; this understanding will be applied to other functions studied hereafter.
- Students should be able to interpret the domain when given a function expressed numerically, algebraically, and graphically.
- Students should apply their understanding of function notation from their work with linear functions to build, evaluate, and interpret quadratic functions using function notation.
- Students should apply their understanding of function notation from their work with non-linear functions when needed to build, evaluate, and interpret functions in authentic contexts.

Progressions

- MP4: mathematical modeling

Examples

- Illustrative Mathematics:
  - Cell phones
  - The Random Walk [Version 1] [Version 2] [Version 3]
  - Using Function Notation [Verson 1] [Version 2]
  - Yam in the Oven
  - 1,000 is half of 2,000
  - A Sum of Functions
  - Kimi and Jordan
  - Lake Algae
  - Skeleton Tower
  - Summer Intern

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AFN.A.3

Standards Statement (2021):

Calculate and interpret the average rate of change of a function over a specified interval.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AFN.B.4	HS.AFN.C.6	HS.DR.D.11	HSF.IF.B.6 HS.AFN.A Crosswalk

Standards Guidance:

Clarifications

- Students should be given opportunities to estimate the rate of change from a graph.
- Students should be able to show that linear functions grow by equal differences over equal intervals and recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
    - Students should be able to compare this behavior to that of the average rate of change of quadratic functions. This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals.

Boundaries

- Work with functions presented as graphs, tables or symbolically.
- Students should choose specified intervals for analysis of functions with substantially varying rates of change.
- Interpreting also includes estimates of the rate of change from a graph.
- MP6: precision
- MP7: structural thinking

Teaching Strategies

- Functions can be presented symbolically, as a graph, or as a table.

Examples

- Illustrative Mathematics:
    - Laptop Battery Charge 2
    - Mathemafish Population
    - Temperature Change
    - The High School Gym

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.AFN.B - Compare and relate functions using common attributes.

STANDARD: HS.AFN.B.4

Standards Statement (2021):

Compare properties of two functions using multiple representations. Distinguish functions as members of the same family using common attributes.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AFN.A.2, HS.AFN.A.1	HS.AFN.D.10	N/A	HSF.IF.C.9 HS.AFN.B Crosswalk

Standards Guidance:

Clarifications

- Students should be able to compare key characteristics of exponential functions with the key characteristics of linear and quadratic function.
- Students should be able to observe using graphs and tables that a quantity is increasing .

Boundaries

- Functions can be represented algebraically, graphically, numerically in tables, or by verbal descriptions.

Examples

- Given a graph of one function and an algebraic expression for another, determine which has the larger y-intercept.
- Given a graph of one quadratic function and an algebraic equation for another, students should be able to determine which has the larger maximum.
- Given a graph of one function and an algebraic equation for another, students should be able to determine which has the larger y-intercept.
- Illustrative Mathematics:
  - Throwing Baseballs
  - Analyzing Graphs

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AFN.B.5

Standards Statement (2021):

Relate the domain of a function to its graph and to its context.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
HS.AFN.A.1	N/A	N/A	HSF.IF.B.5 HS.AFN.B Crosswalk

Standards Guidance:

Boundaries

- Contexts can demand discrete vs. continuous and domain restrictions.
- MP4: mathematical model
- MP6: precision

Terminology

- Use symbolic notation to represent the domain and range of a linear function, considering the specific context.
  - $(-\infty,\infty)$
  - $[3, \infty)$
  - D: $\{x| x \epsilon R\}$
  - D: $\{x| x > 0\}$
  - D: $\{x| x = 1,2,3,4,5,\ldots\}$
  - R: $\{y| y = 10,20,30,\ldots\}$

Examples

- If the function h(n) gives the number of hours it takes a person to assemble n engines in a factory, then the set of positive integers would be an appropriate domain for the function.
- If the function h(t) gives the path of a projectile over time, t, then the set of non- negative real numbers would be an appropriate domain for the function because time does not include negative values.
- A bird is building a nest in a tree 36 feet above the ground. The bird drops a stick from the nest. The function $f(x) = -16x2 + 36$ describes the height of the stick in feet after x seconds. Graph this function. Identify the domain and range of this function. (A student should be able to determine that the appropriate values for the domain and range of this graph are $0 \leq x \leq 1.5$ and $0 \leq y \leq 36$, respectively.)
- Illustrative Mathematics:
  - Average Cost
  - Oakland Coliseum
  - The Canoe Trip, Variation 1
  - The Canoe Trip, Variation 2
  - The restaurant

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.AFN.C - Represent functions graphically and interpret key features in terms of the equivalent symbolic representation.

STANDARD: HS.AFN.C.6

Standards Statement (2021):

Interpret key features of functions, from multiple representations, and conversely predict features of functions from knowledge of context.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
HS.AFN.A.1, HS.AFN.A.3	HS.AFN.D.10	N/A	HSF.IF.B.4 HS.AFN.C Crosswalk

Standards Guidance:

Clarifications

- Students should be able to express characteristics in interval and set notation with linear functions.
- Students should be able to interpret the key characteristics of the graph in a contextual situation.

Boundaries

- Key features include: domain, range, discrete, continuous, intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums.
- Representations include: graphs, tables, spreadsheet representations, as well as symbolic.

Teaching Strategies

- Students should be able to use graphs created by hand and with technology, verbal descriptions, tables, and function notation when analyzing linear functions in context.
- Students should be given opportunities to use interactive graphing technologies to explore and analyze key characteristics of linear functions, including domain, range, intercepts, intervals where the function is increasing or decreasing, positive or negative, maximums and minimums over a specified interval, and end behavior.

Examples

- Illustrative Mathematics:
  - Pizza Place Promotion
  - As the Wheel Turns
  - Containers
  - Influenza epidemic
  - Lake Sonoma
  - Model air plane acrobatics
  - Modeling London's Population
  - Telling a Story With Graphs
  - The Aquarium
  - The story of a flight
  - Warming and Cooling
  - Words - Tables - Graphs

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AFN.C.7

Standards Statement (2021):

Graph functions using technology to show key features.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
HS.AFN.A.1	HS.AFN.D.9	N/A	HSF.IF.C.7 HS.AFN.C Crosswalk

Standards Guidance:

Clarifications

- Students should be able to sketch a graph showing key features including domain, range, and intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; asymptotes; end behavior.
- Key characteristics of the quadratic functions should be expressed in interval and set- builder notation using inequalities.

Boundaries

- Key features include: specific values when context demands; domain and range; discrete or continuous; intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima.
- Use technology to graph functions expressed symbolically or in tables, with intentional choices of window and scale. In some simple cases, graphing functions could by hand or for approximations.
    - Graph linear and quadratic functions and show intercepts, maxima, and minima.★
    - Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.★
    - Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.★
    - Graph exponential and logarithmic functions, showing intercepts and end behavior.★

Teaching Strategies

- Students should be able to use verbal descriptions, tables, and graphs created using interactive technology tools.

Examples

- If the function, h(n), gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
- The function can be presented symbolically, as a graph, or as a table.
- Students should be able to estimate the rate of change from a graph.
- Illustrative Mathematics:
    - Identifying graphs of functions
    - Identifying Exponential Functions
    - Bank Account Balance
    - Exponential Kiss

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.AFN.D - Model a wide variety of authentic situations using functions through the process of making and changing assumptions, assigning variables, and finding solutions to contextual problems.

STANDARD: HS.AFN.D.8

Standards Statement (2021):

Model situations involving arithmetic patterns. Use a variety of representations such as pictures, graphs, or an explicit formula to describe the pattern.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AFN.B.4	HS.AFN.D.10	N/A	HSF.BF.A.2 HS.AFN.D Crosswalk

Standards Guidance:

Clarifications

- Students should be able to:
  - make connections between linear functions and arithmetic sequences presented in contextual situations.
  - build and interpret arithmetic sequences as functions presented graphically and algebraically.
- Sequences can be defined explicitly.
- The focus of this learning objective is on building and interpreting arithmetic sequences.

Examples

- By graphing or calculating terms, students should be able to show how the arithmetic sequence in explicit form $a_1=7$, $a_n = 2_{n-1} + 7$; and the function $f(x) = 2x + 5$ (when x is a natural number) define the same sequence.
- MP2: quantitative and abstract reasoning
- MP4: mathematical modeling
- Illustrative Mathematics:
  - Snake on a Plane

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AFN.D.9

Standards Statement (2021):

Identify and interpret the effect on the graph of a function when the equation has been transformed.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
HS.AFN.A.2, HS.AFN.C.7	N/A	HS.GM.A.1, HS.GM.A.2, HS.GM.A.4	HSF.BF.B.3 HS.AFN.D Crosswalk

Standards Guidance:

Teaching Strategies

- Students should be given opportunities to experiment with cases and illustrate an explanation of the effects on the graph using technology.

Boundaries

- Transformations include translations (f(x)+k, and f(x-h)), reflections (e.g. -f(x) and f(-x), and dilations (e.g. a*f(x)). Interpretations include accounting for different choices of variables, such as initial values or units.
- Full proficiency with linear functions and developing proficiency with exponential functions is expected. Technology provides opportunities for exploration with non-linear functions.
- MP4: mathematical modeling
- MP5: using graphing technology

Examples

- Illustrative Mathematics:
  - Transforming the graph of a function
  - Medieval Archer
  - Building a quadratic function from $f(x)=x^2$
  - Identifying Even and Odd Functions
  - Identifying Quadratic Functions (Standard Form)
  - Identifying Quadratic Functions (Vertex Form)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.AFN.D.10

Standards Statement (2021):

Explain why a situation can be modeled with a linear function, an exponential function, or neither. In a given model, explain the meaning of coefficients and features of functions used, such as slope for a linear model.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.AFN.A.3, 8.AFN.B.4, 8.AFN.B.5, HS.AFN.B.4, HS.AFN.C.6, HS.AFN.D.8	N/A	N/A	HSF.LE.A.1 HS.AFN.D Crosswalk

Standards Guidance:

Clarifications

- Students should be provided with opportunities to learn mathematics in the context of real-life problems.
- Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).

Terminology

- Linear functions grow by equal differences over equal intervals.
- Exponential functions grow by equal factors over equal intervals.

Boundaries

- Identify situations in which one quantity changes at a constant rate per unit interval relative to another.
- Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Teaching Strategies

- Students should be able to fluently navigate between mathematical representations that are presented numerically, algebraically, and graphically.
- For graphical representations, students should be given opportunities to analyze graphs using interactive graphing technologies.

Progressions

- Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
- MP4: Mathematical Modeling

Examples

- Illustrative Mathematics:
  - Basketball Bounces [Assessment Variation 1] [Assessment Variation 2]
  - [Boiling Water] [Choosing an appropriate growth model] [Exponential Functions] [Interesting Interest Rates] [Linear or exponential?]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.NQ.A - Understand and apply the real number system.

STANDARD: HS.NQ.A.1

Standards Statement (2021):

Use reasoning to establish properties of positive integer exponents. Extend the definition of exponentiation to include negative and rational exponents so as to be consistent with these properties. Utilize exponentiation to model authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.NS.A.1, HS.NQ.A.2	N/A	8.AEE.A.2	HSN.RN.A.1 HS.NQ.A Crosswalk

Standards Guidance:

Connections

- Students should be able to use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots and cube roots.
- MP8: generalizing patterns

Catalyzing Change Connection

- Determine precise calculations using rational and irrational numbers to make comparisons and solve problems.
- Use estimation and approximation of calculations to make comparisons and solve problems.

Examples

- Illustrative Mathematics:
  - Evaluating a Special Exponential Expression
  - Evaluating Exponential Expressions
  - Extending the Definitions of Exponents, Variation 2

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.NQ.A.2

Standards Statement (2021):

Compare real numbers presented through different representations, including both rational and irrational numbers. Apply comparisons in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.NS.A.1	HS.NQ.A.1	N/A	HSN.Q.A.1, HSN.Q.A.2 HS.NQ.A Crosswalk

Standards Guidance:

Examples

- Illustrative Mathematics:
  - Felicia's Drive
  - Fuel Efficiency
  - How Much is a Penny Worth?
  - Ice Cream Van
  - Runners' World
  - Selling Fuel Oil at a Loss
  - Solar Radiation Model

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.NQ.B - Attend to units of measurement needed to solve problems through quantitative reasoning and mathematical modeling.

STANDARD: HS.NQ.B.3

Standards Statement (2021):

Use reasoning to choose and interpret measurement units consistently in formulas, graphs, and data displays, as a way to understand problems and to guide the solution of multi-step problems.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
5.GM.C.4	N/A	8.AEE.A.4, 6.RP.A.3, 7.RP.A.1, HS.GM.C.10	HSN.Q.A.1 HS.NQ.B Crosswalk

Standards Guidance:

Clarification

- Identify, use, and record appropriate units of measure within context, within data displays, and on graphs.
- Convert units and rates using proportional reasoning given a conversion factor;
- Use units within multi-step problems and formulas and interpret units of input and resulting units of output

Boundaries

- This standard applies universally in modeling situations.
- Use units of measure (linear, area, capacity, rates, and time) as a way to make sense of conceptual problems
- This includes authentic applications that require changing units to understand a given context.

Teaching Strategies

- Dimensional analysis may be used when converting units and rates.

Examples

- Units of measure may include linear, area, capacity, rates, and time.
- MP2: quantitative and abstract reasoning
- MP4: mathematical modeling
- Illustrative Mathematics:
  - Giving raises
  - New Cuyama
  - Traffic Jam

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.NQ.B.4

Standards Statement (2021):

Define, manipulate, and interpret appropriate quantities using rational and irrational numbers to authentically model situations and use reasoning to justify these choices.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	N/A	HS.GM.C.10, HS.GM.C.11	HSN.Q.A.2 HS.NQ.B Crosswalk

Standards Guidance:

Clarifications

- Given a situation, context, or problem, students should be able to determine, identify, and use appropriate quantities for representing the situation.
- In some cases, students should be translating between different real number representations (for example, if something is to be cut to square root of 31 inches, what is that on a standard ruler, in sixteenths?). Students should draw on fluency with real arithmetic and estimation.

Boundaries

- This standard applies universally in modeling situations.
- Quantities are real number quantities

Progressions

- Use length, area, and volume measurements to solve applied problems.
- Use properties of congruence and similarity to solve applied problems.
- Use graphs and coordinates to solve applied problems.

Examples

- Illustrative Mathematics:
  - Harvesting the Fields
  - Weed killer

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.NQ.B.5

Standards Statement (2021):

Use reasoning to choose a level of accuracy appropriate to limitations on measurement when reporting quantities in modeling situations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
N/A	N/A	HS.GM.C.10, HS.GM.C.11	HSN.Q.A.3 HS.NQ.B Crosswalk

Standards Guidance:

Boundaries

- Note: This standard applies universally in modeling situations.
- MP4: mathematical modeling

Teaching Strategies

- Instruction should include appropriate use of different measurement systems (e.g. feet/inches/yards and meters/centimeters/millimeters).
- Instructional support to convert between measurement units both within a given system (e.g. customary to customary or metric to metric), or across measurement systems (e.g. customary and metric conversions)

Examples

- Illustrative Mathematics:
  - Accuracy of Carbon 14 Dating I
  - Accuracy of Carbon 14 Dating II
  - Bus and Car
  - Calories in a sports drink
  - Dinosaur Bones

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4K: HS Geometry Standards and Guidance

Critical Areas of Focus

Geometric Reasoning

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

Congruence, similarity, and symmetry

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations.

