COMMON CORE STATE STANDARDS FOR

Mathematics (CCSSM)

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High School
Functions

OREGON DEPARTMENT OF EDUCATION

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

★Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.
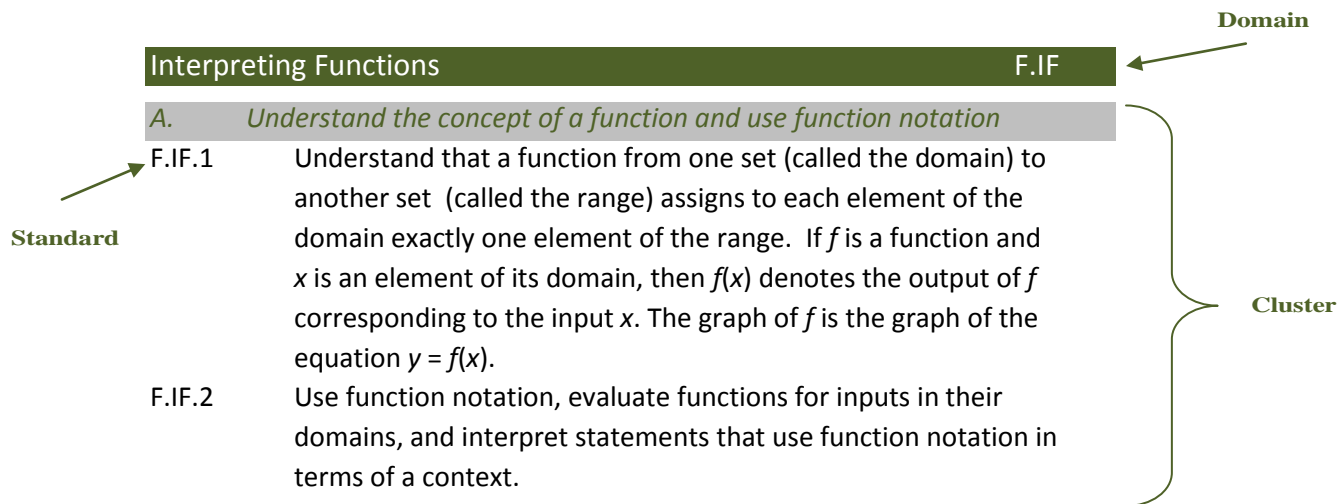
(+) The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards without a (+) symbol may also appear in courses intended for all students.

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How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Domain

Interpreting Functions	F.IF

A.   Understand the concept of a function and use function notation

Standard

F.IF.1   Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F.IF.2   Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Cluster

High School—Functions Overview

Interpreting Functions	IF

A.   Understand the concept of a function and use function notation.
B.   Interpret functions that arise in applications in terms of the context.
C.   Analyze functions using different representations.

Building Functions	BF

D.   Build a function that models a relationship between two quantities.
E.   Build new functions from existing functions.

Linear, Quadratic, and Exponential Models	LE

F.   Construct and compare linear and exponential models and solve problems.
G.   Interpret expressions for functions in terms of the situation they model.

Trigonometric Functions	TF

H.   Extend the domain of trigonometric functions using the unit circle.
I.   Model periodic phenomena with trigonometric functions.
J.   Prove and apply trigonometric identities.

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Mathematical Practices   HS.MP

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

HS.MP.1   Make sense of problems and persevere in solving them.
HS.MP.2   Reason abstractly and quantitatively.
HS.MP.3   Construct viable arguments and critique the reasoning of others.
HS.MP.4   Model with mathematics.
HS.MP.5   Use appropriate tools strategically.
HS.MP.6   Attend to precision.
HS.MP.7   Look for and make use of structure.
HS.MP.8   Look for and express regularity in repeated reasoning.

Interpreting Functions   F.IF

A.   Understand the concept of a function and use function notation

F.IF.1   Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F.IF.2   Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.3   Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*

B.   Interpret functions that arise in applications in terms of the context

F.IF.4   For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F.IF.5   Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F.IF.6   Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

C.   Analyze functions using different representations

F.IF.7   Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

    a.   Graph linear and quadratic functions and show intercepts, maxima, and minima.

    b.   Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

    c.   Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

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    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

    e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.IF.8   Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

    b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.*

F.IF.9   Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions   F.BF

D.   *Build a function that models a relationship between two quantities*

F.BF.1   Write a function that describes a relationship between two quantities.★

    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

    b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

    c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

F.BF.2   Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

E.   *Build new functions from existing functions*

F.BF.3   Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F.BF.4   Find inverse functions.

    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x–1) for x ≠1.*

    b. (+) Verify by composition that one function is the inverse of another.

    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

F.BF.5   (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

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Linear and Exponential Models ★   F.LE

F.   *Construct and compare linear and exponential models and solve problems*

F.LE.1   Distinguish between situations that can be modeled with linear functions and with exponential functions.

       a.   Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

       b.   Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

       c.   Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2   Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.3   Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.LE.4   For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

G.   *Interpret expressions for functions in terms of the situation they model*

F.LE.5   Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions   F.TF

H.   *Extend the domain of trigonometric functions using the unit circle*

F.TF.1   Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.2   Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F.TF.3   (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

F.TF.4   (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

I.   *Model periodic phenomena with trigonometric functions*

F.TF.5   Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

F.TF.6   (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F.TF.7   (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

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J.	*Prove and apply trigonometric identities*
F.TF.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
F.TF.9	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.