HIGHER EDUCATION COORDINATING COMMISSION

OREGON ADULT COLLEGE & CAREER READINESS STANDARDS:

Mathematics Handbook

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Preface

Greetings from the Leadership of Oregon's Adult Basic Skills Programs

After months of hard work among dedicated faculty, program directors, and the Adult Basic Skill (ABS) leadership team at CCWD, our statewide collaborative is pleased to announce the release of the Oregon Adult College and Career Readiness Standards (OACCRS) as the adopted standards for student learning outcomes alignment in Title II programs. ABS programs, including both English language acquisition and secondary credentialing, have long been aligned to the Oregon Adult Leaning Standards (OALS). The OALS provided a well-respected base and we have retained some of their most important elements as we plan for this transition together.

Adult Basic Skills programs play an important role in providing all Oregonians the skills they need for family self-sufficiency, fulfilling careers, community engagement, and continued education. As the primary source for skill building among adult learners, ABS programs help students make connections to the pathways that can lead to better jobs and an advancement of career goals. Our shared commitment to ongoing program improvement remains our greatest strength for serving Oregonians together. With the adoption of OACCRS, the transition from Oregon-focused standards to national standards will strategically position Title II programs to align implementation efforts with our partner initiatives, such as Perkins V, Career Pathways and the WIOA state plan. Additionally, OACCRS will provide a framework for helping both state & local partners coordinate services for reaching the Adult Attainment Goal in Oregon.

Over the course of the next year, CCWD will work closely with local program directors and ABS faculty to develop a two-year implementation plan that will support programs in making a successful transition to CCRS. Using existing resources that have already been aligned to national standards, training will provide access to a variety of instructional materials that build upon a shared understanding of new curricula for ESL and ABE/ASE/GED/AHSD instruction. A training platform will be designed to meet the professional development needs of Title II programs as we move toward implementing OACCRS as the primary standards in Oregon together.

The leadership of Oregon's Adult Basic Skills Programs is committed to providing a solid foundation for the success of all adult learners. Implementation of the OACCRS is a major step towards building that foundation.

Sincerely,

Donna Lewelling	*Darlene Geiger*
Oregon ABS State Director	OCABSD Chair (2018-19)

The Oregon Office of Community Colleges and Workforce Development (CCWD) wishes to acknowledge those who have contributed to the development of the Oregon Adult College and Career Readiness Standards Handbooks for Instruction. Their dedication, expertise, and excellent work are deeply appreciated.

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Introduction: Oregon's Evolving Adult Basic Skills Standards

Standards-based education has been an integral part of national and state initiatives to improve the quality of adult education and literacy programs for over a decade. Content standards define what learners should know and be able to do within a specific content area. Effective standards that are fully-implemented have powerful, positive impact on instruction and learning.

- Standards that are grounded in real expectations can prepare students for success in higher levels of education and employment.
- Agreed-upon standards can provide a common language for the field and guide professional development.
- Rigorous standards that clearly describe expectations for student outcomes can encourage educators to be accountable to stakeholders, students, and each other.
- Clear standards can be the basis for formative and summative assessments which measure student progress and program improvement.
- Standards that are comprehensive and coherent can tell educators how to focus and sequence curriculum and instruction.

Recognizing the value content standards could bring to adult education, the U.S. Department of Education Office of Vocational and Adult Education (OVAE)[1] launched a national standards effort in 2003. This initiative—*Promoting College and Career-Ready Standards in Adult Basic Education*[2]—sought to reinforce the connections between adult education, postsecondary education, and work by articulating the critical skills and knowledge adults must have for success in college, technical training, and careers. The initiative promised to produce a set of college and career readiness standards for adult education and a review of the alignment between the National Reporting System (NRS) and selected standards for adult education.

Oregon also recognized the contributions that content standards could make to the quality of Adult Basic Skills (ABS) programs when the Oregon Council for Adult Basic Skills Development (OCABSD) unanimously agreed in May of 2008 to support the development and implementation of content standards for adult learners. The subsequent process of developing and piloting standards was jointly led by directors, instructors, state staff, and national experts. Their product, the Oregon Adult Basic Skills (ABS) Learning Standards, was deeply rooted in Equipped for the Future (EFF)[3], a framework that outlined what was

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[1] In 2014, OVAE became the Office of Career, Technical and Adult Education (OCTAE).

[2] United States, U.S. Department of Education, Office of Vocational and Adult Education. (n.d.). *Promoting College and Career-Ready Standards in Adult Basic Education*. Retrieved July 1, 2019, from https://www2.ed.gov/about/offices/list/ovae/pi/AdultEd/factsh/promoting-college-career.pdf

[3] EFF Fundamentals. (n.d.). Retrieved July 1, 2019, from https://eff.clee.utk.edu/fundamentals/default.htm Equipped for the Future

important for adults to know and be able to do as lifelong learners, parents and family members, citizens and community members, and workers. After extensive piloting and revision, the Office of Community Colleges and Workforce Development (CCWD) published the *Read with Understanding Framework, the Listen Actively and Speak so Others Can Understand Framework, and the Use Math to Communicate and Solve Problems Framework* in 2010, followed by the *Write to Express Meaning Framework* in 2014.
From 2010 through 2015, the Oregon Professional Development System delivered extensive professional development opportunities for the Oregon Adult Basic Skills (ABS) Learning Standards through face-to-face training, including Orientations, Learning Circles, and Institutes. Beginning in 2016-2017, CCWD offered an online Orientation for Instructors and an online Orientation for Administrators.

From 2010 through 2015, the Oregon Professional Development System delivered extensive professional development opportunities for the Oregon Adult Basic Skills (ABS) Learning Standards through face-to-face training, including Orientations, Learning Circles, and Institutes. Beginning in 2016-2017, CCWD offered an online Orientation for Instructors and an online Orientation for Administrators.

In 2010, as ABS practitioners worked on standards for adult learners in Oregon, the National Governors Association (NGA) and the national Council of Chief State School Officers (CCSSO) published the Common Core State Standards for K-12 (CCSS) in English Language Arts/Literacy and Mathematics.[4]

These CCS Standards became the foundation for the first product from OVAE's initiative which was a report published in 2013, *College and Career Readiness Standards for Adult Education* (CCRS).[5] The report delineates sets of standards in English language arts/literacy and mathematics, specifically identifying what adults need to know to be genuinely ready for postsecondary education, training, and employment. In the introduction to the report, OVAE explained why it made sense to base college and career readiness standards for adult education in standards developed for K-12.

While the academic standards developed by states in recent decades reflected broad agreement among experts about what was desirable for students to learn, they did not necessarily identify what was *essential* for students to know to be prepared for the rigors of postsecondary training, work, or citizenship. It was not until the development of the Common Core State Standards (CCSS) in 2010—to date adopted by 46[6] states for K–12 programs—that such a consensus emerged. Based on evidence from a wide array of sources, including student performance data, academic research, assessment data, and results of large-scale surveys of postsecondary instructors and employers, the CCSS offer clear signposts indicating what is most important for college and career readiness (National Governors Association [NGA] 2010b, 2010c, pp. 91–93).[7]

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[4] Development Process. (n.d.). Retrieved July 1, 2019, from http://www.corestandards.org/about-the-standards/development-process/

[5] Pimentel, S. (2013, April). College and Career Readiness Standards for Adult Education (United States, U.S. Department of Education, Office of Vocational and Adult Education). Retrieved July 1, 2019, from https://lincs.ed.gov/publications/pdf/CCRStandardsAdultEd.pdf

[6] Map: Tracking the Common Core State Standards. (2015, June 29). Retrieved July 1, 2019, from https://www.edweek.org/ew/section/multimedia/map-states-academic-standards-common-core-or.html
As of September, 2017, 34 states and the District of Columbia maintained CCSS adoption and 11 have announced major revisions or replacements. One state, Minnesota, adopted CCSS only in English/Language Arts.

[7] Pimentel, S. (n.d.). College and Career Readiness Standards for Adult Education (United States, U.S. Department of Education, Office of Vocational and Adult Education). Salem, OR.

OVAE appointed a panel of experts from adult education, community colleges, and career and technical training to adapt the CCSS for adult education. The panel was charged with reducing the extensive list of standards in the CCSS to a number manageable for adult education programs and students based on relevance, importance, and "where the evidence for college and career readiness was most compelling."[8] Exact wording of the CCS Standards was maintained except to change references to K-12 grades or children and to create examples more appropriate for adults.

The CCRS report also includes essential stipulations about what the CCR Standards are not—they do not specify how instructors should teach; they are not a curriculum; they do not include the full spectrum of support appropriate for English language learners. (A full list of these statements is included in Appendix A.)

The CCR Standards report fulfilled OVAE's 2003 commitment to strengthening the connections between adult education, postsecondary education, and work. However, the report's introduction clearly passes that challenge forward to local programs which provide students the opportunity to acquire college and career readiness skills to pursue their long-term career aspirations.

Increasingly, students entering the workforce are discovering that they need critical knowledge and skills that are used on a regular basis. They recognize that pursuing a career pathway that pays enough to support a family and provides genuine potential for advancement hinges on being able to perform the complex tasks identified by the CCSS as critical for postsecondary success. Leading economists who have examined labor market projections note that key college and career ready knowledge and skills are closely linked to being able to get the training necessary to earn a living wage in high-growth industries (Carnevale and Desrochers 2002, 2003). [9]

In 2013, the Georgetown Center on Education and the Workforce reemphasized the essential relationship between education and employment, stating that by 2020, 65 percent of all jobs in the U. S. will require postsecondary education and training beyond high school.[10] This need for post-secondary education and training is reflected in the Workforce Innovation and Opportunity Act (WIOA) which became law in 2014. Title II of WIOA, called the Adult Education and Family Literacy Act, ramped up the expectations for students in Adult Basic Education (ABE), Adult Secondary Education (ASE), and English Language Acquisition (ELA) programs. Completion of high school equivalency was redefined from passing the GED Test to passing the GED Test <u>and</u> entering postsecondary education and/or employment. The stronger focus was also apparent in the introduction of a new activity, Integrated Education and Training (IET), which allows use of Title II funding for programs to offer basic skills and workforce training simultaneously.

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[8] Pimentel, S. (n.d.). College and Career Readiness Standards for Adult Education (United States, U.S. Department of Education, Office of Vocational and Adult Education). Salem, OR.
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[9] Pimentel, S. (n.d.). College and Career Readiness Standards for Adult Education (United States, U.S. Department of Education, Office of Vocational and Adult Education). Salem, OR.
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[10] Carnevale, A. P., Smith, V., & Strohl, J. (n.d.). *Recovery: Job growth and education requirements through 2020* [PDF]. Georgetown Pubic Policy Institute: Center on Education & the Workforce. https://1gyhoq479ufd3yna29x7ubjn-wpengine.netdna-ssl.com/wp-content/uploads/2014/11/Recovery2020.ES_.Web_.pdf

WIOA also compelled action on the Oregon ABS Learning Standards by requiring that states describe how the content standards for Title II-funded activities would be aligned with state content standards adopted under the Elementary and Secondary Education Act Education (ESEA). Oregon's alignment was indirect: the national CCRS were clearly aligned with the CCS Standards which Oregon had adopted for K-12, so Oregon ABS was obligated to demonstrate the alignment between the Learning Standards and the CCRS. To do this, CCWD commissioned national CCRS experts to conduct gap analyses of the Oregon ABS Learning Standards. This led to a comprehensive revision of the math standards and addition of the CCRS key advances or shifts to all of the learning standards frameworks. The revised standards frameworks were published on the Higher Education Coordinating Commission (HECC) website in 2016 with a new name, the Oregon Adult Learning Standards (OALS).

Rationale for Adopting the CCCRS in Oregon

In 2017-2018, fulfilling WIOA requirements, CCWD reviewed Title II providers' implementation of OALS. Reports of participation in professional development and an inventory of local learning standard activities and documents revealed significant gaps in implementation.

During discussions regarding the barriers to implementation of OALS with program administrators and instructors, two factors emerged as likely barriers to use of the Learning Standards.

- Lack of resources at the both the state and local levels for professional and curriculum development.