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Domains and Clusters

Geometric Reasoning and Measurement (HS.GM)

- HS.GM.A Apply geometric transformations to figures through analysis of graphs and understanding of functions.
- HS.GM.B Construct and communicate geometric arguments through use of proofs, logical reasoning, and geometric technology.
- HS.GM.C Solve problems and interpret solutions of area and volume of shapes by applying concepts of congruence, similarity, symmetry in authentic contexts.
- HS.GM.D Apply concepts of right triangle trigonometry in authentic contexts to solve problems and interpret solutions.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.GM.A - Apply geometric transformations to figures through analysis of graphs and understanding of functions.

STANDARD: HS.GM.A.1

Standards Statement (2021):

Apply definitions of rotations, reflections, and translations to transform a figure and map between two figures in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.GM.A.2, 8.GM.A.3, 8.GM.A.5, HS.GM.A.2, HS.GM.A.4	N/A	8.AFN.A.1, HS.AFN.A.2, HS.AFN.D.9	HSG.CO.A.2 HSG.CO.A.4 HSG.CO.A.5 HS.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to determine congruency by identifying the rigid transformation(s) that produced the image of a figure.
- Opportunities should be provided for students to write statements of congruency.
- Given two polygons, students should be able to use the definition of congruence in terms of rigid motions to verify congruence if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- Students should be able to use function notation to represent transformations in the coordinate plane.

Terminology

- A rigid transformation that preserves size and shape (e.g. translation, rotation, or reflection).

Boundaries

- Draw the transformation (rotation, reflection, or translation) for a given geometric figure.
- Students should be able to apply definitions of reflections across any line in context or on a coordinate grid.
- Students should be able to apply definitions of rotations around any point of any degree in context or on a coordinate grid.

Teaching Strategies

- Students should have ample opportunities to use geometric tools and/or technology to explore figures created from translations, reflections, and rotations.
- Students should be able to determine images created by a given translations, reflections, or rotations.

Examples

- Illustrative Mathematics: [Dilations and Distances] [Fixed points of rigid motions] [Horizontal Stretch of the Plane] [Defining Reflections] [Reflected Triangles] [Defining Rotations] [Identifying Rotations] [Identifying Translations] [Trigonometric Identities and Rigid Motions]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.GM.A.2

Standards Statement (2021):

Verify experimentally the properties of a dilation given a center and a scale factor. Solve problems in authentic contexts involving similar triangles or dilations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.GM.A.3, 8.GM.A.4, 8.GM.A.5	HS.GM.A.1, HS.GM.A.3, HS.GM.D.12	HS.AFN.D.9	HSG.SRT.B.5, HSG.SRT.A.1, HSG.SRT.A.2, HSG.SRT.A.3 HS.GM.A Crosswalk

Standards Guidance:

Clarifications

- Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- Students should be able to identify dilation as reduction or enlargement depending on scale factor.
- Students should be given opportunities to draw a dilated image given any center and scale factor in context or on a coordinate grid.
- Students should be able to describe properties of dilations, such as center, scale factor, angle measure, parallelism, and collinearity.

Terminology

- A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Teaching Strategies

- Triangles can be shown to be similar using transformations and triangle similarity theorems. Apply theorems of AA similarity, SSS similarity, and SAS similarity to prove similarity of two given triangles.
- Dilations should be limited to those centered at the origin.

Progressions

- Model with mathematics to use similarity to solve authentic problems to measure lengths and distances indirectly.
- Use the properties of similarity transformations could be used to establish the Angle-Angle (AA) criterion for two triangles to be similar.

Examples

- A high school student visits a giant cedar tree near the town of Elk River, Idaho and the end of his shadow lines up with the end of the tree's shadow. The student is 6 feet tall and his shadow is 8 feet long. The cedar tree's shadow is 228 feet long. How tall is the cedar tree?

Examples

- Illustrative Mathematics:
  - [Dilating a Line] [Are They Similar?] [Congruent and Similar Triangles] [Bank Shot] [Congruence of parallelograms] [Finding triangle coordinates] [Points from Directions]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.GM.A.3

Standards Statement (2021):

Use the slopes of segments and the coordinates of the vertices of triangles, parallelograms, and trapezoids to solve problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
HS.GM.A.2	N/A	8.AEE.B.5, 8.AEE.B.6, 8.AEE.C.8	HSG.GPE.B.5 HS.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students should have opportunities to analyze and apply theorems about lines and angles from the context of parallel lines cut by a transversal to make sense of relationships between lines and angles in quadrilaterals and triangles.
- Students should be familiar with triangle congruence theorems (SSS, SAS, ASA, AAS, or HL) to solve problems and to prove relationships in geometric figures by applying geometric and algebraic reasoning.

Progressions

- Possible applications include using slopes to determine parallel sides in parallelograms and trapezoids, perpendicular diagonals in rhombuses, perpendicular sides in a rectangle
- Use slope and coordinates to verify mid-segment properties in triangles and trapezoids.
- Use coordinates of vertices for lengths of sides and diagonals to classify quadrilaterals and triangles.

Examples

- Illustrative Mathematics:
  - Equal Area Triangles on the Same Base I
  - Equal Area Triangles on the Same Base II
  - Parallel Lines in the Coordinate Plane
  - Slope Criterion for Perpendicular Lines
  - Inscribing a Triangle in a Circle
  - When are two lines perpendicular?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.GM.A.4

Standards Statement (2021):

Use definitions of transformations and symmetry relationships to justify the solutions of problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
4.GM.A.1, 4.GM.A.2, 4.GM.A.3, 4.GM.C.7	HS.GM.A.1	HS.AFN.D.9	HSG.CO.A.1 HS.GM.A Crosswalk

Standards Guidance:

Clarifications

- Students should be able to define and identify figures as preimages and images.
- Students use definitions to identify lines of symmetry and angles of rotation to map a figure onto itself.
- Students use definitions to identify angles of rotation, lines of reflection, and directions of translations to map a preimage onto its image.
- Students use definitions to experiment with transformations represented on and off the coordinate plane.

Terminology

- Definitions of geometric figures and geometric relationships could include definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Boundaries

- Definitions should include angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Examples

- Illustrative Mathematics:
  - Defining Parallel Lines
  - Defining Perpendicular Lines

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.GM.B - Construct and communicate geometric arguments through use of proofs, logical reasoning, and geometric technology.

STANDARD: HS.GM.B.5

Standards Statement (2021):

Apply and justify triangle congruence and similarity theorems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.GM.A.2	HS.GM.B.6	N/A	HSG.CO.B.7 HSG.CO.B.8 HS.GM.B Crosswalk

Standards Guidance:

Clarifications

- Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure;
- Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- Students should be able to apply properties of congruence to solve problems with missing values involving corresponding parts. Opportunities should also be available for students to understand when conditions do not result in congruence.

Boundaries

- The focus here is to develop an understanding of techniques for proving that two triangles are congruent.
- Advanced courses could include explanations for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions (HSG.CO.B.8).

Terminology

- Logic statements include conditional, converse, inverse, and contrapositive statements.

Teaching Strategies

- Use of triangle congruence theorems (SSS, SAS, ASA, AAS, or HL) should be used to solve problems in authentic contexts.
- Students' ways of communicating triangle congruence could possibly include formal methods such as: logic statements, two-column proofs, paragraph proofs, and flow proofs.

Examples

- Construct viable arguments and critique the reasoning of others when showing that two triangular roof trusses must be congruent.
- Illustrative Mathematics:
  - Properties of Congruent Triangles
  - Why does ASA Work?
  - Why does SAS work?
  - Why does SSS work?
  - When does SSA Work to Determine Triangle Congruence?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.GM.B.6

Standards Statement (2021):

Justify theorems of line relationships, angles, triangles, and parallelograms; and use them to solve problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.GM.B.4, 8.GM.A.5, HS.GM.B.5	HS.GM.D.12	N/A	HSG.CO.C.9 HSG.CO.C.10 HS.GM.B Crosswalk

Standards Guidance:

Clarification

- Students should be given opportunities to explore using visual tools in order to precisely prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.

Boundaries

- Angle and line relationship theorems include:
  - when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and conversely use to justify lines are parallel;
  - points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
  - vertical angles are congruent;
- Triangle Theorems include:
  - Sum of interior angles 180 degrees
  - Properties of special triangles (isosceles, equilateral, and right).
  - Angle sums formed by polygons
- Parallelogram theorems include:
  - Properties of special quadrilaterals (sides, angles, and diagonals), and
  - Properties of special triangles (isosceles, equilateral, and right).

Clarifications

- Students should be provided opportunities to build a conceptual understanding of a point, line, line segment, plane, arc, and angle through modeling and exploration of authentic phenomena.
- Students should use symbolic notation for point, line, plane, line segment, angle, circle, arc, perpendicular line, and parallel line.

Progressions

- Construct viable arguments and critique the reasoning of others when justifying the congruence of diagonals in a rectangle that is built by a contractor installing a rectangular window.

Examples

- Illustrative Mathematics: [Classifying Triangles] [Congruent angles in isosceles triangles] [Midpoints of Triangle Sides] [Sum of angles in a triangle]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.GM.B.7

Standards Statement (2021):

Perform geometric constructions with a variety of tools and methods.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.GM.A.2	N/A	N/A	HSG.CO.D.12 HS.GM.B Crosswalk

Standards Guidance:

Clarifications

- Student should be able to:
  - Copy a segment and angle.
  - Bisect a segment and angle.
  - Construct perpendicular lines, including the perpendicular bisector of a line segment.
  - Construct a line parallel to a given line through a point not on the line.

Teaching Strategies

- Tools to include compass and straightedge, string, reflective devices, paper folding, and/or dynamic geometric software.
- Constructions to include copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Progressions

- Use appropriate tools strategically when choosing the physical method and appropriate procedures for performing a construction

Examples

- Illustrative Mathematics:
  - Locating Warehouse
  - Origami equilateral triangle
  - Origami regular octagon
  - Origami silver rectangle

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.GM.C - Solve problems and interpret solutions of area and volume of shapes by applying concepts of congruence, similarity, symmetry in authentic contexts.

STANDARD: HS.GM.C.8

Standards Statement (2021):

Solve authentic modeling problems using area formulas for triangles, parallelograms, trapezoids, regular polygons, and circles.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.GM.A.1, 7.GM.B.3, 7.GM.B.5, 8.GM.C.9	HS.GM.C.9	N/A	HSG.GMD.A.1 HS.GM.C Crosswalk

Standards Guidance:

Teaching Strategies

- Students should give informal arguments for area formulas, and combine them to solve problems with composite figures.
- Students should be able to choose the appropriate geometric polygon to approximate the area of irregular objects.

Examples

- Model with Mathematics can be used here to solve a variety of problems involving area.
- Illustrative Mathematics:
  - Area of a circle
  - Circumference of a circle
  - Volume formulas for cylinders and prisms
  - Volume of a Special Pyramid

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.GM.C.9

Standards Statement (2021):

Use volume and surface area formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems and apply to authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.GM.B.5, 8.GM.C.9, HS.GM.C.8	N/A	N/A	HSG.GMD.A.3 HS.GM.C Crosswalk

Standards Guidance:

Teaching Strategies

- Students should be able to choose the appropriate geometric figure or solid to approximate density of irregular objects in a geometric situation.
- Students should give informal arguments for area and volume formulas, and combine them to solve problems with composite figures. This standard is limited to right solids.

Examples

- Make sense of problems and persevere in solving them when finding the volume of prisms and pyramids with regular polygon bases (possibly using trigonometry)
- Persons per square mile, fish per cubic feet of a fish tank
- Illustrative Mathematics:
  - Centerpiece
  - Doctor's Appointment
  - The Great Egyptian Pyramids
  - Volume Estimation

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.GM.C.10

Standards Statement (2021):

Use geometric shapes, their measures, and their properties to describe real world objects, and solve related authentic modeling and design problems.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.GM.A.1, 7.GM.B.3, 7.GM.B.5, 8.GM.C.9	N/A	HS.NQ.B.3, HS.NQ.B.4, HS.NQ.B.5	HSG.MG.A.1 HSG.MG.A.3 HS.GM.C Crosswalk

Standards Guidance:

Clarifications

- This includes the use of volume formulas for prisms, cylinders, pyramids, cones, and spheres.
- Students should be able to verify experimentally the formulas for the volume of a cylinder, pyramid, sphere, prism and cone; emphasize volume as the product of the area of the base and the height for both prisms and cylinders.
- Students should find the volume of solids and composite solids to explain real-life phenomena.

Terminology

- Prism – a solid figure that has the same cross section all along its length

Examples

- Model with Mathematics can be used here to solve a variety of problems such as designing a real world object with CAD design tools for 3D printing or CNC machining.
- Illustrative Mathematics:
  - Coins in a circular pattern
  - Eratosthenes and the circumference of the earth
  - How far is the horizon?
  - Paper Clip
  - Regular Tessellations of the plane
  - Running around a track I
  - Running around a track II

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.GM.C.11

Standards Statement (2021):

Apply concepts of density based on area and volume in authentic modeling situations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.GM.C.9	N/A	HS.AFN.A.3, 7.RP.A.1, HS.NQ.B.4, HS.NQ.B.5	HSG.MG.A.2 HS.GM.C Crosswalk

Standards Guidance:

Clarifications

- The focus is on geometric probability and proportional reasoning.
- This should include an understanding of the ratios of areas (area ratio = (scale factor)^2) and volumes (volume ratio = (scale factor)^3) of similar figures.

Examples

- Model with Mathematics to compute persons per square miles, BTUs per cubic foot, or specimens per acre.
- Illustrative Mathematics:
  - Archimedes and the King's crown
  - A Ton of Snow
  - How many cells are in the human body?
  - How many leaves on a tree?
  - How many leaves on a tree? (Version 2)
  - How thick is a soda can? Variation I
  - How thick is a soda can? Variation II
  - Indiana Jones and the Golden Statue

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.GM.D - Apply concepts of right triangle trigonometry in authentic contexts to solve problems and interpret solutions.

STANDARD: HS.GM.D.12

Standards Statement (2021):

Apply sine, cosine, and tangent ratios, and the Pythagorean Theorem, to solve problems in authentic contexts.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.GM.B.6, 8.GM.B.7, HS.GM.A.2, HS.GM.B.6	N/A	N/A	HSG.SRT.C.8 HSG.SRT.C.6 HSG.SRT.C.7 HS.GM.D Crosswalk

Standards Guidance:

Clarifications

- In seventh grade, students write and solve equations using supplementary, complementary, vertical, and adjacent angles.
- Explain and use the relationship between the sine and cosine of complementary angles (e.g. sin(30) = cos(60) = 0.5).

Teaching Strategies

- Demonstrate understanding that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- Students should be able to use sine, cosine, and tangent to solve real-life problems that require them to find missing side and angle measurements.

Progressions

- Applications should involve finding angle and side measures of right triangles.

Examples

- Illustrative Mathematics:
  - Finding the Area of an Equilateral Triangle
  - Mt. Whitney to Death Valley
  - Seven Circles [Verstion 1] [Version 2] [Version 3]
  - Defining Trigonometric Ratios
  - Setting Up Sprinklers
  - Ask the Pilot
  - Constructing Special Angles

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.GM.D.13

Standards Statement (2021):

Apply the Pythagorean Theorem in authentic contexts, and develop the standard form for the equation of a circle.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.GM.B.8	HS.GM.D.14	HS.AEE.A.2, HS.AEE.D.9	HSG.GPE.A.1 HS.GM.D Crosswalk

Standards Guidance:

Terminology

- The standard form of the equation for a circle is $(x-h)^2 + (y-k)^2 = r^2$.