- Concern about the lack of fit between CASAS and OALS. The concern that CASAS did not measure OALS outcomes influenced both the use of the Learning Standards and the use of CASAS results, which in-turn affected performance on federal targets for measurable skill gain.

In the winter of 2018, a group of instructors, program administrators, and CCWD staff asked two fundamental questions. The first was whether or not CCRS could be part of a solution to reduce these barriers. If yes, how could Oregon retain the unique strengths of the OALS while adopting the CCRS?

Adopting the CCRS could also solve some of the resource issues, addressing the second barrier. As Susan Pimentel pointed out in the CCR Standards report, the close relationship between CCRS and CCSS has an additional advantage.

With 46 states adopting the CCSS, a full range of standards-based resources have been developed from which adult education can benefit. These include formative and summative assessments, instructional materials, teacher preparation, and professional development opportunities. These materials will be more robust than any one state—or any one program—could afford to develop on its own. Many of those resources are available online; they are easy to locate; and they are free.[11]

OCTAE has contributed resources specific to CCRS to the LINCS College and Career Standards Collection.[12] LINCS also provides online access to tools and materials developed by partnerships made possible by the

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[11] Pimentel, S. (n.d.). College and Career Readiness Standards for Adult Education (United States, U.S. Department of Education, Office of Vocational and Adult Education). Salem, OR.
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[12] Resources By Topic: College and Career Standards. (n.d.). Retrieved July 1, 2019, from https://lincs.ed.gov/professional-development/resource-collections/by-topic/College%20and%20Career%20Standards

existence of a common set of standards. At the state and local levels, state adult education units, professional associations, community colleges, and other organizations have developed materials and staff development resources that are accessible online through state adult education offices and literacy resource centers.

Finally, adopting CCRS could make alignment between CASAS and standards more transparent. OCTAE has approved new assessment instruments, *CASAS GOALS Reading* and *CASAS GOALS Math*, which are directly aligned to the CCR Standards.[13] These tests are included in the new test benchmarks and educational functioning levels (EFL) chart which was published in March 2019,[14] fulfilling the second promise of OVAE's *Promoting College and Career-Ready Standards in Adult Basic Education* initiative.

The answer to the second question, how to incorporate the unique strengths of the OALS with CCRS was more complex, but discussions with the directors, trainers, and developers of the Learning Standards led to a vision of new standards that could transparently combine OALS with CCRS.

With encouraging answers to both of the fundamental questions, OCABSD agreed in fall of 2018 to adopt the CCRS as the primary standards for student learning outcomes in Title II programs. Noting the role of OALS in allowing Oregon to reach current levels of success for students, the agreement stipulated that parts of OALS should be retained as long-standing and well-respected guidelines that go beyond CCRS.

Prompted by that agreement, ten consultants, many of whom helped develop the Oregon ABS Learning Standards, produced the Oregon Adult College and Career Readiness Standards (OACCRS) Handbooks for Instruction, therefore realizing the vision of standards that transparently encompass key elements of the standards developed by Oregon and the national CCRS. These new standards, mandated for use by Oregon Title II programs beginning July 1, 2020, aim to achieve revised goals that sustain the original vision for Oregon's first content standards in 2008.

- A statement of key sets of knowledge and skills which support student success in high school equivalency and transition to postsecondary education/training and careers
- A common language for the ABS system to talk about teaching and learning
- A basis for statewide professional development grounded in research and best practice
- Broad access to national, state, and organizational resources
- A clearly-delineated continuum of skills that guide goal-setting, curriculum, instruction, assessment, and accountability
- Explicit alignment with mandated assessment to guide placement and instruction and promote program success in meeting targets

[13] CASAS. (n.d.). Retrieved July 1, 2019, from https://www.casas.org/
Main Website

[14] *Test Benchmarks for NRS Educational Functioning Levels* [Docx]. (2019, March). National Reporting System for Adult Education.
A project of the U.S. Department of Education; https://nrsweb.org/resources/test-benchmarks-nrs-educational-functioning-levels-efl-updated-march-2019

- Consistent expectations across programs for learners who transfer between institutions

- Support for program efforts in course development or revision as part of college institutional effectiveness and program improvement

The OACCR Standards have been developed to meet these goals and to strengthen the place of adult learners in Oregon where 69% of job openings will require education beyond high school by 2020 [15] ABS programs can achieve this potential for student success only if they fully implement the standards. By mandating the OACCRS and publishing this handbook, CCWD has taken the first steps toward implementation. The Handbook provides support with multiple views of standards, specific tools for teaching, and dozens of resources that will assist in curriculum development and delivery. CCWD will reinvigorate statewide and regional professional development opportunities and support local efforts such as Professional Learning Communities and expert coaching for curriculum development. Content standards have their maximum impact when they are the heart of instruction and learning in local programs.

[15] Recovery: Projections of Jobs and Education Requirements Through 2020: State Report, Oregon 2010-2020 Total Job Openings [PDF]. (n.d.). Georgetown Pubic Policy Institute: Center on Education & the Workforce. https://1gyhoq479ufd3yna29x7ubjn-wpengine.netdna-ssl.com/wp-content/uploads/Oregon-Recovery.pdf

Goals

OACCRS will provide:

- A statement of key sets of knowledge and skills which support student success in high school equivalency and transition to postsecondary education/training and careers;
- A common language for the ABS system to talk about teaching and learning;
- A basis for statewide professional development grounded in research and best practice;
- Broad access to national, state, and organizational resources;
- A clearly-delineated continuum of skills that guide goal-setting, curriculum, instruction, assessment, and accountability;
- Explicit alignment with mandated assessment to guide placement and instruction and promote program success in meeting targets;
- Consistent expectations across programs for learners who transfer between institutions;
- Support for program efforts in course development or revision as part of college institutional effectiveness and program improvement;

While the OACCR Standards have been developed to meet these goals, ABS programs can achieve their potential for student success only if they implement the standards. By mandating the OACCRS and publishing this Handbook, CCWD has taken the first steps toward implementation. The Handbook provides support with multiple views of standards, specific tools for teaching, and dozens of resources that will assist in curriculum development and delivery. CCWD continues to offer statewide and regional professional development opportunities and will support local efforts such as Professional Learning Communities and expert coaching for curriculum development. Content standards have their maximum impact when they are the heart of instruction and learning in local programs.

Oregon Adult College and Career Readiness Standards for Mathematics

There are four sections before in the Oregon Adult College and Career Readiness Standards (OACCRS) for Mathematics. Section 1 is an explanation of the **Key Shifts** required for mathematics instruction using the CCR Standards. Section 2 contains the **Standards for Mathematical Practice** with an explanation for each practice. Section 3 presents several views of the **Standards for Mathematical Content** including the major work of each level, information about how the standards are organized including a list of the domains by level, and the standards for all domains in each level. The appendices provide the CCR Standards in varied formats as well as some tools and resources for instruction.

Section 1: Key Shifts in the Standards[16]

Through their selections, CCRS for Adult Education panelists validated three key shifts in instruction prompted by the Common Core State Standards (CCSS) and outlined by Student Achievement Partners (2012). The shifts described below identify the most significant elements of the CCSS for Mathematics. At the heart of these shifts is a focus in mathematics instruction on delving deeply into the key processes and ideas upon which mathematical thinking relies. The shifts below therefore center on the knowledge and skills students must master to be adept at understanding and applying mathematical ideas.

Shift 1 — Focus: Focusing Strongly Where the Standards Focus

Strong instruction narrows the scope of what is taught while centering on a great depth of understanding. Instead of racing to cover topics. Focusing deeply on the major work of each level will allow students to secure the mathematical foundations, conceptual understanding, procedural skill and fluency, and ability to apply the math they have learned to solve all kinds of problems—inside and outside the math classroom. This important shift finds explicit expression in the selection of priority content addressing a clear understanding of place value and its connection to operations in the early levels. The emphasis on numeracy in early grades leads to a deeper understanding of the properties of operations at subsequent levels, encouraging fluency in the application of those properties, eventually for all operations with all number systems in a variety of situations.

Shift 2 — Coherence: Designing Learning Around Coherent Progressions Level to Level

The second key shift required by the CCSS and reflected in panelists' selections is to create coherent progressions in the content within and across levels, so that students can build new understanding onto previous foundations. That way, instructors can count on students having conceptual understanding of core content. Instead of each standard signaling a new concept or idea, standards at higher levels become extensions of previous learning. The focus on understanding numbers and their properties through the levels also exemplifies the progression from number to expressions and equations and then to algebraic thinking. This is seen in the selected standards within and across the levels. For example, an emphasis on

[16] Unless otherwise noted, information about the CCR Standards is taken from College and Career Readiness Standards for Adult Education available at: Pimentel, S. (2013, April). *College and Career Readiness Standards for Adult Education* (United States, U.S. Department of Education, Office of Vocational and Adult Education). Retrieved June 24, 2019, from https://lincs.ed.gov/publications/pdf/CCRStandardsAdultEd.pdf

understanding place value, as indicated above for Shift 1, progresses to using place value to add and subtract two-digit numbers to fluency in addition and subtraction of whole numbers to 1000 (including a requirement to explain why the strategies for addition and subtraction work). An understanding of both the numbers and their operations grows from the emphasis on place value and follows a progression extending beyond operations with numbers to include algebraic expressions and equations and ultimately to a deep understanding of functions. These connections can be further exemplified in applications related to other domains within and across the levels, such as the connection between properties of operations (e.g., multiplication) and geometric applications (e.g., area).

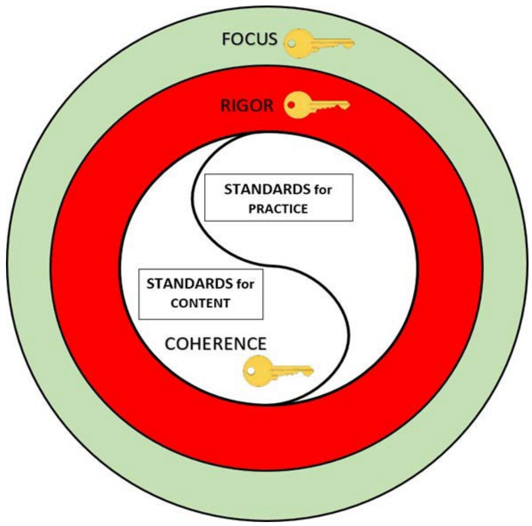
Shift 3 — Rigor: Pursuing Conceptual Understanding, Procedural Skill and Fluency, and Application—All With Equal Intensity

The third key shift required by the CCSS and reinforced in panelists' selections is equal measures of conceptual understanding of key concepts, procedural skill and fluency, and rigorous application of mathematics in real-world contexts. Students with a solid conceptual understanding see mathematics as more than just a set of procedures. They know more than "how to get the answer" and can employ concepts from several perspectives. Students should be able to use appropriate concepts and procedures, even when not prompted, and in content areas outside of mathematics. Panelists therefore selected standards reflecting key concepts used in a variety of contexts, such as place value, ratios and proportional relationships, and linear algebra. They also selected standards calling for speed and accuracy in calculations using all number systems, as well as standards providing opportunities for students to apply math in context, such as calculations related to geometric figures involving rational number measures; calculation of probabilities as fractions, decimals, or percent; and statistical analysis of rational data.

Visualization

In addition to the key shifts explained previously, the OACCRS for Mathematics have two central parts: The Standards for Mathematical Practice and The Standards for Mathematical Content.

THE LINK BETWEEN THE KEY SHIFTS AND THE TWO SETS OF MATHEMATICAL STANDARDS



As the graphic above depicts, in the OACCRS for mathematics there is a strong connection between the Standards for Practice and the Standards for Content. The development of mathematical content is the result of the application of mathematical habits of mind reflected in the practices. Further, the OACCRS require that the content be taught *through* the practices so that the connections are *real* and integrated rather than interspersed. To be faithful to this integration and address the need for delivering **coherent** instruction (that is, deep instruction on interconnected topics rather than "covering" disconnected topics, tricks or mnemonics) both the content and practice are surrounded by increased **focus** (fewer topics, but greater depth) and **rigor (**pursuing conceptual understanding, procedural skills and fluency, and application with equal intensity.) Given the ambitious nature of the OACCRS, there is no random non-contextualized instruction such as obscure fractions operations (e.g., $\frac{11}{14} \pm \frac{17}{32}$) or other non- standards supporting curricula.

To reiterate, the connection between the Standards for Mathematical Practice and the Standards for Mathematical Content might be thought of as follows. The standards for content detail what students should know and be able to do, whereas the standards for mathematical practice describe attributes of mathematically proficient students (that is, they describe how student behaviors demonstrate their learning.)

Please note that the following pages detail the two sets of standards. More information on how to successfully use the Practices and Content standards together can be found in **Appendix G** and **Appendix H**.

THIS PAGE IS NOT PART OF THE ORIGINAL COLLEGE AND CAREER READINESS STANDARDS FOR ADULT EDUCATION

Section 2: Standards for Mathematical Practice

The **Standards for Mathematical Practice**, the "Practices," describe habits of mind that mathematics educators at all levels of learning should seek to develop in their students. These practices rest on "processes and proficiencies" with established significance in mathematics education, including such skills as complex problem solving, reasoning and proof, modeling, precise communication, and making connections.

1. Make sense of problems and persevere in solving them. (MP.1)
2. Reason abstractly and quantitatively. (MP.2)
3. Construct viable arguments and critique the reasoning of others. (MP.3)
4. Model with mathematics. (MP.4)
5. Use appropriate tools strategically. (MP.5)
6. Attend to precision. (MP.6)
7. Look for and make use of structure. (MP.7)
8. Look for and express regularity in repeated reasoning. (MP.8)

Explanation of the Standards for Mathematical Practice

Make sense of problems and persevere in solving them. (MP.1) Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Less experienced students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Reason abstractly and quantitatively. (MP.2) Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Construct viable arguments and critique the reasoning of others. (MP.3) Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Less experienced students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later. Later, students learn to determine domains to which an argument applies. Students at all levels can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Model with mathematics. (MP.4) Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This might be as simple as writing an addition equation to describe a situation. A student might apply proportional reasoning to plan a school event or analyze a problem in the community. A student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Use appropriate tools strategically. (MP.5) Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Attend to precision. (MP.6) Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. Less experienced students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Look for and make use of structure. (MP.7) Mathematically proficient students look closely to discern a pattern or structure. Students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that it's value cannot be more than 5 for any real numbers x and y.

Look for and express regularity in repeated reasoning. (MP.8) Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Early on, students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Section 3: Standards for Mathematical Content

The **Standards for Mathematical Content** are a balanced combination of procedural fluency and conceptual understanding intended to be connected to the Practices across domains and at each level. The Practices define ways students are to engage with the subject matter as they grow in mathematical maturity and expertise across levels. Content expectations that begin with the word "understand" highlight the relationship between the two parts of the CCRS for Mathematics and connect the practices and content standards.

Major Work of the Levels

Following is a general summary of the work at each of the OACCRS levels. It is <u>not</u> a complete list of skills and concepts at each level, but instead provides an overview of the expectations at each level. Instruction at each level should focus on developing fluency with the number sets indicated (e.g. numbers 0 -100 or decimals to the thousandths), with particular attention paid to building on previous learning as well as setting the stage for future learning (cohesion).

The CASAS/GOALS score ranges and NRS levels provide only a general guideline to show how the OACCRS aligns with that assessments. It is necessary for each individual program to determine which level of the standards best fit the needs of their students. It should be noted that many adult learners need instruction beginning at Level C and that Level C and Level D contain the bulk of what is normally taught in adult education programs.

Note that in adult education there is a very strong focus on application – mathematics for adults is taught in terms of contextualized situations and applicability to adult life. Necessary to the development of the mathematics skills listed below is that students apply what they are learning, going far beyond simple rote computation.

LEVEL	SKILLS
Level A includes concepts typically taught in grades K-1. This level might be appropriate for students who score at the very lowest levels on CASAS/GOALS (lower than 194). This is NRS ABE Level 1, Beginning ABE Literacy.	• Works with numbers from 0-100 • Develops the concepts of place value • Compares the size of numbers • Adds and subtracts numbers less than 100 (using mental math, written methods, modeling, and the relationship between addition and subtraction) • Analyzes and compares two- and three-dimensional shapes • Begins measurement and organizing data

Level B includes concepts typically taught in grades 2 – 3. This level might be appropriate for students whose CASAS/GOALS scores are from 194 to 203. This is NRS ABE Level 2, Beginning Basic Education.	• Extends concepts of place value, adding, and subtracting as above to numbers up to 1000 and rounding • Extends operations to multiplication and division • Develops fraction concepts using denominators of 2, 3, 4, 5, 6, and 8 • Begins formalized process of classifying geometric shapes • Identifies number patterns and shape patterns Solves problems with time intervals • Represents data on picture or bar graphs • Understands the basic concepts in area and perimeter
Together, Levels C and D represent the bulk of the adult learning requirements. 89% of the total number of CCS Standards for the equivalent grade levels were selected for inclusion in CCRS.	
Level C Includes concepts typically taught in grades 4-5, as well as lower grade 6. This level might be appropriate for students whose CASAS/GOALS scores are from 204 to 214. This is NRS Level 3, Low Intermediate Basic Education.	• Extends arithmetic of the four basic functions focusing on fluency • Extends the number system to decimal fractions to the 1000ths • Defines and computes greatest common factor and least common multiple • Extends the understanding of fraction equivalence and ordering, computes and solves problems with all four basic functions with common fractions • Develops the concepts of ratio and proportional reasoning • Writes and evaluates numerical expressions involving whole-number exponents • Reads, writes, and evaluates numerical expressions (order of operations and solving equations and inequalities)

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- Graph points on a coordinate plane
- Solves real-world problems involving area, surface area, and volume.
- Uses models and formulas, analyzes geometric properties (e.g., parallel, perpendicular, symmetry);
- Converts measurement units within a system (e.g., feet to miles not feet to kilometers)
- Works with angles and their measurement;
- Works with data sampling, measures of central tendency, and spread

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See the note on the previous page regarding the bulk of the adult learning standards.

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Level D includes concepts typically taught in upper grade 6 – grade 8.

Students at this level should be competent in the four basic operations with whole numbers, fractions, and decimals.

This level might be appropriate for students whose CASAS/GOALS scores are from 215-225.

This is NRS Level 4, Middle Intermediate Basic Education.

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- Extends understanding and operations of the number system to include negatives and absolute value
- Uses ratio concepts, ratio, and unit rate reasoning to solve problems
- Solves real-life problems using numerical and algebraic equations
- Works with radicals and positive and negative exponents
- Uses scientific notation
- Graphs proportional relationships
- Analyzes and solves simultaneous equations
- Compares equations and graphs of linear and non-linear functions
- Interprets and sketches graphs that have particular features (e.g., speed versus time for a person running up a sharp incline)
- Defines and evaluates functions

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	• Calculates with square and cube roots • Solves simple quadratic equations • Solves problems associated with scale drawings • Uses angle vocabulary (complementary, supplementary, vertical, adjacent) and solves equations for unknown angles • Solves real-world problems involving area, volume, and surface area of compound figures • Understands and applies concepts of similarity and congruence • Understands and applies concepts of similarity and congruence • Understands and applies the Pythagorean Theorem • Summarizes and describes data distribution including central tendency and variability • Uses random sampling to draw conclusions • Develops, uses, and evaluates probability models • Investigates patterns in data with two variables
Level E includes concepts typically taught in high school. Students at this level are almost college ready. This level might be appropriate for students whose CASAS/GOALS scores are higher than 226. This is NRS Levels 5 and 6, High Intermediate Basic Education and Adult Secondary Education.	• Rewrites expressions with radicals having positive and negative exponents • Effectively utilizes units of measure as a guide to solve problems and pays attention to and uses correct units in solutions • Rearranges formulas to highlight a quantity of interest • Recognizes and uses patterns in factoring • Factors quadratics to find zeroes of the functions • Represents graphically and solves equations and inequalities in one variable and linear systems

	• Uses function notation • Adds, subtracts, multiplies, and divides polynomials • Applies rates and relationships to density models; • Uses congruence and similarity criteria to prove relationships in geometric figures and determining volumes of cylinders, pyramids, cones, and spheres • Interprets the parameters in a linear or exponential function • Applies knowledge of statistics and probability in a modeling context, interpreting and comparing data distributions • Discriminates between correlation and causation

Organization of the Standards for Mathematical Content

To fully understand the how the CCR Standards for Mathematical Content are organized, it is helpful to know they are derived from the Common Core State Standards (CCSS), which span 22 domains across grades K-8 and high school. CCRS has divided the same domains into five levels, A—E.

Domains for CCRS Levels A—Most Of Level D (From Ccss K- 8)	Domains For CCRS Part Of Level D—Level E (From CCRS High School)
NBT: Number & Operations in Base Ten (K–5)	**N.RN:** The Real Number System
NS: The Number System (6–8)	**N.Q:** Number and Quantity
NF: Number & Operations—Fractions (3–5)	**A.SSE**: Algebra: Seeing Structure in Expressions
RP: Ratios & Proportional Relationships (6–7)	**A.APR**: Algebra: Arithmetic with Polynomials and Rational Expressions
OA: Operations & Algebraic Thinking (K–5)	**A.CED:** Algebra: Creating Equations
EE: Expressions & Equations (6–8)	**A.REI:** Algebra: Reasoning with Equations and Inequalities
F: Functions (8)	**F.IF:** Functions: Interpreting Functions
G: Geometry (K–8)	**F.BF:** Functions: Building Functions
MD: Measurement & Data (K–5)	**F.LE:** Functions: Linear, Quadratic, and Exponential Models
SP: Statistics & Probability (6–8)	**G.CO:** Geometry - Congruence
	G.SRT: Geometry - Similarity, Right Triangles, and Trigonometry
	G.GMD: Geometry - Geometric Measurement and Dimension

	G.M.G Geometry: Modeling with Geometry
	G.GMD: Geometry: Geometric Measurement and Dimension

Appendix B contains a guide to aid understanding of the coding in the standards.

Standards by Level

Mathematics Standards Level A

Level A focuses almost entirely on counting, cardinality, number sense, and base-ten operations. This includes developing an understanding of whole number relationships and two-digit place value, as well as strategies for (and fluency with) addition and subtraction. To provide a foundation for algebra, standards introduce the concept of an equation, a variable, and the meaning of the equal sign, all within the context of addition and subtraction within 20. In addition to number, some attention is given to describing and reasoning about geometric shapes in space as a basis for understanding the properties of congruence, similarity, and symmetry, and developing an understanding of linear measurement (length).

Level A (K-1)

Number and Operations: Base Ten	
Understand place value.	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a “ten.” b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). (1.NBT.2) Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. (1.NBT.3)
Use place value understanding and the properties of operations to add and subtract.	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. (1.NBT.4) Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. (1.NBT.5) Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and

	strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (1.NBT.6)
Operations and Algebraic Thinking	
Represent and solve problems involving addition and subtraction.	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (1.OA.2)
Understand and apply properties of operations and the relationship between addition and subtraction.	Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* (1.OA.3) Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.* (1.OA.4)
Add and subtract with numbers up to 20.	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). (1.OA.5) Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (1.OA.6)
Work with addition and subtraction.	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.* (1.OA.7) Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □* (1.OA.8)

Geometry	
Analyze, compare, create, compose shapes.	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/ "corners") and other attributes (e.g., having sides of equal length). (K.G.4)
Reason with shapes and their attributes.	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter- circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[15] (1.G.2)
Measurement and Data	
Measure lengths indirectly and by iterating length units.	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* (1.MD.2)
Represent and interpret data.	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. (1.MD.4)

[15] Students do not need to learn formal names such as "right rectangular prism.

Mathematics Standards Level B

Level B emphasizes understanding base-ten notation (place value for whole numbers to 1000), developing fluency in addition and subtraction (to 3 digits), understanding and exploring strategies for multiplication and division (within 100), and a foundational understanding of fractions. These skills will prepare students for work with rational numbers, ratios, rates, and proportions in subsequent levels. A critical area of focus is on gaining a foundational understanding of fractions and preparing the way for work with rational numbers. In the areas of measurement and geometry, using standard units of measure and developing understanding of the structure of rectangular arrays and areas are priorities, as well as analyzing two-dimensional shapes as a foundation for understanding area, volume, congruence, similarity and symmetry.