Clarifications

- Students should be able to identify the center and radius of a circle from an equation in standard form or from the graph of a circle.
- Students should be able to write the equation of a circle in standard form given the graph of the circle.
- Students should be able to graph a circle from the standard form equation of a circle.

Teaching Strategies

- Given the coordinates of the center and length of the radius, write the equation of the circle in standard form.
- Given the equation of a circle in standard form, determine the coordinates of its center and the length of its radius.

Progressions

- Use the Pythagorean Theorem to develop and apply the distance formula
- Look for and make use of structure to make connections to the Pythagorean Theorem and distance formula.

Examples

- Illustrative Mathematics:
  - Explaining the equation for a circle
  - Slopes and Circles

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.GM.D.14

Standards Statement (2021):

Use the coordinate plane to determine parallel and perpendicular relationships, and the distance between points.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
8.GM.B.6, 8.GM.B.8, HS.GM.D.13	N/A	N/A	HSG.GPE.B.4 HS.GM.D Crosswalk

Standards Guidance:

Clarifications

- Students should be able to classify quadrilaterals as parallelograms (including rectangles, rhombi, and squares) using sides and diagonals.
- Students should be familiar with the distance formula when calculating the area and perimeter of quadrilaterals and triangles.

Terminology

- Cartesian coordinates refer to (x,y) system on a rectangular grid with the x-coordinate representing horizontal distance from the origin, and the y-coordinate representing vertical distance from the origin.

Boundaries

- Course level expectation is limited to use of a rectangular (Cartesian) coordinate system.

Teaching Strategies

- Applications include the use of coordinates to compute perimeters of polygons and areas of triangles and rectangles. The distance formula will play an important role in these applications.
- Students apply their understanding of linear relationships to derive definitions and to solve problems related to distance, midpoint, slope, area, and perimeter.

Progressions

- Use slope and length of line segments to classify quadrilaterals in the coordinate plane.
- Calculate the area and perimeter of parallelograms, triangles, and regular polygons in the coordinate plane.

Examples

- Use appropriate tools strategically to choose between tools such as the slope formula, distance formula, midpoint formula, or Pythagorean Theorem.
- Find the length of a line segment plotted on the coordinate plane.
- Illustrative Mathematics:
  - A Midpoint Miracle
  - Unit Squares and Triangles
  - Is this a rectangle?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4L: HS Data & Statistics Standards and Guidance

Critical Areas of Focus

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Questioning in Statistics

The statistical problem-solving process typically starts with a statistical investigative question, followed by a study designed to collect data that aligns with answering the question. Analysis of the data is also guided by questioning. Constant questioning and interrogation of the data throughout the statistical problem-solving process can lead to the posing of new statistical investigative questions.

Often when considering secondary data, the data need to first be interrogated – how were measurements made, what type of data were selected, what is the meaning of the data, and what was the study design to collect the data. Once a better understanding of the data has been gained, then one can judge whether the data set is appropriate for exploring the original statistical investigative question or one can pose statistical investigative questions that can be explored with the secondary data set.

Analyze, summarize, and describe data

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Technology in Statistics

The teaching of statistics has been greatly enhanced, moving from teaching with no technology to teaching with integrated technology. The field has evolved from using programming languages in the 1980s to hand-held statistical calculators in the 1990s to online statistical calculators, powerful statistical software packages, and amazing data visualization tools. Simulation is now as easy as accessing a public applet where point-and-click options provide the ability to perform thousands of trials. Computer labs are not necessary – just internet access. Moving to web-based technology allows more access to data visualization, exploration of data, and simulation. However, access to technology varies across school districts. Not all classrooms are equipped with internet access or technology hardware and software. Modern statistical practice is intertwined with technology; thus, it is recommended that technology be embraced to the greatest extent possible within a given circumstance.

Connections to Functions and Modeling.

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Domains and Clusters

Data Reasoning and Probability (HS.DR)

- HS.DR.A Formulate Statistical Investigative Questions.
- HS.DR.B Collect and Consider Data.
- HS.DR.C Analyze, summarize, and describe data.
- HS.DR.D Interpret data and answer investigative questions.
- HS.DR.E Understand independence and conditional probability and use them to interpret data.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.DR.A - Formulate Statistical Investigative Questions

STANDARD: HS.DR.A.1

Standards Statement (2021):

Formulate multivariable statistical investigative questions and determine how data from samples can be collected and analyzed to provide an answer.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.A.1, 7.DR.A.1, 8.DR.A.1	N/A	N/A	HSS.IC.A.1 HS.DR.A Crosswalk

Standards Guidance:

Clarifications

- Focus on supporting students to understand and ask questions about how data could be collected.
- As students engage in multivariable thinking, the types of statistical investigative questions should expand to include questions concerning association and prediction.
- Students pose statistical investigative questions for a particular sample to determine any association of the variables of interest for that sample.

Terminology

- A statistical investigative question is one that requires data that will vary.
  - Statistical questions are set in a context where one wants to know something; are based in variability or uncertainty; are data based; and are approximations/estimates from data analysis.
  - Deterministic questions are based upon exact calculations or theoretical deductions elicited from prior certain knowledge.
- A sample is a subset of a population.
- Samples are taken when examining the entire population is not possible or feasible.

Teaching Strategies

- This is an opportunity for students to create a survey, collect data, and use graphical displays, sample statistics or two way tables to help estimate population parameters which are unknown values.
- It is important to understand samples used on social media or in the news.

Progressions

- CCSS - (HSS.IC.A.1) Understand the process of statistical reasoning, formulate questions, collect, analyze, and interpret data to answer statistical investigative questions.
- GAISE II - (1.C.1) Formulate multivariable statistical investigative questions and determine how data can be collected and analyzed to provide an answer

Examples

- Students can distinguish between situations where a small group (e.g., a classroom) is the entire population (census) and when it is a sample from a larger population (e.g., the classroom is used to answer a question about an entire grade level in a school).
- "Given a list of the arm spans of 9th grade students, what can be predicted about the heights of those students?"
- Illustrative Mathematics: [Haircut Costs] [Speed Trap] [Accuracy of Carbon 14 Dating I]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.DR.A.2

Standards Statement (2021):

Formulate summative, comparative, and associative statistical investigative questions for surveys, observational studies, and experiments using primary or secondary data.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.A.1, 7.DR.A.1, 8.DR.A.1	N/A	N/A	HSS.IC.B.3 HS.DR.A Crosswalk

Standards Guidance:

Clarifications

- Students will draft statistical questions for which appropriate data can be collected and analyzed to answer the statistical investigative question.
- Students will use appropriate sampling techniques, critique a poorly constructed survey, and make suggestions for good questions.
- Students should understand the advantages and disadvantages of each data collection method for specific statistical questions.
- For experimental studies, students are able to identify, discuss, and explain the aspects of best statistical practice for designing an experimental study, including: (1) the clear identification of the statistical question to be investigated; (2) the variables under investigation; and the random selection of experimental units and/or (3) random assignment of treatments for experimental studies.

Terminology

- Types of statistical investigative questions include:
  - Summative questions can be answered using quantitative measures of center and variability for numerical data sets (6.DR.B.2).
  - Comparative questions can be answered using numerical data from random samples to compare between two populations (7.DR.D.4).
  - Associative questions can be answered using bivariate quantitative data to investigate patterns of association between two quantities (8.DR.D.4).
- Types of data collections could include:
  - Surveys involve the collection of data from a pre-defined group to gain insight and information about the statistical investigative question.
  - Observational studies measure a sample as it is without attempting to influence the results.
  - Experiments involve the use of a treatment to explore the effects of the treatment on a sample.
- Types of data include:
  - Primary data is collected through first-hand sources such as surveys, experiments, and other studies.
  - Secondary data is obtained from previously conducted studies or research.

Progressions

- GAISE II - (1.C.2) Pose summary, comparative, and association statistical investigative questions for surveys, observational studies, and experiments using primary or secondary data

Examples

- Illustrative Mathematics: [Types of Statistical Studies] [Strict Parents] [Words and Music II]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.DR.A.3

Standards Statement (2021):

Formulate inferential statistical investigative questions regarding causality and prediction from correlation.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.A.1, 7.DR.A.1, 8.DR.A.1	N/A	N/A	HSS.ID.C.9 HS.DR.A Crosswalk

Standards Guidance:

Clarifications

- Students pose statistical investigative questions for a particular sample to determine any association of the variables of interest for that sample.
- Students should be able to understand the magnitude of a correlation coefficient represents the strength of association; understand and able to calculate a residual; understand that any straight line other than the best fit line (by least squares) will have a larger sum of squared residuals than the best fit line.

Boundaries

- Understand and explain the difference between correlation and causation. It is important for students to discover and understand that strong correlation does not indicate causation.

Progressions

- CCSS - (HSS.ID.C.9) Distinguish between correlation and causation.
- GAISE II - (1.C.3) Pose inferential statistical investigative questions regarding causality and prediction.

Examples

- Determine if statements of causation seem reasonable or unreasonable and justify reasoning.
- Correlation coefficients of r =-.65 and r = .65 indicate the same strength.
- Illustrative Mathematics:
  - Coffee and Crime
  - Golf and Divorce
  - High blood pressure
  - Math test grades

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.DR.A.4

Standards Statement (2021):

Use mathematical and statistical reasoning to formulate questions about data to evaluate conclusions and assess risks.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.A.1, 7.DR.A.1, 8.DR.A.1	N/A	N/A	HSS.IC.B.6 HS.DR.A Crosswalk

Standards Guidance:

Clarifications

- Focus of standard is supporting students to evaluate data presented in reports to evaluate conclusions and/or assess risks.
- Understand different ways in which number appear in everyday discussions of government, business, scientific results, and personal activities.
- Apply mathematical and statistical knowledge to inform and make decisions students face or many need to evaluate in society.

Teaching Strategies

- Generate reasonable estimates and use scale to place quantities in context.
- Interpret visual representations of data to assess conclusions and risks
- Locate data to assess validly of claims and conclusions.

Progressions

- CCSS – (HSS.IC.B.6) Evaluate reports based on data.
- NCTM Essential Skills - Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.

Examples

- Illustrative Mathematics:
  - Used Subaru Foresters I
  - Olympic Men's 100-meter dash
  - Restaurant Bill and Party Size
  - Coffee and Crime

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.DR.B - Collect and Consider Data

STANDARD: HS.DR.B.5

Standards Statement (2021):

Articulate what constitutes good practice in designing a sample survey, an experiment, and an observational study. Understand issues of bias and confounding variables in a study and their implications for interpretation.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.B.2, 7.DR.B.2, 8.DR.B.2	N/A	N/A	n/a HS.DR.B Crosswalk

Standards Guidance:

Clarifications

- Students are able to identify, discuss, and explain the aspects of best statistical practice for designing an experimental study, including:
  - the clear identification of the statistical question to be investigated;
  - the variables under investigation; and
  - the random selection of experimental units and/or the random assignment of treatments to the experimental units.
- Students should be able to describe the ethical consequences of their experiments and analyses.
- Practices for handling data that enhance reproducibility and ensure ethical use include providing descriptions of alterations to collected data, proper treatment of sensitive information, maintaining the confidentiality of data and experimental units, and using Institutional Review Boards to review study designs.

Teaching Strategies

- Students should be able to design and conduct comparative experiments using random assignment and demonstrate correct methods for planning data collection for comparison of treatments.
- Students should be able to randomly assign treatments to experimental units.
- Students provide or select appropriate interpretations of graphical displays and numerical summaries to compare two or more groups in the context of a study.

Progressions

- GAISE II - (2.C.3) Understand what constitutes good practice in designing a sample survey, an experiment, and an observational study
- NCTM Essential Skills –
  - The role of randomization is different in randomly selecting samples and in randomly assigning subjects to experimental treatment groups.
  - The larger the sample size, the less the expected variability in the sampling distribution of a sample statistic.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.DR.B.6

Standards Statement (2021):

Distinguish and choose between surveys, observational studies, and experiments to design an appropriate data collection that answers an investigative question of interest.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.B.2, 7.DR.B.2, 8.DR.B.2	N/A	N/A	HSS.IC.B.4 HS.DR.B Crosswalk

Standards Guidance:

Clarifications

- Students should understand the advantages and disadvantages of each data collection method for specific statistical questions.
- Students should be able to design and conduct comparative experiments using random assignment, or non-experimental designs when random assignment is not possible, and demonstrate correct methods for planning data collection for comparison of treatments.

Terminology

- Surveys involve the collection of data from a pre-defined group to gain insight and information about the statistical investigative question.
- Observational studies measure a sample as it is without attempting to influence the results.
- Experiments involve the use of a treatment to explore the effects of the treatment on a sample.
    - For experimental designs, students should be able to randomly assign treatments to experimental units.
- Nonexperimental research is research that lacks the manipulation of an independent variable, random assignment of participants to conditions or orders of conditions, or both.
    - Examples of non-experimental research could include case studies, focus groups, interviews, correlational or quasi-experimental research, or qualitative studies.

Boundaries

- Limit to population proportion, graphical representations, and visual overlap.

Progressions

- CCSS - (HSS.IC.B.4) Use data from a randomized experiment to compare two treatments to decide if differences between parameters are significant based on the statistics.
- GAISE II - (2.C.2) Distinguish between surveys, observational studies, and experiments.
- NCTM Essential Skills - Study designs are of three main types: sample survey, experiment, and observational study.

Examples

- Illustrative Mathematics:
    - Fred's Flare Formula
    - Margin of Error for Estimating a Population Mean
    - Scratch 'n Win Blues
    - The Marble Jar

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.DR.B.7

Standards Statement (2021):

Apply an appropriate data collection plan when collecting primary data or selecting secondary data for the statistical investigative question of interest.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.B.2, 7.DR.B.2, 8.DR.B.2	N/A	N/A	HSS.ID.B.5 HS.DR.B Crosswalk

Standards Guidance:

Clarifications

- Students will use appropriate sampling techniques, critique a poorly constructed survey, and make suggestions for good questions.
- Students should identify types of displays that are appropriate for categorical data versus quantitative (numerical) data.
- Students should have opportunities to analyze meaningful, real-life data and recognize possible associations and trends in the data.
- Students should understand and apply concepts of sample space to describe categorical data.

Terminology

- Primary data is collected through first-hand sources such as surveys, experiments, and other studies.
- Secondary data is obtained from previously conducted studies or research.

Boundaries

- Students should consider features such as whether the population is well-defined, whether the sampling procedure is random or non-random, and whether the objectivity or bias of questions will result in valid/invalid answers.

Teaching Strategies

- Students may use spreadsheets, graphing calculators, and statistical software to create frequency tables and determine associations or trends in the data.
- Recognize the association between two variables by comparing conditional and marginal percentages.
- Describe patterns observed in the data

Progressions

- GAISE II - (2.C.1) Apply an appropriate data collection plan when collecting primary data or selecting secondary data for the statistical investigative question of interest.
- NCTM Essential Skills - The scope and validity of statistical inferences are dependent on the role of randomization in the study design.

Examples

- Read, interpret and write clear summaries of data displayed in a two-way frequency table.
- Calculate joint, marginal, and conditional relative frequencies.
- Make appropriate displays of joint, marginal, and conditional distributions.

Examples

- Illustrative Mathematics: [Musical Preferences] [Support for a Longer School Day?]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.DR.C - Analyze Data

STANDARD: HS.DR.C.8

Standards Statement (2021):

Identify appropriate ways to summarize and then represent the distribution of univariate and bivariate data multiple ways with graphs and/or tables. Use technology to present data that supports interpretation of tabular and graphical representations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.C.3, 7.DR.C.3, 8.DR.C.3	N/A	N/A	HSS.ID.A.1 HSS.ID.B.6 HS.DR.C Crosswalk

Standards Guidance:

Clarifications

- Students should identify types of displays that are appropriate for categorical data versus quantitative (numerical) data.
- Students should be able to construct scatterplots, and describe positive, negative or no relationship.
- Strength of association is demonstrated by degree of spread about the line of best fit in a scatterplot.
- Numerical data can be displayed visually with graphs, such as using dot plots, histograms, and box plots, to discover patterns and deviations from patterns.
- Students should use spreadsheets, graphing calculators, or statistical software to analyze data.

Terminology

- Univariate data involves describing a single variable, such as student ages or student heights.
- Bivariate data involves relationships between two variables, such as comparing the age of a student and their height.

Teaching Strategies

- This is an extension of middle school expectations where students display data on dot and box plots.
- Opportunity for students to collect and graph their own data and use modeling to fit a function to the data; use a function fitted to data to solve problems in the context of the data. (Emphasize linear models.)
- Students should be able to fluently utilize dot plots, histograms, and box plots to represent data.

Progressions

- GAISE II – (3.C.2) Identify appropriate ways to summarize quantitative or categorical data using tables, graphical displays, and numerical summary statistics, which includes using standard deviation as a measure of variability and a modified boxplot for identifying outliers.

Examples

- Analyze the strengths and weakness inherent in different types of visual data representations.
- Describe and give simple conclusions and interpretations of a graphical representation of data.
- Fit a linear function for a scatter plot that suggests a linear association.
- Illustrative Mathematics:
  - [Laptop Battery Charge 2] [Speed Trap] [Used Subaru Foresters I]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.DR.C.9

Standards Statement (2021):

Use statistics appropriate to the shape of the data distribution to compare the center and spread of two or more different data sets.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.C.3, 7.DR.C.3, 8.DR.C.3	N/A	N/A	HSS.ID.A.2 HSS.ID.A.4 HS.DR.C Crosswalk

Standards Guidance:

Clarifications

- Students should have the opportunity to gain an understanding of this concept through the use of technology tools.
- Students should use the meaning of mean absolute deviation (MAD) learning in sixth grade to interpret the meaning of standard deviation.
- Students were first introduced to the concept of MAD as a tool for comparing variability of multiple data sets in sixth grade mathematics.
- Students should be able to construct scatterplots, and describe positive, negative or no relationship.
- Data may be displayed using histograms, dot plots, or smooth normal curves.

Boundaries

- Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread.
    - Measures of center include the mean, median, and mode.
    - Measures of spread include the range, interquartile range, and standard deviation.
- The shape of a data distribution might be described as symmetric, skewed, uniform, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range).

Progressions

- GAISE II –
    - (3.C.6) Describe associations between two categorical variables using measures such as difference in proportions and relative risk
    - (3.C.7) Describe the relationship between two quantitative variables by interpreting Pearson's correlation coefficient and a least-squares regression line
- NCTM Essential Skills - Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.

Examples

- Illustrative Mathematics:
    - [Measuring Variability in a Data Set] [Understanding the Standard Deviation] [SAT Scores] [Do You Fit In This Car?] [Should We Send Out a Certificate?]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.DR.C.10

Standards Statement (2021):

Use data to compare two groups, describe sample variability, and decide if differences between parameters are significant based on the statistics.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.C.3, 7.DR.C.3, 8.DR.C.3	N/A	N/A	HSS.ID.A.3 HSS.IC.B.5 HS.DR.C Crosswalk

Standards Guidance:

Clarifications

- Students should be able to describe how population estimates may be overstated or understated due to the presence of outliers.
- Students should be able to describe how missing or erroneous values can lead to biased or inaccurate estimations.
- Strength of association is demonstrated by degree of spread about the line of best fit in a scatterplot.
- Students should be able to recognize how sampling variability is influenced by sample size.

Teaching Strategies

- Use data from multiple sources to interpret differences in shape, center and spread
- Discuss the effect of outliers on measures of center and spread.
- Use the 1.5 IQR rule to determine the outliers and analyze their effects on the data set.

Progressions

- CCSS – (HSS.ID.A.3) Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- GAISE II – (3.C.4) Understand how sampling distributions (developed through simulation) are used to describe the sample-to-sample variability of sample statistics
- NCTM Essential Skills - Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line and finding a correlation coefficient, and differentiating between correlation and causation.

Examples

- Students should use spreadsheets, graphing utilities and statistical software to identify outliers and analyze data sets with and without outliers as appropriate.
- Using the 1.5 IQR rule on data set {5,7,8,10,11,12,30}, 30 is determined to be an outlier since it is greater than 19.5, which is the 1.5*IQR +12 (the 3Q).

Examples

- Illustrative Mathematics:
  - [Describing Data Sets with Outliers] [Identifying Outliers] [Musical Preferences] [Measuring Variability in a Data Set]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.DR.D - Interpret data and answer investigative questions

STANDARD: HS.DR.D.11

Standards Statement (2021):

Use statistical evidence from analyses to answer statistical investigative questions, and communicate the findings in a variety of formats (verbal, written, visual) to support informed data-based decisions.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.D.4, 7.DR.D.4, 8.DR.D.4	N/A	8.AFN.B.4, HS.AFN.A.3	HSS.ID.C.7 HS.DR.D Crosswalk

Standards Guidance:

Clarification

- Identify when data can be generalized to a target population.
  - Samples must be randomly selected from the appropriate population to allow for generalizations that extend beyond the sample from which the data were collected.
  - Sampling procedures that are not random do not allow for generalizations to the sampled population because they may be biased.
- Evidence could be interpreted from data displays such as histograms, dot plots, or smooth normal curves.

Teaching Strategies

- Students should be able to recognize that sample statistics vary with repeated sampling.
- Students should be able to interpret the sampling variability in a summary statistic.
- Students should be able to interpret the sampling variability from simulation studies of statistics.
- Students should be able to recognize how sampling variability is influenced by sample size.
- Recognize that there are data sets for which the empirical rule is not appropriate.

Progressions

- GAISE II – (4.C.1) Use statistical evidence from analyses to answer the statistical investigative questions and communicate results through more formal reports and presentations

Examples

- Illustrative Mathematics:
  - Texting and Grades II
  - Used Subaru Foresters II
  - Olympic Men's 100-meter dash

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.DR.D.12

Standards Statement (2021):

Articulate what it means for an outcome or an estimate of a population characteristic to be plausible or not plausible compared to chance variation.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.D.4, 7.DR.D.4, 8.DR.D.4	N/A	N/A	HSS.ID.C.8 HS.DR.D Crosswalk

Standards Guidance:

Clarifications

- Students should be able to decide whether an observed difference is something that would be likely to be observed by chance and whether this difference has any practical meaning.
- Students recognize that significance is demonstrated by a result that is unlikely to occur by chance
- Students recognize that statistical, but not practical, significance is influenced by sample size.

Teaching Strategies

- Students should use spreadsheets, graphing calculators and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals and correlation coefficients.
- Students should be given the opportunity to utilize interactive graphing technologies to interpret the correlation coefficient, r.
    - Students should be able to use the correlation coefficient, r, to make predictions and describe the reasonableness of the prediction in the context of a practical, real-life situation.
    - Explain that the correlation coefficient must be between −1 and 1 inclusive and explain what each of these values means.
    - Determine whether the correlation coefficient shows a weak positive, strong positive, weak negative, strong negative, or no linear correlation. Interpret what the correlation coefficient is telling about the data.

Progressions

- GAISE II – (4.C.3) Understand what it means for an outcome or an estimate of a population characteristic to be plausible or not plausible compared to chance variation
- NCTM Essential Skills - Data-analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.

Examples

- Illustrative Mathematics:
    - Coffee and Crime

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.DR.D.13

Standards Statement (2021):

Use multivariate thinking to articulate how variables impact one another, and measure the strength of association using correlation coefficients for regression curves.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
6.DR.D.4, 7.DR.D.4, 8.DR.D.4	N/A	N/A	HSS.ID.C.9 HS.DR.D Crosswalk

Standards Guidance:

Clarifications

- As students engage in multivariable thinking, the types of statistical investigative questions should expand to include questions concerning association and prediction.
- Students should be able to identify contexts where a change in one attribute may be related to a change in another attribute.
- Students should be able to describe how population estimates may be overstated or understated due to the presence of outliers.
- Students should be able to describe how missing or erroneous values can lead to biased or inaccurate estimations.

Boundaries

- Students should be able to provide a reasonable estimate of the Pearson's correlation coefficient (r) for a scatterplot; identify linear and non-linear relationships in scatterplots; correctly interpret the strength of a linear relationship based on r.
- Students should be able to understand the magnitude of a correlation coefficient represents the strength of association; understand and able to calculate a residual; understand that any straight line other than the best fit line (by least squares) will have a larger sum of squared residuals than the best fit line.

Teaching Strategies

- Opportunity to connect the concept of distinguishing between correlation and causation as students interpret data.
- Understand and explain the difference between correlation and causation. It is important for students to discover and understand that strong correlation does not indicate causation.

Progressions

- GAISE II – (4.C.6) Use multivariate thinking to understand how variables impact one another.
- NCTM Essential Skills - Making and defending informed data-based decisions is a characteristic of a quantitatively literate person.

Examples

- Determine if statements of causation seem reasonable or unreasonable and justify reasoning.
- Correlation coefficients of r =-.65 and r = .65 indicate the same strength.
- Illustrative Mathematics:
  - [Golf and Divorce] [High blood pressure] [Math test grades]

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Cluster: HS.DR.E - Understand independence and conditional probability and use them to interpret data

STANDARD: HS.DR.E.14

Standards Statement (2021):

Describe the possible outcomes for a situation as subsets of a sample space.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.RP.B.4, 7.RP.B.5, 7.RP.B.6, 7.RP.B.7	N/A	N/A	HSS.CP.A.1 HS.DR.E Crosswalk

Standards Guidance:

Progressions

- This provides an opportunity for students to engage with finding the outcomes of situations which include words such as **and**, **or**, **not**, **if**, and **all**, and to grammatical constructions that reflect logical connections.

Examples

- Illustrative Mathematics:
  - Describing Events
  - Return to Fred's Fun Factory (with 50 cents)
  - The Titanic 1

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

STANDARD: HS.DR.E.15

Standards Statement (2021):

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Connections:

Preceding Pathway Content (2021)	Subsequent Pathway Content (2021)	Cross Domain Connections (2021)	Common Core (CCSS) (2010)
7.RP.B.6, 7.RP.B.7	N/A	N/A	HSS.CP.A.5 HS.DR.E Crosswalk

Standards Guidance:

Examples

- Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
- Illustrative Mathematics:
  - Breakfast Before School
  - But mango is my favorite…
  - Rain and Lightning
  - The Titanic 2
  - The Titanic 3

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

SECTION FIVE: Oregon/CCSS Crosswalk

5A: Grade K Math Crosswalk (2021)

K.OA - Algebraic Reasoning: Operations

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
K.OA	Algebraic Reasoning: Operations	K.OA	Operations & Algebraic Thinking
K.OA.A	Understand addition and subtraction.	K.OA.A	Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
K.OA.A.1	Represent addition as putting together and adding to and subtraction as taking apart and taking from using objects, drawings, physical expressions, numbers or equations.	K.OA.A.1	Represent addition and subtraction with objects, fingers, mental images, drawings (drawings need not show details, but should show the mathematics in the problem), sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
K.OA.A.2	Add and subtract within 10. Model authentic contexts and solve problems that use addition and subtraction within 10.	K.OA.A.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
K.OA.A.3	Using objects or drawings, and equations, decompose numbers less than or equal to 10 into pairs in more than one way.	K.OA.A.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
K.OA.A.4	By using objects, drawings, or equations, find the unknown number that makes 10 when added to a given number from 1 - 9.	K.OA.A.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
K.OA.A.5	Fluently add and subtract within 5 with accurate, efficient, and flexible strategies.	K.OA.A.5	Fluently add and subtract within 5.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

K.NCC Numeric Reasoning: Counting and Cardinality

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
K.NCC	Numeric Reasoning: Counting and Cardinality	K.CC	Counting & Cardinality
K.NCC.A	Know number names and the count sequence.	K.CC.A	Know number names and the count sequence.
K.NCC.A.1	Orally count to 100 by ones and by tens in sequential order.	K.CC.A.1	Count to 100 by ones and by tens.
K.NCC.A.2	Count forward beginning from a given number within 100 of a known sequence.	K.CC.A.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
K.NCC.A.3	Identify number names, write numbers, and the count sequence from 0-20. Represent a number of objects with a written number 0-20.	K.CC.A.3	Know number names and the count sequence. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
K.NCC.B	Count to tell the number of objects.	K.CC.B	Count to tell the number of objects.
K.NCC.B.4	Understand the relationship between numbers and quantities; connect counting to cardinality.	K.CC.B.4	Understand the relationship between numbers and quantities; connect counting to cardinality. --(4.a) When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. --(4.b) Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. --(4.c) Understand that each successive number name refers to a quantity that is one larger.
K.NCC.B.5	Count to answer “how many?” questions using up to 20 objects arranged in a variety of configurations or as 10 objects in a scattered configuration. Given a number from 1-20, count out that many objects.	K.CC.B.5	Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
K.NCC.C	Compare numbers.	K.CC.C	Compare numbers
K.NCC.C.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.	K.CC.C.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.)
K.NCC.C.7	Compare two numbers between 1 and 10 presented as written numerals.	K.CC.C.7	Compare two numbers between 1 and 10 presented as written numerals.

K.NBT Numeric Reasoning: Base Ten Arithmetic

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
K.NBT	Numeric Reasoning: Base Ten Arithmetic	K.NBT	Number & Operations in Base Ten
K.NBT.A	Work with numbers 11-19 to gain foundations for place value.	K.NBT.A	Work with numbers 11-19 to gain foundations for place value.
K.NBT.A.1	Compose and decompose from 11 to 19 into groups of ten ones and some further ones using objects, drawings, or equations.	K.NBT.A.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

K.GM – Geometric Reasoning and Measurement

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
K.GM	Geometric Reasoning and Measurement	K.G	Geometry
K.GM.A	Identify and describe shapes.	K.G.A	Identify and describe shapes.
K.GM.A.1	Describe objects in the environment using names of shapes and describe the relative positions of these objects in their environment.	K.G.A.1	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
K.GM.A.2	Correctly name common two-dimensional and three-dimensional geometric shapes regardless of their orientations or overall size.	K.G.A.2	Correctly name shapes regardless of their orientations or overall size.
K.GM.A.3	Identify shapes as two-dimensional or three-dimensional.	K.G.A.3	Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
K.GM.B	Analyze, compare, create, and compose shapes.	K.G.B	Analyze, compare, create, and compose shapes.
K.GM.B.4	Analyze and compare two and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts and attributes.	K.G.B.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).
K.GM.B.5	Represent shapes in the world by building shapes from components and drawing shapes.	K.G.B.5	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
K.GM.B.6	Compose common shapes to form larger shapes.	K.G.B.6	Compose simple shapes to form larger shapes. For example, "can you join these two triangles with full sides touching to make a rectangle?”