LEVEL B (2-3)

Number & Operations: Base Ten	
Understand place value.	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (2.NBT.1) Count within 1000; skip-count by 5s, 10s, and 100s. (2.NBT.2) Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (2.NBT.3) Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (2.NBT.4)
Use place value understanding and the properties of operations to add and subtract.	Add up to four two-digit numbers using strategies based on place value and properties of operations. (2.NBT.6) Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (2.NBT.7)

	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (2.NBT.8) Explain why addition and subtraction strategies work, using place value and the properties of operations. (2.NBT.9)
Use place value understanding and properties of operations to perform multi-digit arithmetic. [16]	Use place value understanding to round whole numbers to the nearest 10 or 100. (3.NBT.1) Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (3.NBT.2) Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. (3.NBT.3)
Number and Operations: Fractions[17]	
Develop understanding of fractions as numbers.	Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$. (3.NF.1) Understand a fraction as a number on the number line; represent fractions on a number line diagram. (3.NF.2) • Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line. (3.NF.2a) • Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. (3.NF.2b) Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (3.NF.3) • Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (3.NF.3a)

[16] A range of algorithms may be used.

[17] Expectations at this level in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.

	• Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. (3.NF.3b) • Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*. (3.NF.3c) • Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (3.NF.3d)
Operations and Algebraic Thinking	
Represent and solve problems involving addition and subtraction.	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (2.OA.1)
Add and subtract with 20.	Fluently add and subtract within 20 using mental strategies. Know from memory all sums of two one- digit numbers. (2.OA.2)
Represent and solve problems involving multiplication and division.	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* (3.OA.1) Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* (3.OA.2) Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (3.OA.3) Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$. (3.OA.4)

Understand properties of multiplication and the relationship between multiplication and division.	Apply properties of operations as strategies to multiply and divide.[18] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* (3.OA.5) Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* (3.OA.6)
Multiply and divide within 100.	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. Know from memory all products of two one-digit numbers. (3.OA.7)
Solve problems involving the four operations, and identify and explain patterns in arithmetic.	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[19] (3.OA.8) Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* (3.OA.9)
Geometry	
Reason with shapes and their attributes.	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[20] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (2.G.1) Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. (2.G.3)

[18] Students need not use formal terms for these properties.

[19] This standard is limited to problems posed with whole numbers having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Reason with shapes and their attributes.	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (3.G.1) Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.* (3.G.2)
Measurement and Data	
Measure and estimate lengths in standard units.	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (2.MD.2) Estimate lengths using units of inches, feet, centimeters, and meters. (2.MD.3) Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (2.MD.4)
Relate addition and subtraction to length.	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. (2.MD.6)

[20] Sizes are compared directly or visually, not compared by measuring.

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. (3.MD.1) Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[21] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[22] (3.MD.2)
Represent and interpret data.	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (2.MD.10) Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* (3.MD.3) Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. (3.MD.4)
Geometric measurement: understand concepts of area and relate to area of multiplication and addition.	Recognize area as an attribute of plane figures and understand concepts of area measurement. • A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. • A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. (3.MD.5) Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units). (3.MD.6) Relate area to the operations of multiplication and addition. (3.MD.7)

[21] Excludes compound units such as cm3 and finding geometric volume of a container.

[22] Excludes multiplicative comparison problems (problems involving notions of "times as much").

	- Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (3.MD.7a) - Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (3.MD.7b) - Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. (3.MD.7c) Recognize area as additive. Find areas of rectilinear figures by decomposing them into non- overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. (3.MD.7d)
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. (3.MD.8)

Mathematics Standards Level C

More than any other, Level C provides the foundation for all future mathematical studies. Fluency with multi-digit whole and decimal numbers as well as calculations with fractions (and the relationships between them) carry the most weight at this level. This extends to working with the concept of ratio and rates, addition and subtraction of fractions, and understanding why the procedures for multiplying and dividing fractions make sense. While the greatest emphasis is still on standards for numbers and operations, attention to algebra and geometry increases considerably in Level C. Reading, writing, and interpreting expressions and equations and generating patterns in numbers and shapes provide a conceptual foundation for functions. In addition, analyzing geometric properties, such as parallelism, perpendicularity, and symmetry, and developing and finding volumes of right rectangular prisms take precedence. Level C also emphasizes sampling techniques and data collection through statistical questioning; to previous standards about data, it adds the understanding of measures of center and spread and display of collected data with line plots.

Level C (4-5, +6)

Number and Operations: Base Ten	
Generalize place value understanding for multi-digit whole numbers.	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* (4.NBT.1) Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (4.NBT.2) Use place value understanding to round multi-digit whole numbers to any place. (4.NBT.3)
Use place value understanding and properties of operations to perform multi-digit arithmetic.	Fluently add and subtract multi-digit whole numbers using the standard algorithm. (4.NBT.4) Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (4.NBT.5) Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (4.NBT.6)

Understand the place value system.	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. (5.NBT.1) Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. (5.NBT.2) Read, write, and compare decimals to thousandths. (5.NBT.3) • Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 ×10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). (5.NBT.3a) • Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (5.NBT.3b) Use place value understanding to round decimals to any place. (5.NBT.4)
Perform operations with multi-digit whole numbers and with decimals to hundredths.	Fluently multiply multi-digit whole numbers using the standard algorithm. (5.NBT.5) Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (5.NBT.6) Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (5.NBT.7) [Note from panel: Applications involving financial literacy should be used.]
The Number System	
Compute fluently with multi-digit numbers and find common factors and multiples.	Fluently divide multi-digit numbers using the standard algorithm. (6.NS.2) Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. (6.NS.3) Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with

	a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* (6.NS.4)
Number and Operations: Fractions[23]	
Extend understanding of fraction equivalence and ordering.	Explain why *a* fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (4.NF.1) Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (4.NF.2)
Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.	Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. (4.NF.3) • Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (4.NF.3a) • Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* (4.NF.3b) • Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (4.NF.3c) • Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. (4.NF.3d) Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (4.NF.4)

[23] Expectations at this level in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

	• Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* (4.NF.4a) • Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* (4.NF.4b) • Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (4.NF.4c)
Understand decimal notation for fractions, and compare decimal fractions.	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* (5.NF.1) Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* (5.NF.2)
Apply and extend previous understanding of multiplication and division to multiply and divide fractions.	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* (5.NF.3) Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (5.NF.4) Interpret multiplication as scaling (resizing), by: • Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

	• Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. (5.NF.5) Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (5.NF.6) Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (5.NF.7) • Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* (5.NF.7a) • Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* (5.NF.7b) • Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lbs. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* (5.NF.7c)
The Number System	
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lbs. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mile?* (6.NS.1)

Ratios and Proportional Relationships	
Understand ratio concepts and use ratio reasoning to solve problems.	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* (6.RP.1) Understand the concept of a unit rate a/b associated with a ratio $a{:}b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* [24] (6.RP.2)
Operations and Algebraic Thinking	
Use the four operations with whole numbers to solve problems.	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. (4.OA.1) Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (4.OA.2) Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (4.OA.3)
Gain familiarity with factors and multiple.	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (4.OA.4)

[24] Expectations for unit rates at this level are limited to non-complex fractions.

Generate and analyze pattern.	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*. (4.OA.5)
Write and interpret numerical expressions.	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. (5.OA.1) Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (2100 + 425) is three times as large as the 2100 + 425, without having to calculate the indicated sum or product.* (5.OA.2)
Expressions and Equations	
Apply and extend previous understandings of arithmetic to algebraic expressions.	Write and evaluate numerical expressions involving whole-number exponents. (6.EE.1) Write, read, and evaluate expressions in which letters stand for numbers. (6.EE.2) • Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y*. (6.EE.2a) • Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* (6.EE.2b) • Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.* (6.EE.2c)

	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*. (6.EE.3) Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.* (6.EE.4)
Reason about and solve one-variable equations and inequalities.	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (6.EE.5) Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (6.EE.6) Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (6.EE.7) Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (6.EE.8)
Represent and analyze quantitative relationships between dependent and independent variables.	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.* (6.EE.9)

Geometry	
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (4.G.1)
Graph points on the coordinate plane to solve real-world and mathematical problems.	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). (5.G.1) Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (5.G.2)
Classify two-dimensional figures into categories based on their properties.	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* (5.G.3)
Solve real-world and mathematical problems involving area, surface area, and volume.	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (6.G.1) Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.3) Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.4)

Measurement and Data	
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (4.MD.2) Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* (4.MD.3)
Geometric measurement: understand concepts of angle and measure angles.	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: • An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. • An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. (4.MD.5) Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (4.MD.6) Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (4.MD.7)
Convert like measurement units within a given measurement system.	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. (5.MD.1)

Represent and interpret data.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally*. (5.MD.2) [Note from panel: Plots of numbers other than measurements also should be encouraged.]
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. • A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. • A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. (5.MD.3) Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (5.MD.4) Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. (5.MD.5) • Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole- number products as volumes, e.g., to represent the associative property of multiplication. (5.MD.5a) • Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. (5.MD.5b) Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. (5.MD.5c)

Statistics and Probability	
Develop understanding of statistical variability.	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* (6.SP.1) Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (6.SP.2) Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (6.SP.3)
Summarize and describe distributions.	Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (6.SP.4) [Also see S.ID.1]

Mathematics Standards Level D

Like preceding levels, Level D also emphasizes number sense and operations, but here the attention is on fluency with all four operations with rational numbers—both negative and positive. The foundation for understanding of irrational numbers is built here, including calculation with square and cube roots and solving simple quadratic equations. Another keen area of concentration is algebra and functions: formulating and reasoning about expressions, equations, and inequalities; solving linear equations and systems of linear equations; grasping the concept of a function; and using functions to describe quantitative relationships. Level D is also where understanding and applying ratios, rates, and proportional reasoning—forming a bridge between rational number operations and algebraic relationships—are developed.

Building on the geometric analysis in Level C, the focus turns to analyzing two- and three- dimensional figures using distance, angle, similarity, and congruence, and understanding basic right triangle trigonometry. Having worked with measurement data in previous levels, students at this level develop notions of statistical variability and learn to understand summary statistics and distributions. The concept of probability is introduced and developed at this level.

Level D (+6, 7-8)

<table>
<tr><th colspan="2">The Number System</th></tr>
<tr><td>Apply and extend previous understandings of numbers to the system of rational numbers.</td><td>Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (6.NS.5)

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous levels to represent points on the line and in the plane with negative number coordinates. (6.NS.6)

• Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. (6.NS.6a)

• Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (6.NS.6b)

• Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. (6.NS.6c)</td></tr>
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	Understand ordering and absolute value of rational numbers. (6.NS.7) Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right. (6.NS.7a) • Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3°$ C > $-7°$ C to express the fact that $-3°$ C is warmer than $-7°$ C.* (6.NS.7b) • Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write $\|-30\| = 30$ to describe the size of the debt in dollars.* (6.NS.7c) • Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.* (6.NS.7d) Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (6.NS.8)
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. (7.NS.1) • Describe situations in which opposite quantities combine to make 0. *For example, if a check is written for the same amount as a deposit, made to the same checking account, the result is a zero increase or decrease in the account balance.* (7.NS.1a) • Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. (7.NS.1b) • Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (7.NS.1c)

	• Apply properties of operations as strategies to add and subtract rational numbers. (7.NS.1d) Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. (7.NS.2) • Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. (7.NS.2a) • Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. (7.NS.2b)
Know that there are numbers that are not rational, and approximate them by rational numbers.	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$, *show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations*. (8.NS.2)
Understand ratio concepts and use ratio reasoning to solve problems.	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (6.RP.3) a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (6.RP.3a) b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* (6.RP.3b) c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. (6.RP.3c) Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (6.RP.3d)

Analyze proportional relationships and use them to solve real-world and mathematical problems.	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.* (7.RP.1) Recognize and represent proportional relationships between quantities. (7.RP.2) • Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (7.RP.2a) • Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (7.RP.2b) [Also see 8.EE.5] • Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. (7.RP.2c) • Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. (7.RP.2d) • Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. (7.RP.3) [Also see 7.G.1 and G.MG.2]
Expressions and Equations	
Use properties of operations to generate equivalent expressions.	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (7.EE.1) Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* (7.EE.2) [Also see A.SSE.2, A.SSE.3, A.SSE.3a, A.CED.4]

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* (7.EE.3) Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (7.EE.4) [Also see A.CED.1 and A.REI.3] a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* (7.EE.4a) [Also see A.CED.1 and A.REI.3] b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* (7.EE.4b) [Also see A.CED.1 and A.REI.3]
Work with radicals and integer exponents.	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{(-5)} = 3^{(-3)} = (1/3)^3 = 1/27$. (8.EE.1) [Also see F.IF.8b] Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. (8.EE.2) [Also see A.REI.2] Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.* (8.EE.3)

	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. (8.EE.4) [Also see N.Q.3]
Understand the connections between proportional relationships, lines, and linear equations.	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (8.EE.5) [Also see 7.RP.2b]
Analyze and solve linear equations and pairs of simultaneous linear equations.	Solve linear equations in one variable. (8.EE.7) [Also see A.REI.3] • Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (8.EE.7a) • Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (8.EE.7b) Analyze and solve pairs of simultaneous linear equations. (8.EE.8) • Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (8.EE.8a) • Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* (8.EE.8b) [Also see A.REI.6] Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* (8.EE.8c)

Functions	
Define, evaluate, and compare functions.	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[25] (8.F.1) [Also see F.IF.1] Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* (8.F.3)
Use functions to model Relationships between quantities.	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4) [Also see F.BF.1 and F.LE.5] Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (8.F.5) [Also see A.REI.10 and F.IF.7]
Geometry	
Draw, construct, and describe geometrical figures and describe the relationships between them.	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (7.G.1) [Also see 7.RP.3]

[25] Function notation is not required at this level.