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
K.GM.C	Describe and compare measurable attributes.	K.MD.A	Describe and compare measurable attributes.
K.GM.C.7	Describe several measurable attributes of a single object using measurable terms, such as length or weight.	K.MD.A.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
K.GM.C.8	Directly compare two objects with a measurable attribute in common, and describe which object has “more” or “less” of the attribute.	K.MD.A.2	Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

K.DR – Data Reasoning

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
K.DR	Data Reasoning	K.MD	Measurement and Data
K.DR.A	Pose investigative questions and collect/consider data.		
K.DR.A.1	Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by sorting and counting.		[new content]

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
K.DR.B	Analyze, represent, and interpret data.		
K.DR.B.2	Analyze data sets by counting the number of objects in each category and interpret results by classifying and sorting objects by count.	K.MD.B.3	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5B: Grade 1 Math Crosswalk (2021)

1.OA - Algebraic Reasoning: Operations

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.OA	Algebraic Reasoning: Operations	1.OA	Operations & Algebraic Thinking
1.OA.A	Represent and solve problems involving addition and subtraction.	1.OA.A	Represent and solve problems involving addition and subtraction.
1.OA.A.1	Use addition and subtraction within 20 to solve and represent problems in authentic contexts involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.	1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
1.OA.A.2	Solve problems that call for addition of three whole numbers whose sum is less than or equal to 20 using objects, drawings or equations.	1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.OA.B	Understand and apply properties of operations and the relationship between addition and subtraction.	1.OA.B	Understand and apply properties of operations and the relationship between addition and subtraction.
1.OA.B.3	Apply properties of operations as strategies to add and subtract.	1.OA.B.3	Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) (Students need not use formal terms for these properties.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.OA.B.4	Understand subtraction as an unknown-addend problem.	1.OA.B.4	Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.OA.C	Add and subtract within 20.	1.OA.C	Add and subtract within 20.
1.OA.C.5	Relate counting to addition and subtraction.	1.OA.C.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
1.OA.C.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10 with accurate, efficient, and flexible strategies.	1.OA.C.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.OA.D	Work with addition and subtraction equations.	1.OA.D	Work with addition and subtraction equations.
1.OA.D.7	Use the meaning of the equal sign to determine whether equations involving addition and subtraction are true or false.	1.OA.D.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.	1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = __ – 3, 6 + 6 = __.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

1.NBT - Numeric Reasoning: Base Ten Arithmetic

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.NBT	Numeric Reasoning: Base Ten Arithmetic	1.NBT	Number & Operations in Base Ten
1.NBT.A	Extend the counting sequence.	1.NBT.A	Extend the counting sequence.
1.NBT.A.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	1.NBT.A.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.NBT.B	Understand place value.	1.NBT.B	Understand place value.
1.NBT.B.2	Understand 10 as a bundle of ten ones and that the two digits of a two-digit number represent amounts of tens and ones.	1.NBT.B.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: -- a. 10 can be thought of as a bundle of ten ones — called a “ten.” -- b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. -- c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
1.NBT.B.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	1.NBT.B.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.NBT.C	Use place value understanding and properties of operations to add and subtract.	1.NBT.C	Use place value understanding and properties of operations to add and subtract.
1.NBT.C.4	Add within 100 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain why sometimes it is necessary to compose a ten.	1.NBT.C.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
1.NBT.C.5	Without having to count, mentally find 10 more or 10 less than a given two-digit number and explain the reasoning used.	1.NBT.C.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
1.NBT.C.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy and model used to a written method and explain the reasoning used.	1.NBT.C.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

1.GM - Geometric Reasoning and Measurement

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.GM	Geometric Reasoning and Measurement	1.G	Geometry
1.GM.A	Reason with shapes and their attributes.	1.G.A	Reason with shapes and their attributes.
1.GM.A.1	Distinguish between defining attributes versus non-defining attributes for a wide variety of shapes. Build and draw shapes to possess defining attributes.	1.G.A.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); for a wide variety of shapes; build and draw shapes to possess defining attributes.
1.GM.A.2	Compose common two-dimensional shapes or three-dimensional shapes to create a composite shape, and create additional new shapes from composite shapes.	1.G.A.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names such as “right rectangular prism.”)
1.GM.A.3	Partition circles and rectangles into two and four equal shares. Describe the equal shares and understand that partitioning into more equal shares creates smaller shares.	1.G.A.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.GM.B	Describe and compare measurable attributes.	1.MD.A	Describe and compare measurable attributes.
1.GM.B.4	Order three objects by length; compare the lengths of two objects indirectly by using a third object.	1.MD.A.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.
1.GM.B.5	Express the length of an object as a whole number of non-standard length units, by laying multiple copies of a shorter object (the length unit) end to end.	1.MD.A.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.GM.C	Tell and write time.	1.MD.B	Tell and write time.
1.GM.C.6	Tell and write time in hours and half-hours using analog and digital clocks.		Tell and write time in hours and half-hours using analog and digital clocks.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

1.DR - Data Reasoning

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.DR	Data Reasoning	1.MD	Measurement and Data
1.DR.A	Pose investigative questions and collect/consider data.		
1.DR.A.1	Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by representing data visually.		[new content]

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
1.DR.B	Analyze, represent, and interpret data.		
1.DR.B.2	Analyze data sets with up to three categories by representing data visually, such as with graphs and charts, and interpret information presented to answer investigative questions.	1.MD.C.3	(1.MD.C.4) Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5C: Grade 2 Math Crosswalk (2021)

2.OA - Algebraic Reasoning: Operations

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.OA	Algebraic Reasoning: Operations	2.OA	Operations & Algebraic Thinking
2.OA.A	Represent and solve problems involving addition and subtraction.	2.OA.A	Represent and solve problems involving addition and subtraction.
2.OA.A.1	Use addition and subtraction within 100 to solve one- and two-step problems in authentic contexts by using drawings and equations with a symbol for the unknown.	2.OA.A.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.OA.B	Add and subtract within 20.	2.OA.B	Add and subtract within 20.
2.OA.B.2	Fluently add and subtract within 20 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.	2.OA.B.2	Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.OA.C	Work with equal groups of objects to gain foundations for multiplication.	2.OA.C	Work with equal groups of objects to gain foundations for multiplication.
2.OA.C.3	Determine whether a group up to 20 objects has an odd or even number by pairing objects or counting them by 2s; record using drawings and equations including expressing an even number as a sum of two equal addends.	2.OA.C.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
2.OA.C.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	2.OA.C.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

2.NBT - Numeric Reasoning: Base Ten Arithmetic

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.NBT	Numeric Reasoning: Base Ten Arithmetic	2.NBT	Number & Operations in Base Ten
2.NBT.A	Understand place value.	2.NBT.A	Understand place value.
2.NBT.A.1	Understand 100 as a bundle of ten tens and that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.	2.NBT.A.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: --(1.a)100 can be thought of as a bundle of ten tens — called a “hundred.” --(1.b) The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2.NBT.A.2	Count within 1000; skip-count by 5's, 10's, and 100's.	2.NBT.A.2	Count within 1000; skip-count by 5s, 10s, and 100s.
2.NBT.A.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	2.NBT.A.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.NBT.B	Use place value understanding and properties of operations to add and subtract.	2.NBT.B	Use place value understanding and properties of operations to add and subtract.
2.NBT.B.5	Fluently add & subtract within 100 using accurate, efficient, & flexible strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	2.NBT.B.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
2.NBT.B.6	Add up to four two-digit numbers using strategies based on place value and properties of operations and describe how two different strategies result in the same sum.	2.NBT.B.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.
2.NBT.B.7	Add and subtract within 1000 using concrete or visual representations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain why sometimes it is necessary to compose or decompose tens or hundreds.	2.NBT.B.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
2.NBT.B.8	Without having to count, mentally find 10 more or 10 less and 100 more or 100 less than a given three-digit number.	2.NBT.B.8	Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.
2.NBT.B.9	Explain why strategies to add and subtract work using properties of operations and the relationship between addition and subtraction.	2.NBT.B.9	Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

2.GM - Geometric Reasoning and Measurement

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.GM	Geometric Reasoning and Measurement	2.G	Geometry
2.GM.A	Reason with shapes and their attributes.	2.G.A	Reason with shapes and their attributes.
2.GM.A.1	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.	2.G.A.1	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (Sizes are compared directly or visually, not compared by measuring.)
2.GM.A.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	2.G.A.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
2.GM.A.3	Partition circles and rectangles into two, three, or four equal parts. Recognize that equal parts of identical wholes need not have the same shape.	2.G.A.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.GM.B	Measure and estimate lengths in standard units.	2.MD.A	Measure and estimate lengths in standard units.
2.GM.B.4	Measure the length of an object by selecting and using appropriate measurement tools.	2.MD.A.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2.GM.B.5	Measure the length of an object using two different length units and describe how the measurements relate to the size of the unit chosen.	2.MD.A.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
2.GM.B.6	Estimate lengths using units of inches, feet, yards, centimeters, and meters.	2.MD.A.3	Estimate lengths using units of inches, feet, centimeters, and meters.
2.GM.B.7	Measure two objects and determine the difference in their lengths in terms of a standard length unit.	2.MD.A.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.GM.C	Relate addition and subtraction to length.	2.MD.B	Relate addition and subtraction to length.
2.GM.C.8	Use addition and subtraction within 100 to solve problems in authentic contexts involving lengths that are given in the same units.	2.MD.B.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
2.GM.C.9	Represent whole number lengths on a number line diagram; use number lines to find sums and differences within 100.	2.MD.B.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, … , and represent whole-number sums and differences within 100 on a number line diagram.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.GM.D	Work with time and money.	2.MD.C	Work with time and money.
2.GM.D.10	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	2.MD.C.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
2.GM.D.11	Solve problems in authentic contexts involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and c (cents) symbols appropriately.	2.MD.C.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ (dollars) and ¢ (cents) symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

2.DR - Data Reasoning

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.DR	Data Reasoning	2.MD	Measurement and Data
2.DR.A	Pose investigative questions and collect/consider data.	2.MD.D	Represent and interpret data.
2.DR.A.1	Generate questions to investigate situations within the classroom. Collect or consider data that can naturally answer questions by using measurements with whole-number units.	2.MD.D.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
2.DR.B	Analyze, represent, and interpret data.	2.MD.D	Represent and interpret data.
2.DR.B.2	Analyze data with a single-unit scale and interpret information presented to answer investigative questions.	2.MD.D.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5D: Grade 3 Math Crosswalk (2021)

3.OA - Algebraic Reasoning: Operations

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.OA	Algebraic Reasoning: Operations	3.OA	Operations & Algebraic Thinking
3.OA.A	Represent and solve problems involving multiplication and division.	3.OA.A	Represent and solve problems involving multiplication and division.
3.OA.A.1	Represent and interpret multiplication of two factors as repeated addition of equal groups.	3.OA.A.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.
3.OA.A.2	Represent and interpret whole-number quotients as dividing an amount into equal sized groups.	3.OA.A.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
3.OA.A.3	Use multiplication and division within 100 to solve problems in authentic contexts involving equal groups, arrays, and/or measurement quantities.	3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
3.OA.A.4	Determine the unknown number in a multiplication or division equation relating three whole numbers by applying the understanding of the inverse relationship of multiplication and division.	3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = __ ÷ 3, 6 × 6 = ?.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.OA.B	Understand properties of multiplication and the relationship between multiplication and division.	3.OA.B	Understand properties of multiplication and the relationship between multiplication and division.
3.OA.B.5	Apply properties of operations as strategies to multiply and divide.	3.OA.B.5	Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15 then 15 × 2 = 30, or by 5 × 2 = 10 then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.) (Students need not use formal terms for these properties.)
3.OA.B.6	Understand division as an unknown-factor in a multiplication problem.	3.OA.B.6	Understand division as an unknown-factor problem. For example, divide 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.OA.C	Multiply and divide within 100.	3.OA.C	Multiply and divide within 100.
3.OA.C.7	Fluently multiply and divide within 100 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.	3.OA.C.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of one-digit numbers.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.OA.D	Solve problems involving the four operations, and identify and explain patterns in arithmetic.	3.OA.D	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
3.OA.D.8	Solve two-step problems in authentic contexts that use addition, subtraction, multiplication, and division in equations with a letter standing for the unknown quantity.	3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).)
3.OA.D.9	Identify and explain arithmetic patterns using properties of operations, including patterns in the addition table or multiplication table.	3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3.NBT - Numeric Reasoning: Base Ten Arithmetic

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.NBT	Numeric Reasoning: Base Ten Arithmetic	3.NBT	Number & Operations in Base Ten
3.NBT.A	Use place value understanding and properties of operations to perform multi-digit arithmetic.	3.NBT.A	Use place value understanding and properties of operations to perform multi-digit arithmetic.[1]
3.NBT.A.1	Use place value understanding to round whole numbers within 1000 to the nearest 10 or 100.	3.NBT.A.1	Use place value understanding to round whole numbers to the nearest 10 or 100.
3.NBT.A.2	Fluently add and subtract within 1000 using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.	3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (A range of algorithms may be used.)
3.NBT.A.3	Find the product of one-digit whole numbers by multiples of 10 in the range 10-90, such as 9 x 80. Students use a range of strategies and algorithms based on place value and properties of operations.	3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. (A range of algorithms may be used.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3.NF - Numeric Reasoning: Fractions

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.NF	Numeric Reasoning: Fractions	3.NF	Number & Operations—Fractions
3.NF.A	Develop understanding of fractions as numbers.	3.NF.A	Develop understanding of fractions as numbers.
3.NF.A.1	Understand the concept of a unit fraction and explain how multiple copies of a unit fraction form a non-unit fraction.	3.NF.A.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)
3.NF.A.2	Understand a fraction as a number on the number line; Represent fractions on a number line diagram.	3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.) --(2.a) Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.) --(2.b) Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.	3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.) --(3.a) Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.) --(3.b) Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3), Explain why the fractions are equivalent, e.g., by using a visual fraction model. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.) --(3.c) Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.) --(3.d) Compare two fractions with the same numerator or the same denominator, by reasoning about their size, Recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3.GM - Geometric Reasoning and Measurement

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.GM	Geometric Reasoning and Measurement	3.G	Geometry
3.GM.A	Reason with shapes and their attributes.	3.G.A	Reason with shapes and their attributes.
3.GM.A.1	Understand that shapes in different categories may share attributes and that shared attributes can define a larger category.	3.G.A.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
3.GM.A.2	Partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.	3.G.A.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part is 1/4 of the area of the shape.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.GM.B	Solve problems involving measurement and estimation.	3.MD.A	Solve problems involving measurement and estimation.
3.GM.B.3	Tell, write, and measure time to the nearest minute. Solve problems in authentic contexts that involve addition and subtraction of time intervals in minutes.	3.MD.A.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.GM.B.4	Measure, estimate and solve problems in authentic contexts that involve liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).	3.MD.A.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems (problems involving notions of "times as much.")

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.GM.C	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	3.MD.C	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.GM.C.5	Recognize area as an attribute of plane figures and understand concepts of area measurement presented in authentic contexts by tiling and counting unit squares.	3.MD.C.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. --(5.a) A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. --(5.b) A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
3.GM.C.6	Measure areas by counting standard and non-standard unit squares.	3.MD.C.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.GM.C.7	Relate area to multiplication and addition. Use relevant representations to solve problems in authentic contexts.	3.MD.C.7	Relate area to the operations of multiplication and addition. --(7.a) Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. --(7.b) Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. --(7.c) Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. --(7.d) Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.GM.D	Geometric measurement: recognize perimeter.	3.MD.D	Geometric measurement: recognize perimeter.
3.GM.D.8	Solve problems involving authentic contexts for perimeters of polygons.	3.MD.D.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different area or with the same area and different perimeter.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

3.DR Data Reasoning

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.DR	Data Reasoning	3.MD	Measurement and Data
3.DR.A	Pose investigative questions and collect/consider data.		
3.DR.A.1	Generate questions to investigate situations within the classroom, school or community. Collect or consider measurement data that can naturally answer questions by using information presented in a scaled picture and/or bar graph.	3.MD.B.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
3.DR.B	Analyze, represent, and interpret data.		
3.DR.B.2	Analyze measurement data with a scaled picture graph or a scaled bar graph to represent a data set with several categories. Interpret information presented to answer investigative questions.	3.MD.B.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5E: Grade 4 Math Crosswalk (2021)

4.OA - Algebraic Reasoning: Operations

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.OA	Algebraic Reasoning: Operations	4.OA	Operations & Algebraic Thinking
4.OA.A	Use the four operations with whole numbers to solve problems.	4.OA.A	Use the four operations with whole numbers to solve problems.
4.OA.A.1	Interpret a multiplication equation as comparing quantities. Represent verbal statements of multiplicative comparisons as equations.	4.OA.A.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
4.OA.A.2	Multiply or divide to solve problems in authentic contexts involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison.	4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
4.OA.A.3	Solve multistep problems in authentic contexts using whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.	4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.OA.B	Gain familiarity with factors and multiples.	4.OA.B	Gain familiarity with factors and multiples.
4.OA.B.4	Find all factor pairs for a whole number in the range 1-100. Determine whether a given whole number in the range of 1-100 is a multiple of a given one-digit number, and whether it is prime or composite.	4.OA.B.4	Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.OA.C	Generate and analyze patterns.	4.OA.C	Generate and analyze patterns.
4.OA.C.5	Analyze a number, visual, or contextual pattern that follows a given rule.	4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4.NBT - Numeric Reasoning: Base Ten Arithmetic

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.NBT	Numeric Reasoning: Base Ten Arithmetic	4.NBT	Number & Operations in Base Ten
4.NBT.A	Generalize place value understanding for multi-digit whole numbers.	4.NBT.A	Generalize place value understanding for multi-digit whole numbers.
4.NBT.A.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	4.NBT.A.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)
4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Use understandings of place value within these forms to compare two multi-digit numbers using >, =, and < symbols.	4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)
4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.	4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.NBT.B	Use place value understanding and properties of operations to perform multi-digit arithmetic.	4.NBT.B	Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT.B.4	Fluently add and subtract multi-digit whole numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.	4.NBT.B.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.)
4.NBT.B.5	Use representations and strategies to multiply a whole number of up to four digits by a one-digit number, and a two-digit number by a two-digit number using strategies based on place value and the properties of operations.	4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.)
4.NBT.B.6	Use representations and strategies to find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.	4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4.NF - Numeric Reasoning: Fractions

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.NF	Numeric Reasoning: Fractions	4.NF	Number & Operations—Fractions
4.NF.A	Extend understanding of fraction equivalence and ordering.	4.NF.A	Extend understanding of fraction equivalence and ordering.
4.NF.A.1	Use visual fraction representations to recognize, generate, and explain relationships between equivalent fractions.	4.NF.A.1	Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)
4.NF.A.2	Compare two fractions with different numerators and/or different denominators, record the results with the symbols >, =, or <, and justify the conclusions.	4.NF.A.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.NF.B	Build fractions from unit fractions.	4.NF.B	Build fractions from unit fractions.
4.NF.B.3	Understand a fraction (a/b) as the sum (a) of fractions of the same denominator (1/b). Solve problems in authentic contexts involving addition and subtraction of fractions referring to the same whole and having like denominators.	4.NF.B.3	Understand a fraction a/b with a > 1 as a sum of fractions 1/b. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.) --(3.a) Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. --(3.b) Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify compositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8. --(3.c) Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. --(3.d) Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. Represent and solve problems in authentic contexts involving multiplication of a fraction by a whole number.	4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.) --(4.a) Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4). --(4.b) Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.) --(4.c) Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.NF.C	Understand decimal notation for fractions, and compare decimal fractions.	4.NF.C	Understand decimal notation for fractions, and compare decimal fractions.
4.NF.C.5	Demonstrate and explain the concept of equivalent fractions with denominators of 10 and 100, using concrete materials and visual models. Add two fractions with denominators of 10 and 100.	4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100 and add 3/10 + 4/100 = 34/100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)
4.NF.C.6	Use and interpret decimal notation for fractions with denominators 10 or 100.	4.NF.C.6	Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100 ; describe a length as 0.62 meters; locate 0.62 on a number line diagram. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)
4.NF.C.7	Use decimal notation for fractions with denominators 10 or 100. Compare two decimals to hundredths place by reasoning about their size, and record the comparison using the symbols >, =, or <.	4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4.GM - Geometric Reasoning and Measurement

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.GM	Geometric Reasoning and Measurement	4.G	Geometry
4.GM.A	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	4.G.A	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
4.GM.A.1	Explore, investigate, and draw points, lines, line segments, rays, angles, and perpendicular and parallel lines. Identify these in two-dimensional figures.	4.G.A.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
4.GM.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size.	4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
4.GM.A.3	Recognize and draw a line of symmetry for a two dimensional figure.	4.G.A.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.GM.B	Solve problems involving measurement and conversion of measurements.	4.MD.A	Solve problems involving measurement and conversion of measurements.
4.GM.B.4	Know relative sizes of measurement units and express measurements in a larger unit in terms of a smaller unit.	4.MD.A.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example: Know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ….
4.GM.B.5	Apply knowledge of the four operations and relative size of measurement units to solve problems in authentic contexts that include familiar fractions or decimals.	4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
4.GM.B.6	Apply the area and perimeter formulas for rectangles in authentic contexts and mathematical problems.	4.MD.A.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.GM.C	Geometric measurement: understand concepts of angle and measure angles.	4.MD.C	Geometric measurement: understand concepts of angle and measure angles.
4.GM.C.7	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. Understand and apply concepts of angle measurement.	4.MD.C.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: --(5.a) An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. --(5.b) An angle that turns through n one-degree angles is said to have an angle measure of n degrees.
4.GM.C.8	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	4.MD.C.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
4.GM.C.9	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.	4.MD.C.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

4.DR - Data Reasoning

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.DR	Data Reasoning	4.MD	Measurement and Data
4.DR.A	Pose investigative questions and collect/consider data.		
4.DR.A.1	Generate questions to investigate situations within the classroom, school or community. Determine strategies for collecting or considering data involving addition and subtraction of fractions that can naturally answer questions by using information presented in line plots.		[new content]

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
4.DR.B	Analyze, represent, and interpret data.	4.MD.B	Represent and interpret data.
4.DR.B.2	Analyze line plots to display a distribution of numerical measurement data, which include displays of data sets of fractional measurements with the same denominator. Interpret information presented to answer investigative questions.	4.MD.B.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5F: Grade 5 Math Crosswalk (2021)

5.OA - Algebraic Reasoning: Operations

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.OA	Algebraic Reasoning: Operations	5.OA	Operations & Algebraic Thinking
5.OA.A	Write and interpret numerical expressions.	5.OA.A	Write and interpret numerical expressions.
5.OA.A.1	Write and evaluate numerical expressions that include parentheses.	5.OA.A.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
5.OA.A.2	Write expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.OA.B	Analyze patterns and relationships.	5.OA.B	Analyze patterns and relationships.
5.OA.B.3	Generate two numerical patterns using two given rules. Identify and analyze relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph them on a coordinate plane.	5.OA.B.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5.NBT - Numeric Reasoning: Base Ten Arithmetic

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.NBT	Numeric Reasoning: Base Ten Arithmetic	5.NBT	Number & Operations in Base Ten
5.NBT.A	Understand the place value system.	5.NBT.A	Understand the place value system.
5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
5.NBT.A.2	Use whole number exponents to denote powers of 10 and explain the patterns in placement of digits that occur when multiplying and/or dividing whole numbers and decimals by powers of 10.	5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.
5.NBT.A.3	Read, write, and compare decimals to thousandths.	5.NBT.A.3	Read, write, and compare decimals to thousandths. --(3.a) Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$. --(3.b) Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
5.NBT.A.4	Use place value understanding to round decimals to any place.	5.NBT.A.4	Use place value understanding to round decimals to any place.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.NBT.B	Perform operations with multi-digit whole numbers and with decimals to hundredths.	5.NBT.B	Perform operations with multi-digit whole numbers and with decimals to hundredths.
5.NBT.B.5	Fluently multiply multi-digit whole numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.	5.NBT.B.5	Fluently multiply multi-digit whole numbers using the standard algorithm.
5.NBT.B.6	Use a variety of representations and strategies to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.	5.NBT.B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
5.NBT.B.7	Use a variety of representations and strategies to add, subtract, multiply, and divide decimals to hundredths. Relate the strategy to a written method and explain the reasoning used.	5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5.NF - Numeric Reasoning: Fractions

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.NF	Numeric Reasoning: Fractions	5.NF	Number & Operations—Fractions
5.NF.A	Use equivalent fractions as a strategy to add and subtract fractions.	5.NF.A	Use equivalent fractions as a strategy to add and subtract fractions.
5.NF.A.1	Add and subtract fractions with unlike denominators, including common fractions larger than one and mixed numbers.	5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)
5.NF.A.2	Solve problems in authentic contexts involving addition and subtraction of fractions with unlike denominators, including common fractions larger than one and mixed numbers.	5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7 by observing that 3/7 < 1/2.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.NF.B	Apply and extend previous understandings of multiplication and division.	5.NF.B	Apply and extend previous understandings of multiplication and division.
5.NF.B.3	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve problems in authentic contexts involving division of whole numbers that result in answers that are common fractions or mixed numbers.	5.NF.B.3	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
5.NF.B.4	Apply and extend previous understanding and strategies of multiplication to multiply a fraction or whole number by a fraction. Multiply fractional side lengths to find areas of rectangles, and represent fractional products as rectangular areas.	5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. --(4.a) Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.) --(4.b) Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.NF.B.5	Apply and extend previous understandings of multiplication and division to represent and calculate multiplication and division of fractions. Interpret multiplication as scaling (resizing) by comparing the size of products of two factors.	5.NF.B.5	Apply and extend previous understandings of multiplication and division to multiply and divide fractions. Interpret multiplication as scaling (resizing) by: --(5.a) Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. --(5.b) Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1.
5.NF.B.6	Solve problems in authentic contexts involving multiplication of common fractions and mixed numbers.	5.NF.B.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions, including solving problems in authentic contexts.	5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) --(7.a) Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4 and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. --(7.b) Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. --(7.c) Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5.GM - Geometric Reasoning and Measurement

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.GM	Geometric Reasoning and Measurement	5.G	Geometry
5.GM.A	Graph points on the coordinate plane to solve real world and mathematical problems.	5.G.A	Graph points on the coordinate plane to solve real world and mathematical problems.
5.GM.A.1	Graph and name coordinate points in the first quadrant using the standard (x, y) notation. Understand the coordinate points values represent the distance traveled along the horizontal x-axis and vertical y-axis.	5.G.A.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
5.GM.A.2	Represent authentic contexts and mathematical problems by graphing points in the first quadrant of the coordinate plane. Interpret the meaning of the coordinate values based on the context of a given situation.	5.G.A.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.GM.B	Classify two-dimensional figures into categories based on their properties.	5.G.B	Classify two-dimensional figures into categories based on their properties.
[n/a]	*[Merged with 5.GM.B.3]*	*5.G.B.3*	*Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
5.GM.B.3	Classify two-dimensional figures within a hierarchy based on their geometrical properties, and explain the relationship across and within different categories of these figures.	5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.GM.C	Convert like measurement units within a given measurement system.	5.MD.A	Convert like measurement units within a given measurement system.
5.GM.C.4	Convert between different-sized standard measurement units within a given measurement system. Use these conversions in solving multi-step problems in authentic contexts.	5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step real world problems.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.GM.D	Geometric measurement: understand concepts of volume.	5.MD.C	Geometric measurement: understand concepts of volume.
5.GM.D.5	Recognize that volume is a measurable attribute of solid figures.	5.MD.C.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. --(3.a) A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. --(3.b) A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
5.GM.D.6	Measure the volume of a rectangular prism by counting unit cubes using standard and nonstandard units.	5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
5.GM.D.7	Relate volume of rectangular prisms to the operations of multiplication and addition. Solve problems in authentic contexts involving volume using a variety of strategies.	5.MD.C.5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. --(5.a) Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent three-fold whole-number products as volumes, e.g., to represent the associative property of multiplication. --(5.b) Apply the formulas V =(l)(w)(h) and V = (b)(h) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. --(5.c) Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
			overlapping parts, applying this technique to solve real world problems.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5.DR - Data Reasoning

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.DR	Data Reasoning	5.MD	Measurement and Data
5.DR.A	Pose investigative questions and collect/consider data.		
5.DR.A.1	Generate questions to investigate situations within the classroom, school or community. Determine strategies for collecting or considering data involving operations with fractions for this grade that can naturally answer questions by using information presented in line plots.		[new content]

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
5.DR.B	Analyze, represent, and interpret data.	5.MD.B	Represent and interpret data.
5.DR.B.2	Analyze graphical representations and describe the distribution of the numerical data through line plots or categorical data through bar graphs. Interpret information presented to answer investigative questions.	5.MD.B.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5G: Grade 6 Math Crosswalk (2021)

6.AEE - Algebraic Reasoning: Expressions and Equations

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.AEE	Algebraic Reasoning: Expressions and Equations	6.EE	Expressions & Equations
6.AEE.A	Apply and extend previous understandings of arithmetic to algebraic expressions.	6.EE.A	Apply and extend previous understandings of arithmetic to algebraic expressions.
6.AEE.A.1	Write and evaluate numerical expressions involving whole-number bases and exponents.	6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.
6.AEE.A.2	Write, read, and evaluate expressions in which letters stand for numbers. Apply knowledge of common mathematical terms to move between the verbal and mathematical forms of an expression including expressions that arise from authentic contexts.	6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers. --(2.a) Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y. --(2.b) Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. --(2.c) Evaluate expressions at specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s^3 and A = 6 s^2 to find the volume and surface area of a cube with sides of length s = 1/2.
6.AEE.A.3	Apply the properties of operations to generate equivalent expressions and to determine when two expressions are equivalent.	6.EE.A.3	Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.
[n/a]	*Merge with 6.AEE.A.3*	*6.EE.A.4*	*Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.AEE.B	Reason about and solve one-variable equations and inequalities.	6.EE.B	Reason about and solve one-variable equations and inequalities.
6.AEE.B.4	Understand solving an equation or inequality as a process of answering which values from a specified set, if any, make the equation or inequality true. Use substitution to determine which number(s) in a given set make an equation or inequality true.	6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6.AEE.B.5	Use variables to represent numbers and write expressions when solving problems in authentic contexts.	6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.AEE.B.6	Write and solve equations of the form x + p = q and px = q in problems that arise from authentic contexts for cases in which p, q and x are all nonnegative rational numbers.	6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.
6.AEE.B.7	Write inequalities of the form x > c and x < c to represent constraints or conditions to solve problems in authentic contexts. Describe and graph on a number line solutions of inequalities of the form x > c and x < c.	6.EE.B.8	Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.AEE.C	Represent and analyze quantitative relationships between dependent and independent variables.	6.EE.C	Represent and analyze quantitative relationships between dependent and independent variables.
6.AEE.C.8	Use variables to represent and analyze two quantities to solve problems in authentic contexts. Including those that change in relationship to one another; write an equation to express one quantity in terms of the other quantity.	6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