Solve real-life and mathematical problems involving angle, measure, area, surface area, and volume.	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. (7.G.4) Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (7.G.5) Solve real-world and mathematical problems involving area, volume and surface area of two- and three- dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (7.G.6) [Also see G.GMD.3]
Understand congruence and similarity using physical models, transparencies, or geometry software.	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (8.G.2) [Also see G.SRT.5] Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (8.G.4) [Also see G.SRT.5] Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* (8.G.5)
Understand and apply the Pythagorean Theorem.	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (8.G.7) Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (8.G.8)

Statistics and Probability	
Summarize and describe distributions.	Summarize numerical data sets in relation to their context, such as by: • Reporting the number of observations. • Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. • Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (6.SP.5)
Use random sampling to draw inferences about a population.	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. (7.SP.1) Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. (7.SP.2)*
Draw informal comparative inferences about two populations.	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* (7.SP.3) Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in one chapter of a science book are generally longer or shorter than the words in another chapter of a lower level science book.* (7.SP.4) [Also see S.ID.3]

Investigate chance processes and develop, use, and evaluate probability models.	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (7.SP.5) Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* (7.SP.6) Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (7.SP.7) • Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* (7.SP.7a) • Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* (7.SP.7b) Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (7.SP.8a) Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. (7.SP.8b)
Investigate patterns of association in bivariate data.	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (8.SP.1) [Also see S.ID.1]

	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (8.SP.2) Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (8.SP.3) [Also see S.ID.7] Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they like to cook and whether they participate actively in a sport. Is there evidence that those who like to cook also tend to play sports?* (8.SP.4) [Also see S.ID.5]

Mathematics Standards Level E

Themes introduced and developed in earlier levels continue and deepen in Level E. Having already extended arithmetic calculations from whole numbers to fractions and from fractions to rational and irrational numbers, understanding the real number system comes to the fore.

Understanding radical expressions, using and interpreting units in problem solving, and attending to precision are important areas of focus. Prior work with proportional relationships and functions expands from linear expressions, equations, and functions to quadratic, rational, exponential, and polynomial. To bridge the gap between algebra and geometry, rates and relationships are applied to density models. Work also advances in geometry, including using congruence and similarity criteria to prove relationships in geometric figures and determining volumes of cylinders, pyramids, cones, and spheres. Basic skills and knowledge of statistics and probability are applied in a modeling context, in which students interpret and compare data distributions and understand issues of correlation and causation.

Note: Making mathematical models is a Standard for Mathematical Practice (MP.4), and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).

Level E (High School)

Number and Quantity: The Real Number System	
Extend the properties of exponents to rational exponents.	Rewrite expressions involving radicals and rational exponents using the properties of exponents. (N.RN.2)
Number and Quantity: Quantities	
Reason quantitatively and use units to solve problems.	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.* (N.Q.1) Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.* (N.Q.3) [Also see 8.EE.4]
Algebra: Seeing Structure in Expressions	
Interpret the structure of expressions.	Interpret expressions that represent a quantity in terms of its context.* (A.SSE.1) Interpret parts of an expression, such as terms, factors, and coefficients.* (A.SSE.1a)

	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* (A.SSE.2) [Also see 7.EE.2]
Write expressions in equivalent forms to solve problems.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* (A.SSE.3) [Also see 7.EE.2] Factor a quadratic expression to reveal the zeros of the function it defines.* (A.SSE.3a) [Also see 7.EE.2]
Algebra: Arithmetic With Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials.	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A.APR.1) [Note from panel: Emphasis should be on operations with polynomials.]
Rewrite rational expressions.	Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$ where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. (A.APR.6)
Algebra: Creating Equations	
Create equations that describe numbers or relationships.	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.* (A.CED.1) [Also see 7.EE.4, 7.EE.4a, and 7.EE.4b] Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.* (A.CED.2) Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.** (A.CED.3)

	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.** (A.CED.4) [Also see 7.EE.2]
	Algebra: Reasoning With Equations & Inequalities
Understand solving equations as a process of reasoning and explain the reasoning.	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A.REI.1) Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (A.REI.2) [Also see 8.EE.2]
Solve equations and inequalities in one variable.	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A.REI.3) [Also see 7.EE.4, 7.EE.4a, 7.EE.4b, and 8.EE.7] Solve quadratic equations in one variable. (A.REI.4)
Solve systems of equations.	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (A.REI.6) [Also see 8.EE.8b]
Represent and solve equations and inequalities graphically.	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10) [Also see 8.F.5]
	Functions: Interpreting Functions
Understand the concept of a function and use function notation.	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. (F.IF.1) [Also see 8.F.1] Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (F.IF.2)

Interpret functions that arise in applications in terms of the context.	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *For example, for a quadratic function modeling a projectile in motion, interpret the intercepts and the vertex of the function in the context of the problem*.* (F.IF.4) [*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.]* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *(F.IF.5)* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph * (F.IF.6) [NOTE: See conceptual modeling categories.]
Analyze functions using different representations.	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* (F.IF.7) [Also see 8.F.5] Use properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in an exponential function and then classify it as representing exponential growth or decay*. (F.IF.8b) [Also see 8.EE.1] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (F.IF.9)
Functions: Building Functions	
Build a function that models a relationship between two quantities.	Write a function that describes a relationship between two quantities. * (F.BF.1) [Also see 8.F.4

Functions: Linear, Quadratic, and Exponential Models	
Construct and compare linear, quadratic, and exponential models and solve problems.	Distinguish between situations that can be modeled with linear functions and with exponential functions.* (F.LE.1) • Recognize situations in which one quantity changes at a constant rate per unit interval relative to another *(F.LE.1b) Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. * (F.LE.1c)
Interpret expressions for functions in terms of the situation they model.	Interpret the parameters in a linear or exponential function in terms of a context. * (F.LE.5) [Also see 8.F.4]
Geometry: Congruence	
Experiment with transformations in the plane.	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (G.CO.1)
Geometry: Similarity, Right Triangles, & Trigonometry	
Prove theorems involving similarity.	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G.SRT.5) [Also see 8.G.2 and 8.G.4]
Geometry: Geometric Measurement & Dimension	
Explain volume formulas and use them to solve problems.	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. * (G.GMD.3) [Also see 7.G.6]

Geometry: Modeling With Geometry	
Apply geometric concepts in modeling situations.	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* (G.MG.2) [Also see 7.RP.3]
Statistics and Probability: Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on a single count or measurable variable.	Represent data with plots on the real number line (dot plots, histograms, and box plots). (S.ID.1) [Also see 6.SP.4 and 8.SP.1] Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (S.ID.3) [Also see 7.SP.4]
Summarize, represent, and interpret data on two categorical and quantitative variables.	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (S.ID.5) [Also see 8.SP.4]
Interpret linear models.	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S.ID.7) [Also see 8.SP.3] Distinguish between correlation and causation. (S.ID.9)

Appendix A: What the CCR Standards Are Not

The standards should be recognized for what they *are not* as well as what they are. The central design parameters that guided the work of the panels include[26]:

- First and foremost, the selected standards do not specify a national or federal set of mandates, but rather articulate a framework of standards for states to employ voluntarily in strengthening their adult education programs with respect to college and career readiness. (This statement points out that OCTAE has left the decision about a specific set of standards to each state.

  Oregon has mandated use of the OACCRS by Title II-funded ABS programs no later than July 1, 2020.)

- Second, the order of the selected standards within a level does not represent an order in which they are to be taught or a hierarchy of importance.

- Third, the selected standards do not specify how instructors should teach, but rather merely define what all students should be expected to know and be able to do to be prepared for postsecondary success.

- Fourth, the standards are not a curriculum, and states or programs choosing to adopt them will need to complement the standards with high-quality curricula that align with the content and expectations.

- Fifth, the standards are not meant to specify the full spectrum of support and interventions appropriate for English language learners and students with special needs to meet these standards, nor do they mirror the significant diversity of students' learning needs, abilities, and achievement levels.

- Sixth, the standards do not offer an exhaustive list of what can be taught beyond the fundamentals specified within these CCR standards; much is purposefully left to the discretion of teachers, curriculum developers, program administrators, and states in deciding what (if any) content to add.

- Finally, while the mathematics and ELA/literacy components in this report are crucial to college and career readiness, they do not define the whole of such preparedness; students depend on a variety of readiness skills and preparation, including habits of mind such as stamina, persistence, punctuality, and time and workload management skills.

* These reflect many of the same design parameters for K-12 standards set out in the introduction to the Ccss for ELA/literacy and mathematics (NGA 2010a).

[26] Pimentel, S. (2013, April). *College and Career Readiness Standards for Adult Education* (United States, U.S. Department of Education, Office of Vocational and Adult Education). Retrieved June 24, 2019, from https://lincs.ed.gov/publications/pdf/CCRStandardsAdultEd.pdf

Appendix B: Guide to Aid Understanding Coding in the Standards

Below is a guide to aid understanding of the coding in the standards that follow. The text in **green** print is explanatory and not in the actual document.

The Number System (The Domain)	
Compute fluently with multi-digit numbers and find common factors and multiples.	- **The big concept or overarching standards statement**
• Fluently divide multi-digit numbers using the standard algorithm. (6.NS.2) • Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. (6.NS.3) • Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* (6.NS.4)	**Skills that support understanding The big concept**
The citation at the end of each standard (circled in red above) identifies the CCSS grade, domain, and standard number (or standard number and letter, where applicable). So, 6.NS.6a, for example, stands for Grade 6, Number Sense domain, Standard 5.OA.2 stands for Grade 5, Operations and Algebraic Thinking domain, Standard 2.	

Instructors who wish to see examples of lessons on each coded standard can simply paste the code (e.g., 6.NS.3) into their browsers and find an abundance of teaching ideas. Care should be taken that activities chosen are appropriate for adults rather than young children.

The overarching standard statement or big concept for this example, **Compute fluently with multi-digit numbers and find common factors and multiples,** may be deceptively simple. To understand this idea fully, the instructor must refer to the lower levels to find that up to this point, students may have worked only with numbers up to 1000 in addition and subtraction and have worked with multiplication and division up to 100. There are many gaps to be filled. Also, the big concept addresses fluency and that would imply that students must have a good grasp of the basic number facts and families as well as the relationships between the operations (cohesion). Decisions about calculator use must be made (Mathematical Practice 5 – use appropriate tools) and there are many ways to incorporate modeling (Mathematical Practice 4) to deepen learners' understanding.

Additionally, the second bulleted supporting skill refers to "multi-digit decimals." Does that mean students have conquered whole number computation? How many digits are in a "multi-digit" decimal? The instructor must consider the whole of the content progression to see that in preceding standards it is stated that students will perform operations with multi-digit whole numbers and decimals to hundredths. The two sub-benchmarks here add fluency and "standard algorithm" to the expectation. And there is the term "standard algorithm" - is the standard algorithm in the US the same as the standard algorithm world-wide? When, if any time, would calculator use be both beneficial?