6.RP - Proportional Reasoning: Ratios and Proportions

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.RP	Proportional Reasoning: Ratios and Proportions	6.RP	Ratios & Proportional Relationships
6.RP.A	Understand ratio concepts and use ratio reasoning to solve problems.	6.RP.A	Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.A.1	Understand the concept of a ratio in authentic contexts, and use ratio language to describe a ratio relationship between two quantities.	6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
6.RP.A.2	Understand the concept of a unit rate in authentic contexts and use rate language in the context of a ratio relationship.	6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0 (b not equal to zero), and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)
6.RP.A.3	Use ratio and rate reasoning to solve problems in authentic contexts that use equivalent ratios, unit rates, percents, and/or measurement units.	6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. --(3.a) Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. --(3.b) Solve unit rate problems including those involving unit pricing and constant speed. For example, If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? --(3.c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole given a part and the percent. --(3.d) Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

6.NS - Numeric Reasoning: Number Systems

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.NS	Numeric Reasoning: Number Systems	6.NS	The Number System
6.NS.A	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	6.NS.A.1	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
6.NS.A.1	Represent, interpret, and compute quotients of fractions to solve problems in authentic contexts involving division of fractions by fractions.	6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.NS.B	Compute fluently with multi-digit numbers and find common factors and multiples.	6.NS.B	Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.B.2	Fluently divide multi-digit numbers using accurate, efficient, and flexible strategies and algorithms based on place value and properties of operations.	6.NS.B.2	Fluently divide multi-digit numbers using the standard algorithm.
6.NS.B.3	Fluently add, subtract, multiply, and divide positive rational numbers using accurate, efficient, and flexible strategies and algorithms.	6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
6.NS.B.4	Determine greatest common factors and least common multiples using a variety of strategies. Apply the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.	6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.NS.C	Apply and extend previous understandings of numbers to the system of rational numbers.	6.NS.C	Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in authentic contexts, explaining the meaning of zero in each situation.	6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6.NS.C.6	Represent a rational number as a point on the number line. Extend number line diagrams and coordinate axes to represent points on the line and in the coordinate plane with negative number coordinates.	6.NS.C.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. --(6.a) Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. --(6.b) Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. --(6.c) Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.NS.C.7	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. Write, interpret, and explain statements of order for rational numbers and absolute value in authentic applications.	6.NS.C.7	Understand ordering and absolute value of rational numbers. --(7.a) Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. --(7.b) Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C. --(7.c) Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars. --(7.d) Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.
6.NS.C.8	Graph points in all four quadrants of the coordinate plane to solve problems in authentic contexts. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

6.GM Geometric Reasoning and Measurement

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.GM	Geometric Reasoning and Measurement	6.G	Geometry
6.GM.A	Solve real-world and mathematical problems involving area, surface area, and volume.	6.G.A	Solve real-world and mathematical problems involving area, surface area, and volume.
6.GM.A.1	Find the area of triangles, quadrilaterals, and other polygons by composing into rectangles or decomposing into triangles and other shapes. Apply these techniques to solve problems in authentic contexts.	6.G.A.1	Find area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
6.GM.A.2	Find the volume of a right rectangular prism with fractional edge lengths by filling it with unit cubes of appropriate unit fraction edge lengths. Connect and apply to the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths to solve problems in authentic contexts.	6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
6.GM.A.3	Draw polygons in the four quadrant coordinate plane given coordinates for the vertices and find the length of a side. Apply these techniques to solve problems in authentic contexts.	6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
6.GM.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures, including those from authentic contexts.	6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

6.DR - Data Reasoning

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.DR	Data Reasoning	6.SP	Statistics and Probability
6.DR.A	Formulate Statistical Investigative Questions		Original CCSS
6.DR.A.1	Formulate and recognize statistical investigative questions as those that anticipate changes in descriptive data related to the question and account for it in the answers.	6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.DR.B	Collect and Consider Data		Original CCSS
6.DR.B.2	Collect and record data with technology to identify and describe the characteristics of numerical data sets using quantitative measures of center and variability.	6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.DR.C	Analyze, summarize, and describe data		Original CCSS
6.DR.C.3	Analyze data representations and describe measures of center and variability of quantitative data using appropriate displays.	6.SP.A.3 6.SP.B.4	(A.3) Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (B.4) Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
6.DR.D	Interpret data and answer investigative questions		Original CCSS
6.DR.D.4	Interpret quantitative measures of center to describe differences between groups from data collected to answer investigative questions.	6.SP.B.5	Summarize numerical data sets in relation to their context, such as by: --(5.a) Reporting the number of observations. --(5.b) Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. --(5.c) Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. --(5.d) Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5H: Grade 7 Math Crosswalk (2021)

7.AEE - Algebraic Reasoning: Expressions and Equations

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.AEE	Algebraic Reasoning: Expressions and Equations	7.EE	Expressions & Equations
7.AEE.A	Use properties of operations to generate equivalent expressions.	7.EE.A	Use properties of operations to generate equivalent expressions.
7.AEE.A.1	Identify and write equivalent expressions with rational numbers by applying associative, commutative, and distributive properties.	7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
7.AEE.A.2	Understand that rewriting an expression in different forms in a contextual problem can show how quantities are related.	7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.AEE.B	Solve mathematical problems in authentic contexts using numerical and algebraic expressions and equations.	7.EE.B	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
7.AEE.B.3	Write and solve problems in authentic contexts using expressions and equations with positive and negative rational numbers in any form. Contexts can be limited to those that can be solved with one or two-step linear equations.	7.EE.B.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations as strategies to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
7.AEE.B.4	Use variables to represent quantities and construct one- and two-step linear inequalities with positive rational numbers to solve authentic problems by reasoning about the quantities.	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. --(4.a) Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? --(4.b) Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example, As a salesperson, you are paid $50 per week plus $3 per sale. This week you want

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
			your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

7.RP - Proportional Reasoning: Ratios and Proportions

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.RP	Proportional Reasoning: Ratios and Proportions	7.RP	Ratios & Proportional Relationships
7.RP.A	Analyze proportional relationships and use them to solve mathematical problems in authentic contexts.	7.RP.A	Analyze proportional relationships and use them to solve real-world and mathematical problems.
7.RP.A.1	Solve problems in authentic contexts involving unit rates associated with ratios of fractions.	7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.
7.RP.A.2	Recognize and represent proportional relationships between quantities in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Identify the constant of proportionality (unit rate) within various representations.	7.RP.A.2	Recognize and represent proportional relationships between quantities. --(2.a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. --(2.b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. --(2.c) Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. --(2.d) Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.RP.A.3	Use proportional relationships to solve ratio and percent problems in authentic contexts.	7.RP.A.3	Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.RP.B	Investigate chance processes and develop, use, and evaluate probability models.	7.SP.C	Investigate chance processes and develop, use, and evaluate probability models.
7.RP.B.4	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Represent probabilities as fractions, decimals, and percents.	7.SP.C.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
7.RP.B.5	Use experimental data and theoretical probability to make predictions. Understand the probability predictions may not be exact.	7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7.RP.B.6	Develop a probability model and use it to find probabilities of events. Compare theoretical and experimental probabilities and explain possible sources of discrepancy if any exists.	7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. --(7.a) Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. --(7.b) Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
			for the spinning penny appear to be equally likely based on the observed frequencies?
7.RP.B.7	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.	7.SP.C.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. --(8.a) Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. --(8.b) Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. --(8.c) Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

7.NS - Numeric Reasoning: Number Systems

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.NS	Numeric Reasoning: Number Systems	7.NS	The Number System
7.NS.A	Apply and extend previous understandings of operations with fractions.	7.NS.A	Apply and extend previous understandings of operations with fractions.
7.NS.A.1	Apply and extend previous understandings of addition, subtraction and absolute value to add and subtract rational numbers in authentic contexts. Understand subtraction as adding the additive inverse, p – q = p + (–q).	7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. --(1.a) Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. --(1.b) Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. --(1c) Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. --(1.d) Apply properties of operations as strategies to add and subtract rational numbers.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. Interpret operations of rational numbers solving problems in authentic contexts.	7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. --(2.a) Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. --(2.b) Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. --(2.c) Apply properties of operations as strategies to multiply and divide rational numbers. --(2.d) Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
7.NS.A.3	Understand that equivalent rational numbers can be written as fractions, decimals and percents.	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

7.GM - Geometric Reasoning and Measurement

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.GM	Geometric Reasoning and Measurement	7.G	Geometry
7.GM.A	Draw, construct, and describe geometrical figures and describe the relationships between them.	7.G.A	Draw construct, and describe geometrical figures and describe the relationships between them.
7.GM.A.1	Solve problems involving scale drawings of geometric figures. Reproduce a scale drawing at a different scale and compute actual lengths and areas from a scale drawing.	7.G.A.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
7.GM.A.2	Draw triangles from three measures of angles or sides. Understand the possible side lengths and angle measures that determine a unique triangle, more than one triangle, or no triangle.	7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
[n/a]	*Merge with 7.GM.A.2*	*7.G.A.3*	*Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.*

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.GM.B	Solve mathematical problems in authentic contexts involving angle measure, area, surface area, and volume.	7.G.B	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
7.GM.B.3	Understand the relationship between area and circumference of circles. Choose and use the appropriate formula to solve problems with radius, diameter, circumference and area of circles.	7.G.B.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
7.GM.B.4	Apply facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to determine an unknown angle in a figure.	7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
7.GM.B.5	Solve problems in authentic contexts involving two- and three-dimensional figures. Given formulas, calculate area, volume and surface area.	7.G.B.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

7.DR - Data Reasoning

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.DR	Data Reasoning	7.SP	Statistics and Probability
7.DR.A	Formulate Statistical Investigative Questions		Original CCSS
7.DR.A.1	Formulate summary, comparative investigative questions to gain information about a population and that a sample is valid only if the sample is representative of that population.	7.SP.A.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.DR.B	Collect and Consider Data		Original CCSS
7.DR.B.2	Collect or consider data from a random sample to compare and draw inferences about a population with an unknown characteristic of interest.	7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.DR.C	Analyze, summarize, and describe data		Original CCSS
7.DR.C.3	Analyze two data distributions visually to compare multiple measures of center and variability.	7.SP.B.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
7.DR.D	Interpret data and answer investigative questions		Original CCSS
7.DR.D.4	Interpret measures of center and measures of variability for numerical data from random samples to compare between two populations, and to answer investigative questions.	7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5I: Grade 8 Math Crosswalk (2021)

8.AEE - Algebraic Reasoning: Expressions and Equations

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.AEE	Algebraic Reasoning: Expressions and Equations	8.EE	Expressions & Equations
8.AEE.A	Expressions and Equations Work with radicals and integer exponents.	8.EE.A	Expressions and Equations Work with radicals and integer exponents.
8.AEE.A.1	Apply the properties of integer exponents using powers of 10 to generate equivalent numerical expressions.	8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 3^2 × 3^(–5) = 3^(–3) = 1/(3^3) = 1/27.
8.AEE.A.2	Represent solutions to equations using square root and cube root symbols.	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form x^2 = p and x^3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.
8.AEE.A.3	Estimate very large or very small quantities using scientific notation with a single digit times an integer power of ten.	8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 × 10^8 and the population of the world as 7 × 10^9, and determine that the world population is more than 20 times larger.
8.AEE.A.4	Perform operations with numbers expressed in scientific notation.	8.EE.A.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.AEE.B	Understand the connections between proportional relationships, lines, and linear equations.	8.EE.B	Understand the connections between proportional relationships, lines, and linear equations.
8.AEE.B.5	Graph proportional relationships in authentic contexts. Interpret the unit rate as the slope of the graph, and compare two different proportional relationships represented in different ways.	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
8.AEE.B.6	Write the equation for a line in slope intercept form y = mx + b, where m and b are rational numbers, and explain in context why the slope m is the same between any two distinct points.	8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y =mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.AEE.C	Analyze and solve linear equations and pairs of simultaneous linear equations.	8.EE.C	Analyze and solve linear equations and pairs of simultaneous linear equations.
8.AEE.C.7	Solve linear equations with one variable including equations with rational number coefficients, with the variable on both sides, or whose solutions require using the distributive property and/or combining like terms.	8.EE.C.7	Solve linear equations in one variable. --(7.a) Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). --(7.b) Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8.AEE.C.8	Find, analyze, and interpret solutions to pairs of simultaneous linear equations using graphs or tables.	8.EE.C.8	Analyze and solve pairs of simultaneous linear equations. --(8.a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. --(8.b) Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. --(8.c) Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

8.AFN - Algebraic Reasoning: Functions

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.AFN	Algebraic Reasoning: Functions	8.F	Functions
8.AFN.A	Define, evaluate, and compare functions.	8.F.A	Define, evaluate, and compare functions.
8.AFN.A.1	Understand in authentic contexts, that the graph of a function is the set of ordered pairs consisting of an input and a corresponding output.	8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)
8.AFN.A.2	Compare the properties of two functions represented algebraically, graphically, numerically in tables, or verbally by description.	8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
8.AFN.A.3	Understand and identify linear functions, whose graph is a straight line, and identify examples of functions that are not linear.	8.F.A.3	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.AFN.B	Use functions to model relationships between quantities.	8.F.B	Use functions to model relationships between quantities.
8.AFN.B.4	Construct a function to model a linear relationship in authentic contexts between two quantities.	8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
8.AFN.B.5	Describe qualitatively the functional relationship between two quantities in authentic contexts by analyzing a graph.	8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

8.NS - Numeric Reasoning: Number Systems

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.NS	Numeric Reasoning: Number Systems	8.NS	DOMAIN: The Number System
8.NS.A	Know that there are numbers that are not rational, and approximate them by rational numbers.	8.NS.A	Know that there are numbers that are not rational, and approximate them by rational numbers.
8.NS.A.1	Know that real numbers that are not rational are called irrational.	8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
8.NS.A.2	Use rational approximations of irrational numbers to compare size and locate on a number line.	8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of √2 (square root of 2), show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

8.GM - Geometric Reasoning and Measurement

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.GM	Geometric Reasoning and Measurement	8.G	Geometry
8.GM.A	Understand congruence and similarity using physical models, transparencies, or geometry software.	8.G.A	Understand congruence and similarity using physical models, transparencies, or geometry software.
8.GM.A.1	Verify experimentally the properties of rotations, reflections, and translations.	8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations: --(1.a) Lines are taken to lines, and line segments to line segments of the same length. --(1.b) Angles are taken to angles of the same measure. --(1.c) Parallel lines are taken to parallel lines.
8.GM.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.	8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
8.GM.A.3	Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.	8.G.A.3	Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.
8.GM.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and/or dilations.	8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
8.GM.A.5	Use informal arguments to establish facts about interior and exterior angles of triangles and angles formed by parallel lines cut with a transversal.	8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
			the three angles appear to form a line, and give an argument in terms of transversals why this is so.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.GM.B	Understand and apply the Pythagorean Theorem.	8.G.B	Understand and apply the Pythagorean Theorem.
8.GM.B.6	Distinguish between applications of the Pythagorean Theorem and its Converse in authentic contexts.	8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.
8.GM.B.7	Apply the Pythagorean Theorem in authentic contexts to determine unknown side lengths in right triangles.	8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8.GM.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.GM.C	Solve mathematical problems in authentic contexts involving volume of cylinders, cones, and spheres.	8.G.C	Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
8.GM.C.9	Choose and use the appropriate formula for the volume of cones, cylinders, and spheres to solve problems in authentic contexts.	8.G.C.9	Know the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