The first part of the third bulleted supporting skill addresses finding common factors and multiples (Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.) A logical thought might be, "Isn't that a totally different objective?" The answer to that question lies in the fact that those skills were linked purposely to the two previous bullets: factors and multiples arise naturally from multiplication and division and instructors may make an effort to use those terms in their teaching. (e.g., If we have two factors, 48 and 207, what is their product? OR: Any column in the multiplication table lists the first 10 or so multiples of the number at the top. OR: If we go down the 5's column and the 3's column, notice that 15 is in each column...and so forth).

The third bulleted supporting skill also addresses applying the distributive property. Why is that included? How might understanding use of the distributive property contribute to computational fluency? (How do you multiply $5.97 x 4? Perhaps you think 4 x ($6 - $.03) = 4 x $6 – 4 x $.03? = $24 -$0.12 = $23.88). How might understanding the distributive property with whole numbers build cohesion as learners progress to higher levels?

Appendix C: OACCRS Mathematics Content Standards Progression by Domain[27]

OACCRS Mathematics Content Progressions

This document contains the College and Career Readiness Standards for Adult Education, organized by level and also by domain, to highlight the important progressions in the CCR Standards associated with the Major Work of the Level (MWOTL) are identified in plain type and standards that support the MWOTL are in italics.

Number and Ratios: Understanding and Operations

Level A – Number Base Ten
1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two- digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

[27] Appendix C is from College and Career Readiness Standards-in-Action, LINCS Training Module, March 2015; College and Career Standards. (n.d.). Retrieved from https://lincs.ed.gov/professional-development/resource-collections/by-topic/College and Career Standards

1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Level B – Number Base Ten

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

***2.NBT.6** Add up to four two-digit numbers using strategies based on place value and properties of operations.*

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three- digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.

3.NBT.1 *Use place value understanding to round whole numbers to the nearest 10 or 100.*
3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.
Level B – Fractions
3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.
3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.
3.NF.2a Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
3.NF.2b Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
3.NF.3 *Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.*
3.NF.3a *Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.*
3.NF.3b *Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.*
3.NF.3c *Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

3.NF.3d *Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.*

Level C – Number Base Ten

4.NBT.1 *Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*

4.NBT.2 *Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.*

4.NBT.3 *Use place value understanding to round multi-digit whole numbers to any place.*

4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.1 *Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.*

5.NBT.2 *Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.*

5.NBT.3 Read, write, and compare decimals to thousandths.
5.NBT.3a Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000).
5.NBT.3b Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
5.NBT.4 *Use place value understanding to round decimals to any place.*
5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm. [NOTE: A “standard algorithm” might be any accepted algorithm that fits the experience and needs of the students.]
5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. [NOTE: Applications involving financial literacy should be used.]
6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.
6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
6.NS.4 *Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).*

Level C – Fractions

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

***4.NF.3** Understand a fraction a/b with a > 1 as a sum of fractions 1/b.*

***4.NF.3a** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.*

***4.NF.3b** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.*

Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

4.NF.3c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

4.NF.3d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

4.NF.4a Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).

4.NF.4b Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)

4.NF.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

4.NF.6 *Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

4.NF.7 *Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.*

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$).

5.NF.2 *Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.*

For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
5.NF.5 *Interpret multiplication as scaling (resizing), by:* a. *Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.* b. *Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (nxa)/(nxb) to the effect of multiplying a/b by 1.*
5.NF.6 *Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.*
5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
5.NF.7a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.
5.NF.7b Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.
5.NF.7c *Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

6.NS.1 *Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mile and area 1/2 square mile?*

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."

Level D – Number Systems

6.NS.5 *Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.*

6.NS.6 *Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous levels to represent points on the line and in the plane with negative number coordinates.*

6.NS.6a *Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.*

6.NS.6b *Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.*

6.NS.6c *Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.*
6.NS.7 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous levels to represent points on the line and in the plane with negative number coordinates.
6.NS.7a *Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
6.NS.7b *Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3° C > –7° C to express the fact that –3 degrees C is warmer than –7 degrees C.*
6.NS.7c *Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*
6.NS.7d *Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*
6.NS.8 *Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.*
7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
7.NS.1a *Describe situations in which opposite quantities combine to make 0.* *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

7.NS.1b Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.
7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real- world contexts.
7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.
7.NS.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.
8.NS.2 *Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Level D – Ratio and Proportional Reasoning
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.3a Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
6.RP.3c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
6.RP.3d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.
7.RP.2 Recognize and represent proportional relationships between quantities.
7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (Also see 8.EE.5)

7.RP.2c Represent proportional relationships by equations. For example, if total cost *t* is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.
7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. (Also see 7.G.1 and G.MG.2)
Level E – Number and Quantity
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*

Algebra and Functions

Level A – Operations & Algebraic Thinking
1.OA.2 *Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings,and equations with a symbol for the unknown number to represent the problem.*
1.OA.3 Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)
1.OA.4 Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.
1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
1.OA.7 *Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.* *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*
1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.

Level B – Operations and Algebraic Thinking
2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
2.OA.2 Fluently add and subtract within 20 using mental strategies. Know from memory all sums of two one-digit numbers.
3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 x 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?.
3.OA.5 Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

3.OA.6 Understand division as an unknown-factor problem.

For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 x 5 = 8) or properties of operations. Know from memory all products of two one-digit numbers.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Level C – Operations and Algebraic Thinking

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 *Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.*

4.OA.3 *Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.*

4.OA.4 *Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole-number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.*

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.
6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.
6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.
6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as $5 - y$.
6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2 (8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.
6.EE.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole- number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.

6.EE.3 *Apply the properties of operations to generate equivalent expressions.*

For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.

6.EE.4 *Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).*

For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

6.EE.5 *Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.*

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.7 *Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.*

6.EE.8 *Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.*

Level D – Expressions and Equations

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 *Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.*

For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05." [Also see A.SSE.2, A.SSE .3, A.SSE .3a, A.CED.4]

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

7.EE.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

7.EE.4b Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

***8.EE.1**. Know and apply the properties of integer exponents to generate equivalent numerical expressions.*

For example, $3^2 \times 3^{-5} = 3^{-3} = (1/3)^3 = 1/27$.

***8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.*

8.EE.3 *Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 × 10^8 and the population of the world as 7 × 10^9, and determine that the world population is more than 20 times larger.*
8.EE.4 *Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.*
8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. [Also see 7.RP.2b]
8.EE.7 Solve linear equations in one variable.
8.EE.7a Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).
8.EE.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8.EE.8 Analyze and solve pairs of simultaneous linear equations.
8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.

8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Level D – Functions

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. [Also see F.IF.1]

8.F.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

8.F.4 *Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. [Also see F.BF.1 and F.LE.5]*

8.F.5 *Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. [Also see A.REI.10 and F.IF.7]*

Level E – Algebra

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*

A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.*

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

For example, see x^4 – y^4 as (x^2)^2 – (y^2^2, thus recognizing it as a difference of squares that can be factored as (x^2 – y^2)(x^2 + y^2). [Also see 7.EE.2]

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* [Also see 7.EE.2]

A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.* [Also see 7.EE.2]

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [NOTE: Emphasis should be on operations with polynomials.]

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. For example, represent inequalities describing nutritional and c ost constraints on combinations of different foods.*

A.CED.4 *Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.)* [Also see 7.EE.2 and F.IF.8]*

A.REI.1 *Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.*

A.REI.2 *Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.*

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A.REI.4 Solve quadratic equations in one variable.

A.REI.6 *Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.*

A.REI.10 *Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). [Also see 8.F.5]*

Level E – Functions

F.IF.1 *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). [Also see 8.F.1]*

F.IF.2 *Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.*

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

F.IF.6 *Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* [NOTE: See modeling conceptual categories]*

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [Also see 8.F.5]

F.IF.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{(t/10)}$, and classify them as representing exponential growth or decay.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.BF.1 Write a function that describes a relationship between two quantities.* [Also see 8.F.4]

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.*

F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*

F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.* [Also see 8.F.4]

GEOMETRY

Level A – Geometry and Geometric Measurement
K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter- circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.
1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Level B – Geometry and Geometric Measurement
2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
3.G.1 *Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.*

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.
2.MD.2 *Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.*
2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.
2.MD.6 *Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.*
3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
3.MD.7 Relate area to the operations of multiplication and addition.
3.MD.7a Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
3.MD.7b Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
3.MD.7c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a x b and a x c. Use area models to represent the distributive property in mathematical reasoning.

3.MD.7d Recognize area as additive. Find areas of rectilinear figures by decomposing them into non- overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

3.MD.8 *Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.*

Level C – Geometry and Geometric Measurement

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real- world and mathematical problems.

5.G.3 Understand that attributes belonging to a category of two- dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

6.G.4 *Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.*

4.MD.2 *Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.*

4.MD.3 *Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

4.MD.5 *Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:*

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measureangles.

b. An angle that turns through n one-degree angles is said to have an angle measure of ndegrees.

4.MD.6 *Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.*

4.MD.7 *Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.*

5.MD.1 *Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.*

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

5.MD.5a Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

5.MD.5b Apply the formulas V = l x w x h and V = B x h for rectangular prisms to find volumes of right rectangular prisms with whole- number edge lengths in the context of solving real world and mathematical problems.

5.MD.5c Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Level D – Geometry

7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. [Also see 7.RP.3]

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.5 *Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.*
7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three- dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them.
8.G.5 *Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Level E – Geometry
G.CO.1 *Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
G.MG.2 *Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* [Also see 7.RP.3]*

Data, Probability, and Statistical Measurement

Level A – Measurement and Data
1.MD.4 *Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.*
Level B – Measurement and Data
2.MD.10 *Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put- together, take-apart, and compare problems using information presented in a bar graph.*
3.MD.3 *Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
3.MD.4 *Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.*
Level C – Measurement & Data
5.MD.2 *Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. [NOTE: Plots of numbers other than measurements should also be encouraged.]*
6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

6.SP.4 *Display numerical data in plots on a number line, including dot plots, histograms, and box plots. [Also see S.ID.1]*

Level D – Statistics and Probability

6.SP.5 *Summarize numerical data sets in relation to their context, such as by:*

a. *Reporting the number of observations.*

b. *Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.*

c. *Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.*

d. *Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.*

7.SP.1 *Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.*

7.SP.2 *Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

7.SP.3 *Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
7.SP.4 *Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.* *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
7.SP.5 *Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.*
7.SP.6 *Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
7.SP.7 *Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.*
7.SP.7a *Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
7.SP.7b *Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.8a *Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.*
7.SP.8b *Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.*
8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?
Level E – Interpreting Data
S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). [Also see 6.SP.4]
S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S.ID.9 *Distinguish between correlation and causation.*

Appendix D: Major Work of the Levels by Domain[28]

This document is not meant to be a substitute for the CCR Standards for Adult Education; rather, it is meant to be used in conjunction with the CCR Standards for Adult Education, where full descriptions of the major work can be found in the introductions for each level.

Level A (Beginning Literacy/ CCRS Grades K-1):

- Number: Developing understanding of whole number place value for tens and ones;
- Number: Developing understanding of addition and subtraction and the properties of these operations;
- Geometry: Describing and reasoning about shapes and their attributes;
- Geometry: Developing understanding of linear measurement.

Level B (Beginning Basic/CCRS Grades 2-3):

- Number: Extending understanding of base-10 notation;
- Number: Adding and subtracting to 1,000; fluency and application to 100;
- Number: Understanding multiplication and division of whole numbers to 100;
- Number: Understanding division as inverse of multiplication; single-digit divisors;
- Number: Developing understanding of fractions, especially unit fractions;
- Geometry: Using standard units of measure for length, time, liquid volume, and mass;
- Geometry: Developing understanding of area and its relationship to addition and multiplication;
- Geometry: Analyzing and partitioning 2-dimensional shapes.