8.DR - Data Reasoning

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.DR	Data Reasoning	8.SP	Statistics and Probability
8.DR.A	Formulate Statistical Investigative Questions		Original CCSS
8.DR.A.1	Formulate statistical investigative questions to articulate research topics and uncover patterns of association seen in bivariate categorical data.	8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.DR.B	Collect and Consider Data		Original CCSS
8.DR.B.2	Collect or consider data using surveys and measurements to capture patterns of association, and critically analyze data collection methods.	8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.DR.C	Analyze, summarize, and describe data		Original CCSS
8.DR.C.3	Analyze patterns of association between two quantitative or categorical variables and reason about distributions to compare groups.	8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
8.DR.D	Interpret data and answer investigative questions		Original CCSS
8.DR.D.4	Interpret scatter plots for bivariate quantitative data to investigate patterns of association between two quantities to answer investigative questions.	8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5J: High School Algebra Crosswalk (2021)

HS.AEE - Algebraic Reasoning: Expressions and Equations

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.AEE	Algebraic Reasoning: Expressions and Equations	HSA	High School Algebra
HS.AEE.A	Use algebraic reasoning to rewrite expressions in equivalent forms.	[n/a]	[new cluster]
HS.AEE.A.1	Interpret an expression which models a quantity by viewing one or more of its parts as a single entity. Reason about how changes in parts of the expression impact the whole, and vice versa.	HSA.SSE.A.1	Interpret expressions that represent a quantity in terms of its context.* ---HSA.SSE.A.1.A Interpret parts of an expression, such as terms, factors, and coefficients. - ---HSA.SSE.A.1.B Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.
HS.AEE.A.2	Create and recognize an equivalent form of an expression to understand the quantity represented in an authentic context.	HSA.SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* --HSA.SSE.B.3.A Factor a quadratic expression to reveal the zeros of the function it defines. --HSA.SSE.B.3.B Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. --HSA.SSE.B.3.C Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
HS.AEE.A.3	Rearrange formulas and equations to highlight a specific quantity.	HSA.CED.A.4 HSF.IF.C.8	(A.4) Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. (C.8) Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.AEE.B	Use algebraic reasoning to find solutions to an equation, inequality, and systems of equations or inequalities.	[n/a]	[new cluster]
HS.AEE.B.4	Define variables and create equations with two or more variables to represent relationships between quantities in order to solve problems in authentic contexts.	HSA.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
HS.AEE.B.5	Define variables and create inequalities with one or more variables and use them to solve problems in authentic contexts.	HSA.CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
HS.AEE.B.6	Solve systems of linear equations and systems of linear inequalities in authentic contexts through reasoning, algebraic means, or strategically using technology.	HSA.REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.AEE.C	Analyze the structure of an equation or inequality to determine an efficient strategy to find and justify a solution.	[n/a]	[new cluster]
HS.AEE.C.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities; interpret solutions as viable or nonviable options in authentic contexts.	HSA.CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
HS.AEE.C.8	Construct a viable argument to justify a method for solving equations or inequalities.	HSA.REI.C.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.AEE.D	Make predictions in different applications using expressions, equations, and inequalities to analyze authentic contexts.	[n/a]	[new cluster]
HS.AEE.D.9	Understand that the solution to an equation in two variables is a set of points in the coordinate plane that form a curve, which could be a line.	HSA.REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
HS.AEE.D.10	Recognize and explain why the point(s) of intersection of the graphs of f(x) and g(x) are solutions to the equation f(x)=g(x). Interpret the meaning of the coordinates of these points in authentic contexts.	HSA.REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
HS.AEE.D.11	Graph and explain why the points in a half plane are solutions to a linear inequality and the solutions to a system of inequalities are the points in the intersection of corresponding half planes. Interpret the meaning of the coordinates of these points in authentic contexts.	HSA.REI.D.12 HSA.REI.B.3	(D.12) Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (B.3) Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

HS.AFN - Algebraic Reasoning: Functions

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.AFN	Algebraic Reasoning: Functions	HSF	High School Functions
HS.AFN.A	Describe functions by using both symbolic and graphical representations.	[n/a]	[new cluster]
HS.AFN.A.1	Understand a function as a rule that assigns a unique output for every input and that functions model situations where one quantity determines another.	HSF.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
HS.AFN.A.2	Use function notation and interpret statements that use function notation in terms of the context and the relationship it describes.	HSF.IF.A.2 HSF.BF.A.1	(A.2) Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (A.1) Write a function that describes a relationship between two quantities.*
HS.AFN.A.3	Calculate and interpret the average rate of change of a function over a specified interval.	HSF.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.AFN.B	Compare and relate functions using common attributes.	[n/a]	[new cluster]
HS.AFN.B.4	Compare properties of two functions using multiple representations. Distinguish functions as members of the same family using common attributes.	HSF.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
HS.AFN.B.5	Relate the domain of a function to its graph and to its context.	HSF.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.AFN.C	Represent functions graphically and interpret key features in terms of the equivalent symbolic representation.	[n/a]	[new cluster]
HS.AFN.C.6	Interpret key features of functions, from multiple representations, and conversely predict features of functions from knowledge of context.	HSF.IF.B.4	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
HS.AFN.C.7	Graph functions using technology to show key features.	HSF.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* --HSF.IF.C.7.A Graph linear and quadratic functions and show intercepts, maxima, and minima. --HSF.IF.C.7.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. --HSF.IF.C.7.C Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. --HSF.IF.C.7.D (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. --HSF.IF.C.7.E Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.AFN.D	Model a wide variety of authentic situations using functions through the process of making and changing assumptions, assigning variables, and finding solutions to contextual problems.	[n/a]	[new cluster]
HS.AFN.D.8	Model situations involving arithmetic patterns. Use a variety of representations such as pictures, graphs, or an explicit formula to describe the pattern.	HSF.BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*
HS.AFN.D.9	Identify and interpret the effect on the graph of a function when the equation has been transformed.	HSF.BF.B.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
HS.AFN.D.10	Explain why a situation can be modeled with a linear function, an exponential function, or neither. In a given model, explain the meaning of coefficients and features of functions used, such as slope for a linear model.	HSF.LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. --HSF.LE.A.1.A Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. --HSF.LE.A.1.B Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. --HSF.LE.A.1.C Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

HS.NQ - Numeric Reasoning: Number and Quantity

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.NQ	Numeric Reasoning: Number and Quantity	HSN	High School Number
HS.NQ.A	Understand and apply the real number system.	[n/a]	[new cluster]
HS.NQ.A.1	Use reasoning to establish properties of positive integer exponents. Extend the definition of exponentiation to include negative and rational exponents so as to be consistent with these properties. Utilize exponentiation to model authentic contexts.	HSN.RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.
HS.NQ.A.2	Compare real numbers presented through different representations, including both rational and irrational numbers. Apply comparisons in authentic contexts.	[n/a]	[new content]

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.NQ.B	Attend to units of measurement needed to solve problems through quantitative reasoning and mathematical modeling.	[n/a]	[new cluster]
HS.NQ.B.3	Use reasoning to choose and interpret measurement units consistently in formulas, graphs, and data displays, as a way to understand problems and to guide the solution of multi-step problems.	HSN.Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
HS.NQ.B.4	Define, manipulate, and interpret appropriate quantities using rational and irrational numbers to authentically model situations and use reasoning to justify these choices.	HSN.Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
HS.NQ.B.5	Use reasoning to choose a level of accuracy appropriate to limitations on measurement when reporting quantities in modeling situations.	HSN.Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5K: High School Geometry Crosswalk (2021)

HS.GM - Geometric Reasoning and Measurement

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.GM	Geometric Reasoning and Measurement	HSG	High School Geometry
HS.GM.A	Apply geometric transformations to figures through analysis of graphs and understanding of functions.	[n/a]	[new cluster]
HS.GM.A.1	Apply definitions of rotations, reflections, and translations to transform a figure and map between two congruent figures in authentic contexts.	HSG.CO.A.2 HSG.CO.A.4 HSG.CO.A.5	(A.2) Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). (A.4) Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (A.5) Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.GM.A.2	Verify experimentally the properties of a dilation given a center and a scale factor. Solve problems in authentic contexts involving similar triangles or dilations.	HSG.SRT.B.5 HSG.SRT.A.1 HSG.SRT.A.2 HSG.SRT.A.3	(B.5) Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (A.1) Verify experimentally the properties of dilations given by a center and a scale factor: HSG.SRT.A.1a A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. HSG.SRT.A.1b The dilation of a line segment is longer or shorter in the ratio given by the scale factor. (A.2) Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (A.3) Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
HS.GM.A.3	Use the slopes of segments and the coordinates of the vertices of triangles, parallelograms, and trapezoids to solve problems in authentic contexts.	HSG.GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
HS.GM.A.4	Use definitions of transformations and symmetry relationships to justify the solutions of problems in authentic contexts.	HSG.CO.A.1	Use definitions of geometric figures and geometric relationships to justify the solutions of problems.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.GM.B	Construct and communicate geometric arguments through use of proofs, logical reasoning, and geometric technology.	[n/a]	[new cluster]
HS.GM.B.5	Apply and justify triangle congruence and similarity theorems in authentic contexts.	HSG.CO.B.7 HSG.CO.B.8	(B.7) Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (B.8) Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
HS.GM.B.6	Justify theorems of line relationships, angles, triangles, and parallelograms; and use them to solve problems in authentic contexts.	HSG.CO.C.9 HSG.CO.C.10	(C.9) Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (C.10) Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
HS.GM.B.7	Perform geometric constructions with a variety of tools and methods.	HSG.CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.GM.C	Solve problems and interpret solutions of area and volume of shapes by applying concepts of congruence, similarity, symmetry in authentic contexts.	[n/a]	[new cluster]
HS.GM.C.8	Solve authentic modeling problems using area formulas for triangles, parallelograms, trapezoids, regular polygons, and circles.	HSG.GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
HS.GM.C.9	Use volume and surface area formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems and apply to authentic contexts.	HSG.GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
HS.GM.C.10	Use geometric shapes, their measures, and their properties to describe real world objects, and solve related authentic modeling and design problems.	HSG.MG.A.1 HSG.MG.A.3	(A.1) Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* (A.3) Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
HS.GM.C.11	Apply concepts of density based on area and volume in authentic modeling situations.	HSG.MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.GM.D	Apply concepts of right triangle trigonometry in authentic contexts to solve problems and interpret solutions.	[n/a]	[new cluster]
HS.GM.D.12	Apply sine, cosine, and tangent ratios, and the Pythagorean Theorem, to solve problems in authentic contexts.	HSG.SRT.C.8 HSG.SRT.C.6 HSG.SRT.C.7	(C.8) Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* (C.6) Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (C.7) Explain and use the relationship between the sine and cosine of complementary angles.
HS.GM.D.13	Apply the Pythagorean Theorem in authentic contexts, and develop the standard form for the equation of a circle.	HSG.GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
HS.GM.D.14	Use the coordinate plane to determine parallel and perpendicular relationships, and the distance between points.	HSG.GPE.B.4	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

5L: High School Data Reasoning and Statistics Crosswalk (2021)

HS.DR - Data Reasoning and Probability

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.DR	**Data Reasoning and Probability**	HSS	High School Statistics and Probability
HS.DR.A	Formulate Statistical Investigative Questions	[n/a]	[new cluster]
HS.DR.A.1	Formulate multivariable statistical investigative questions and determine how data from samples can be collected and analyzed to provide an answer.	HSS.IC.A.1	Understand the process of statistical reasoning, formulate questions, collect, analyze, and interpret data to answer statistical investigative questions.
HS.DR.A.2	Formulate summative, comparative, and associative statistical investigative questions for surveys, observational studies, and experiments using primary or secondary data.	HSS.IC.B.3	Recognize the difference between sample surveys, experiments and observational studies and understand the role of randomization in each.
HS.DR.A.3	Formulate inferential statistical investigative questions regarding causality and prediction from correlation.	HSS.ID.C.9	Distinguish between correlation and causation.
HS.DR.A.4	Use mathematical and statistical reasoning to formulate questions about data to evaluate conclusions and assess risks.	HSS.IC.B.6	Evaluate reports based on data.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.DR.B	Collect and Consider Data	[n/a]	[new cluster]
HS.DR.B.5	Articulate what constitutes good practice in designing a sample survey, an experiment, and an observational study. Understand issues of bias and confounding variables in a study and their implications for interpretation.	[n/a]	[new content]
HS.DR.B.6	Distinguish and choose between surveys, observational studies, and experiments to design an appropriate data collection that answers an investigative question of interest.	HSS.IC.B.4	Use data from a sample survey to estimate a population parameter.
HS.DR.B.7	Apply an appropriate data collection plan when collecting primary data or selecting secondary data for the statistical investigative question of interest.	HSS.ID.B.5	Analyze the association between two categorical variables by using two-way tables and comparative bar graphs.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.DR.C	Analyze, summarize, and describe data	[n/a]	[new cluster]
HS.DR.C.8	Identify appropriate ways to summarize and then represent the distribution of univariate and bivariate data multiple ways with graphs and/or tables. Use technology to present data that supports interpretation of tabular and graphical representations.	HSS.ID.A.1 HSS.ID.B.6	(A.1) Represent the distribution of data multiple ways with plots on the real number line. (B.6) Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
HS.DR.C.9	Use statistics appropriate to the shape of the data distribution to compare the center and spread of two or more different data sets.	HSS.ID.A.2 HSS.ID.A.4	(A.2) Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets. (A.4)Use the mean and standard deviation of an approximately normally distributed data set to estimate population percentages.
HS.DR.C.10	Use data to compare two groups, describe sample variability, and decide if differences between parameters are significant based on the statistics.	HSS.ID.A.3 HSS.IC.B.5	(ID.A.3) Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (IC.B.5)Use data from a randomized experiment to compare two

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
			treatments to decide if differences between parameters are significant based on the statistics.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.DR.D	Interpret data and answer investigative questions	[n/a]	[new cluster]
HS.DR.D.11	Use statistical evidence from analyses to answer statistical investigative questions, and communicate the findings in a variety of formats (verbal, written, visual) to support informed data-based decisions.	HSS.ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
HS.DR.D.12	Articulate what it means for an outcome or an estimate of a population characteristic to be plausible or not plausible compared to chance variation.	HSS.ID.C.8	Compute, using technology, and interpret the correlation coefficient of a linear fit.
HS.DR.D.13	Use multivariate thinking to articulate how variables impact one another, and measure the strength of association using correlation coefficients for regression curves.	HSS.ID.C.9	Distinguish between correlation and causation.

OR INDEX	Standards Statement (2021)	CCSS INDEX	Previous Standards Statement (CCSS, 2010)
HS.DR.E	Understand independence and conditional probability and use them to interpret data	[n/a]	[new cluster]
HS.DR.E.14	Describe the possible outcomes for a situation as subsets of a sample space.	HSS.CP.A.1	Describe the possible outcomes for a situation as subsets of a sample space.
HS.DR.E.15	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	HSS.CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.