Level C (Low Intermediate/CCRS Grades 4-5 + 6):

- Number: Extending the number system to positive rational numbers;
- Number: Extending place value understanding for decimals to thousandth;

[28] This appendix is from LINCS Foundational Unit 1; United States, U.S. Department of Education. (2016). College and Career Readiness Standards In-Action. Retrieved June 26, 2019, from https://lincs.ed.gov/publications/pdf/ccr/Math_Unit_1_Materials/Math_1_part_mat.pdf

- Number: Attaining fluency with operations, using multi-digit whole numbers and decimals;
- Number: Understanding fraction equivalence and comparison;
- Number: Developing fluency with sums and differences of fractions;
- Number: Connecting ratio and rate to whole number multiplication and division;
- Algebra: Writing, evaluating, and interpreting expressions and equations;
- Geometry: Developing understanding of the coordinate plane;
- Geometry: Classifying geometric 2-dimensional figures based on properties;
- Geometry: Developing an understanding and solving problems involving volume and surface area;
- Statistics and Probability: Developing understanding of statistical variability;
- Number: Extending number sense and fluency with operations to all rational numbers;
- Number: Understanding ratio and rate and using them to solve problems.

Level D (Middle Intermediate to High Intermediate CCRS Grades 6 + 7-8):

- Algebra: Applying proportional relationships
- Algebra: Working with expressions and linear equations
- Algebra: Solving linear equations and systems of linear equations
- Algebra: Developing the concept of function
- Algebra: Graphing functions in the coordinate plane and analyzing their graphs
- Geometry: Solving problems involving scale drawings
- Geometry: Solving problems involving 2- and 3-dimensional figures: area, surface area, and volume
- Geometry: Analyzing 2- and 3-dimensional shapes using side length and angle measurements, similarity, and congruence
- Geometry: Applying the Pythagorean theorem
- Statistics and Probability: Understanding patterns of association for bivariate data and describing them with a linear equation, when appropriate

- Statistics and Probability: Summarizing and interpreting data and data distributions Statistics and Probability: Understanding and applying probability concepts

- Statistics and Probability: Drawing inferences about populations based on random samples (probability distributions)

Level E (Adult Secondary CCRS Grades 9-12):

- Number: Extending understanding of number systems to the set of real numbers

- Number: Writing equivalent expressions involving radicals and rational exponents

- Number: Reasoning quantitatively and the use of units and appropriate levels of precision

- Algebra: Defining, evaluating, comparing, and modeling with linear, quadratic, and exponential functions and equations

- Algebra: Building, interpreting, and analyzing functions using different representations

- Algebra: Reasoning with and solving linear, quadratic, and exponential equations and linear inequalities

- Algebra: Interpreting and using the structure of expressions to solve problems

- Algebra: Operating with algebraic expressions, including polynomials and rational expressions

- Geometry: Applying similarity and congruence concepts to geometric figures, including triangles

- Geometry: Using geometric models and volume formulas to solve measurement problems

- Statistics and Probability: Summarizing, representing, and interpreting one- and two-variable data, including using frequency tables

Appendix E: OACCRS Standards Progression Matrix

This matrix is meant to be an auxiliary resource for instructors, administrators, and programs to help them understand the full flow of the mathematics standards. To allow understanding of “what came before” and “what comes after” the matrix is presented in a different format than what is found in Appendix C. Those new to learning standards may decide to file this part of the OACCRS Handbook in the "Information to Study Later” drawer.

What is this Standards Progression Matrix?

The Standards Progression Matrix indicates the learning path students should follow to establish deep understanding of the topics in focus. These Standards Progressions organize the OACCRS Standards for Mathematics vertically by concept and horizontally by educational functioning level (EFL). They may aid recognizing the connections between what comes before and what comes after achievement of a specific standard or learning goal. That recognition is crucial for the following reasons:

- Planning the sequence of instruction for a specific skill or set of skills;
- Identifying specific trouble areas along the learning continuum for struggling students and facilitating learning by focusing on requisite skills;
- Creating formative assessment tools for monitoring student progress;
- Differentiating instruction or varying instructional strategies, learning activities, resources, and assessments in the multi-level classroom.

Further, the Progression Matrix supports the key shift of coherence in instruction. As stated earlier, mathematics is not a list of disconnected topics, instead it is a coherent body of knowledge made up of interconnected concepts. It is important for instruction to be based on progressions that build from level to level. Carefully connecting learning across levels allows students to build new understanding on the foundations previously attained. **Each standard is not a new event, but an extension of previous learning.**

How is the Matrix Read?

If one goes down any column, the matrix is much like Appendix C. Reading across a row reveals the connection between the levels. For instance, suppose an instructor is teaching Level 3 students and needs to teach 5.NBT.3 (Read, write, and compare decimals to thousandths.) It is important to know that for students to understand standard 5.NBT.3, they must understand standard 5.NBT.1 (Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.) Backtracking further, the same students must have a good grasp of (2.NBT.1) Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.

The matrix is designed with many empty spaces for example, on page 19 level 3, there is nothing to the left of 6.RP.1. Of course, that means that the concept (in this case, ratios) is first introduced to students at level 3. Notice also that under 5.RP.1 is 5.RP.2 (unit rates). As one moves horizontally on the matrix, it is seen that at level 4 students begin using ratios and rate reasoning to solve a variety of problems.

Please note that this matrix is best viewed online. The nature of this chart requires it to be very wide, the reader may have to zoom in or out on the page to view areas of interest. Printing of this matrix is best done by going to the pdf for the document found on the Adult Basic Skills (ABS) Policies, Training, Standards, Reporting Forms and Resources page and choosing “Shrink oversized pages” from the print menu. The print will then be very small.

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
NUMBERS AND OPERATIONS					
Understand place value.		**Understand the place value system.**			
(1.NBT.2) Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases. a. 10 can be thought of as a bundle of ten ones — called a “ten.” b. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). c. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.	**(2.NBT.1)** Understand that the three digits of a three-digit number repre-sent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases. 100 can be thought of as a bundle of ten tens — called a “hundred.” he numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	**(5.NBT.1)** Recognize that in a multi- digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.			
		(5.NBT.3) Read, write, and compare decimals to thousandths.			
		(5.NBT.3a) Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 ×1 + 3 ×(1/10) + 9 × (1/100) + 2 × (1/1000).			
	(2.NBT.2) Count within 1000; skip- count by 5s, 10s, and 100s;	**(5.NBT.3b)** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.			
	(2.NBT.3) Read and write numbers to 1000 using base-ten numerals, number names, and expanded form				

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
(1.NBT.3) Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**(2.NBT.4)** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	**(5.NBT.2)** Explain patterns in the number of zeroes of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **(5.NBT.4)** Use place value understanding to round decimals to any place.			
Use place value understanding and the properties of operations to add and subtract.					
(1.NBT.4) Add within 100, including adding a two-digit number and a one- digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **(1.NBT.5)** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	**(2.NBT.6)** Add up to four two-digit numbers using strategies based on place value and properties of operations. **(2.NBT.7)** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **(2.NBT.8)** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.				

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
(1.NBT.6) Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**(2.NBT.9)** Explain why addition and subtraction strategies work, using place value and the properties of operations.				

Generalize place value understanding for multi-digit whole numbers.

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
		(4.NBT.1) Recognize that in a multi- digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*			
		(4.NBT.2) Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi- digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.			
		(4.NBT.3) Use place value understanding to round multi-digit whole numbers to any place.			

Use place value understanding and properties of operations to perform multi-digit arithmetic.			
	(3.NBT.1) Use place value understanding to round whole numbers to the nearest 10 or 100. **(3.NBT.2)** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	**(4.NBT.4)** Fluently add and subtract multi- digit whole numbers using the standard algorithm.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
	(3.NBT.3) Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	**(4.NBT.5)** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.			
		(4.NBT.6) Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.			
Perform operations with multi-digit whole numbers and with decimals to hundredths.					
		(5.NBT.5) Fluently multiply multi-digit whole numbers using the standard algorithm.			
		(5.NBT.6) Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.			

	(5.NBT.7) Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. [Note from panel: Applications involving financial literacy should be used.]	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Develop understanding of fractions as numbers.		**Extend understanding of fraction equivalence and ordering.**			
	(3.NF.1). Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.	**(4.NF.1)** Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.			
	(3.NF.2) Understand a fraction as a number on the number line; represent fractions on a number line diagram.				
	(3.NF.2a) Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/ b and that the endpoint of the part based at 0 locates the number 1/b on the number line.				
	(3.NF.2b) Represent a fraction a/b on a number line diagram by marking off a length 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.				

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
	(3.NF.3) Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.	**(4.NF.2)** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.			
	(3.NF.3a) Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.				
	(3.NF.3b) Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction mode).				
	(3.NF.3c) Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.				
	(3.NF.3d) Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.				

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.					
		(4.NF.3) Understand a fraction a/b with a > 1 as a sum of fractions 1/b.			
		(4.NF.3a) Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.			
		(4.NF.3b) Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1+1 + 1/8 = 8/8 +8/8 + 1/8.			
		(4.NF.3c) Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.			
		(4.NF.3d) Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
		(4.NF.4) Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.			
		(4.NF.4a) Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4= 5 × (1/4).			
		(4.NF.4b) Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b).			
		(4.NF.4c) Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?			
Understand decimal notation for fractions, and compare decimal fractions.					
		(4.NF.6) Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
		(4.NF.7) Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.			
Use equivalent fractions as strategy to add and subtract fractions.					
		(5.NF.1) Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general,* a/b + c/d = *(*ad + bc*)*/bd.*)*			
		(5.NF.2) Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 + 3/7, by observing that 3/7 < 1/2.*			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Apply and extend previous understanding of multiplication and division to multiply and divide fractions.					
		(5.NF.3) Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as theresult of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*			
		(5.NF.4) Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.			
		(5.NF.5) Interpret multiplication as scaling (resizing), by: comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication; and explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1,			

	results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	
	(5.NF.6) Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
		(5.NF.7) *Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.*			
		(5.NF.7a) *Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to showthe quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.*			
		(5.NF.7b) *Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.*			
		(5.NF.7c) *Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
THE NUMBER SYSTEM					
Compute fluently with multi-digit numbers and find common factors and multiples.					
		(6.NS.2) Fluently divide multi-digit numbers using the standard algorithm.			
		(6.NS.3) Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.			
		(6.NS.4) Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*			
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.					
		(6.NS.1) Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will*			

	each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mile and area 1/2 square mile?	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Apply and extend previous understandings of numbers to the system of rational numbers.					
			(6.NS.5) Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/ negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.		
			(6.NS.6) Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous levels to represent points on the line and in the plane with negative number coordinates.		
			(6.NS.6a) Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g.,-(-3) = 3, and that 0 is its own opposite.		
			(6.NS.6b) Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.		

	(6.NS.6c) Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. (6.NS.6c) people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.					
			(7.NS.1) Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.		
			(7.NS.1a) Describe situations in which opposite quantities combine to make 0. For example, if a check is written for the same amount as a deposit, made to the same checking account, the result is a zero increase or decrease in the account balance.		
			(7.NS.1b) Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real- world contexts.		
			(7.NS.1c) Understand subtraction of rational numbers as adding the additive inverse, p – q = p+ -q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.		

	(7.NS.1d) Apply properties of operations as strategies to add and subtract rational numbers.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
			(7.NS.2) Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.		
			(7.NS.2a) Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.		
			(7.NS.2b) Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non- zero divisor) is a rational number. If p and q are integers, then -(p/q) = (-p)/q = p/(-q). Interpret quotients of rational numbers by describing real-world contexts.		
			(7.NS.2c) Apply properties of operations as strategies to multiply and divide rational numbers.		
			(7.NS.2d) Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.		

	(7.NS.3) Solve real-world and mathematical problems involving the four operations with rational numbers.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Know that there are numbers that are not rational, and approximate them by rational numbers.					
			(8.NS.2) Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$, *show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*		

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
RATIOS AND PROPORTIONAL RELATIONSHIPS					
Understand ratio concepts and use ratio reasoning to solve problems.					
		(6.RP.1) Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**(6.RP.3)** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.		
			(6.RP.3a) Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.		
		(6.RP.2) Understand the concept of a unit rate a/b associated with a ratio $a{:}b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*	**(6.RP.3b)** Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?		
			(6.RP.3c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.		
			(6.RP.3d) Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.		

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Analyze proportional relationships and use them to solve real-world and mathematical problems.					
			(7.RP.1) Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*		
			(7.RP.2) Recognize and represent proportional relationshipsbetween quantities.		
			(7.RP.2a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.		
			(7.RP.2b) [Also see 8.EE.5] Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.		
			(7.RP.2c) Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.		

	(7.RP.2d) Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
			(7.RP.3) [Also see 7.G.1 and G.MG.2] Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*		
NUMBER AND QUANTITY					
Extend the properties of exponents to rational exponents.					
				(N.RN.2) Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
Reason quantitatively and use units to solve problems.					
					(N.Q.1) Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*
					(N.Q.3) [Also see 8.EE.4] Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*

Note: Making mathematical models is a Standard for Mathematical Practice (MP.4), and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
OPERATIONS AND ALGEBRAIC THINKING					
Represent and solve problems involving addition and subtraction.					
(1.OA.2) Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**(2.OA.1)** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.				
Understand and apply properties of operations and the relationship between addition and subtraction.	**Understand properties of multiplication and the relationship between multiplication and division.**				
(1.OA.3) *Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (propertyof addition.)*	**(3.OA.5) A**pply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 ×5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2). (Distributive property.)*				
(1.OA.4) *Understand subtraction as an unknown-addend problem. For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.*	**(A3.OA.6)** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*				

Add and subtract within 20.	
(1.OA.5) Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
(1. OA.6) Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1= 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	**(2.OA.2)** Fluently add and subtract within 20 using mental strategies. Know from memory all sums of two one-digit numbers.				
Work with addition and subtraction.	**Represent and solve problems involving multiplication and division.**	**Use the four operations with whole numbers to solve problems.**			
(1.OA.7) Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*	**(3.OA.1)** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	**(4.OA.1)** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.			
	(3.OA.2) Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	**(4.OA.2)** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
(1.OA.8) Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ϒ − 3, 6 + 6 = ϒ.*	**(3.OA.3)** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	**(4.OA.3)** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.			
	(3.OA.4) Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ϒ ÷ 3, 6 × 6 = ?.*				
Multiply and divide within 100.					
	(3.OA.7) Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. Know from memory all products of two one-digit numbers.				
Solve problems involving the four operations, and identify and explain patterns in arithmetic.		**Generate and analyze patterns.**			
	(3.OA.8) Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.				

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
	(3.OA.9) Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	**(4.OA.5)** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*			
Gain familiarity with factors and multiples.					
		(4.OA.4) Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.			
Write and interpret numerical expressions.					
		(5.OA.1) Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.			
		(5.OA.2) Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (2100 + 425) is three times as large as the 2100 + 425, without having to calculate the indicated sum or product.*			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
EXPRESSIONS AND EQUATIONS					
Apply and extend previous understandings of arithmetic to algebraic expressions.			**Use properties of operations to generate equivalent expressions.**	**Interpret the structure of expressions.**	
		(6.EE.1) Write and evaluate numerical expressions involving whole-number exponents.	**(7.EE.2) [Also see A.SSE.2, A.SSE.3, A.SSE.3a, A.CED.4]** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* a + *0.05*a = *1.05*a *means that "increase by 5%" is the same as "multiply by 1.05."*	**(A.SSE.1)** Interpret expressions that represent a quantity in terms of its context.*	
				(A.SSE.1a) Interpret parts of an expression, such as terms, factors, and coefficients.*	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
		(6.EE.2) Write, read, and evaluate expressions in which letters stand for numbers.		**(A.SSE.2) [Also see 7.EE.2]** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	
		(6.EE.2a) Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as 5 – y.			
		(6.EE.2b) Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 * 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.			
		(6.EE.2c) Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
		(6.EE.3) Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.	**(7.EE.1)** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.		
		(6.EE.4) Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.			
Write expressions in equivalent forms to solve problems.					
				(A.SSE.3) [Also see 7.EE.2] Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*	
				(A.SSE.3a) [Also see 7.EE.2] Factor a quadratic expression to reveal the zeros of the function it defines.*	

Perform arithmetic operations on polynomials.	
	(A.6.1) Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A.APR.1) [Note from panel: Emphasis should be on operations with polynomials.]

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Reason about and solve one-variable equations and inequalities.			**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**		
		(6.EE.5) Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	**(7.EE.4) [Also see A.CED.1 and A.REI.3]** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.		
		(6.EE.6) Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	**(7.EE.4a) [Also seeA.CED.1 and A.REI.3]** Solve word problems leading to equations of the form px + q = r and p(x + q) r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?		
		(7.EE.4b) [Also see A.CED.1 and A.REI.3] Solve word problems leading to inequalities of the form px+ q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
		(6.EE.7) Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. **(6.EE.8)** Write an inequality of the form $x > c$ or $x < c$ torepresent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	**(7.EE.3)** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*		

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Represent and analyze quantitative relationships between dependent and independent variables.					
		(6.EE.9) Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* d = 65t *to represent the relationship between distance and time.*			
Work with radicals and integer exponents.					
			(8.EE.1) [Also see F.IF.8b] Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{(-5)} = 3^{(-3)} = (1/3)^3 = 1/27$.		
			(8.EE.2) [Also see A.REI.2] Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.		

	(8.EE.3) Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
			(8.EE.4) [Also see N.Q.3] Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose unitsof appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for sea floor spreading). Interpret scientific notation that has been generated by technology.		
Understand the connections between proportional relationships, lines, and linear equations.					
			(8.EE.5) [Also see 7.RP.2b] Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*		
Analyze and solve linear equations and pairs of simultaneous linear equations.					
			(8.EE.7) [Also see A.REI.3] Solve linear equations in one variable.		
			(8.EE.7a) Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).		

	(8.EE.7b) Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
			(8.EE.8) Analyze and solve pairs of simultaneous linear equations.		
			(8.EE.8a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.		
			(8.EE.8b) [Also see A.REI.6] Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x +2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.		
			(8.EE.8c) Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.		
Rewrite rational expressions.					
				(A.APR.6) Rewrite simple rational expressions in different forms; write $a(x)/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of *r x* less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Create equations that describe numbers or relationships.					
				(A.CED.1) [Also see 7.EE.4, 7.EE.4a, and 7.EE.4b] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.*	
				(A.CED.2) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	
				(A.CED.3) Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**	
				(A.CED.4) [Also see 7.EE.2] Rearrange formulas to highlight a quantity of interest, using thesame reasoning as in solving equations. *For example, rearrange Ohm's law* V = IR *to highlight resistance* R.*	

Understand solving equations as a process of reasoning and explain the reasoning.	
	(A.REI.1) Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
	(A.REI.2) [Also see 8.EE.2] Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Solve equations and inequalities in one equation.					
				(A.REI.3) [Also see 7.EE.4 7.EE.4a, 7.EE.4b, and 8.EE.7] Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
				(A.REI.4) Solve quadratic equations in one variable.	
Solve systems of equations.					
					(A.REI.6) [Also see 8.EE.8b] Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
Represent and solve equations and inequalities graphically.					
					(A.REI.10) [Also see 8.F.5] Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
FUNCTIONS					
Define, evaluate, and compare functions.				**Understand the concept of a function and use function notation.**	
			(8.F.1) [Also see F.IF.1] Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	**(F.IF.1) [Also see 8.F.1]** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	

	(8.F.3) Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line.*	**(F.IF.2)** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Use functions to model relationships between quantities.				**Interpret functions that arise in applications in terms of the context.**	
			(8.F.4) [Also see F.BF.1 and F.LE.5] Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.		***(F.IF.4)*** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *For example, for a quadratic function modeling a projectile in motion, interpret the intercepts and the vertex of the function in the context of the problem*.* [*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.]*
			(8.F.5) [Also see A.REI.10 and F.IF.7] Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.		***(F.IF.5)*** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person- hours it takes to assemble* n *engines in a factory, then the positive integers would be an appropriate domain for the function.**

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
					(F.IF.6) Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* [NOTE: See conceptual modeling categories.]
Analyze functions using different representations.					
					(F.IF.7) [Also see 8.F.5] Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
					(F.IF.8b) [Also see 8.EE.1] Use properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in an exponential function and then classify it as representing exponential growth or decay.*
					(F.IF.9) Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
Build a function that models a relationship between two quantities.					
				(F.BF.1) [Also see 8.F.4] Write a function that describes a relationship between two quantities.*	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Construct and compare linear, quadratic, and exponential models and solve problems.					
				(F.LE.1) Distinguish between situations that can be modeled with linear functions and with exponential functions.*	
				(F.LE.1b) Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*	
				(F.LE.1c) Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*	
Interpret expressions for functions in terms of the situation they model.					
					(F.LE.5) [Also see8.F.4] Interpret the parameters in a linear or exponential function in terms of a context.*

Note: Making mathematical models is a Standard for Mathematical Practice (MP.4), and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).

GEOMETRY (G)

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
GEOMETRIC SHAPES AND FIGURES					
Analyze, compare, create, and compose shapes.		**Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**	**Draw, construct, and describe geometrical figures and describe the relationships between them.**		
(K.G.4) Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).		**(4.G.1)** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	**(7.G.1) [Also see7.RP.3]** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.		
Classify two-dimensional figures into categories based on their properties.					
		(5.G.3) Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*			
Reason with shapes and their attributes.					
(1.G.2) Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.	**(2.G.1)** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.				

	(2.G.3) Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
	(3.G.1) Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these.				
	(3.G.2) Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*				
Graph points on the coordinate plane to solve real-world and mathematical problems.					
		(5.G.1) Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).			

	(5.G.2) Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Solve real-world and mathematical problems involving area, surface area, and volume.			**Solve real-life and mathematical problems involving angle, measure, area, surface area, and volume.**		
		(6.G.1) Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	**(7.G.4)** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.		
		(6.G.3) Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	**(7.G.5)** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.		
		(6.G.4) Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**(7.G.6) [Also see G.GMD.3]** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.		

CONGRUENCE		
Understand congruence and similarity using physical models, transparencies, or geometry software.		**Experiment with transformations in the plane.**
	(8.G.2) [Also see G.SRT.5] Understand that a two- dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	
	(8.G.4) [Also see G.SRT.5] Understand that a two- dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
			(8.G.5) Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	**(G.CO.1)** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY					
Understand and apply the Pythagorean Theorem.				**Prove theorems involving similarity.**	
			(8.G.7) Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.		**(G.SRT.5) [Also see 8.G.2 and 8.G.4]** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
			(8.G.8) Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.		
GEOMETRIC MEASUREMENT AND DIMENSION					
Explain volume formulas and use them to solve problems.					
				(G.GMD.3) [Also see 7.G.6] Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*	

MODELING WITH GEOMETRY	
Apply geometric concepts in modeling situations.	
	(G.MG.2) [Also see 7.RP.3] Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTU's per cubic foot).*

DATA (D)

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
MEASUREMENT AND DATA					
Measure lengths indirectly and by iterating length units.	**Measure and estimate lengths in standard units.**				
(1.MD.2) Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	**(2.MD.2)** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **(2.MD.3)** Estimate lengths using units of inches, feet, centimeters, and meters. **(2.MD.4)** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (2.MD.4)				
Represent and interpret data.					
(1.MD.4) Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**(2.MD.10)** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	**(5.MD.2)** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this level to solve problems involving information presented in line plots. *For example, given different measurements ofliquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally*. [Note from panel: Plots of numbers other than measurements also should be encouraged.]			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
	(3.MD.3) Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*				
	(3.MD.4) Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.				
Relate addition and subtraction to length.					
	(2.MD.6) Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 ona number line diagram.				
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.		**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**			
	(3.MD.1) Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	**(4.MD.2)** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
	(3.MD.2) Measure and estimate liquid volumes and masses of objects using standard units of grams (g),kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.	**(4.MD.3)** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*			
Understand concepts of angle and measure angles.					
		(4.MD.5) Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.			
		(4.MD.6) Measure angles in whole- number degrees using a protractor. Sketch angles of specified measure.			

	(4.MD.7) Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Convert like measurement units within a given measurement system.					
		(5.MD.1) Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi- step, real-world problems.			
Understand concepts of area and relate to area of multiplication and addition.		**Understand concepts of volume and relate volume to multiplication and to addition.**			
	(3.MD.5) Recognize area as an attribute of plane figures and understand concepts of area measurement. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measurearea. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	**(5.MD.3)** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. A cube with side length 1unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.			
	(3.MD.6) Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised	**(5.MD.4)** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units			

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
	(3.MD.7) Relate area to the operations of multiplication and addition.	**(5.MD.5)** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.			
	(3.MD.7a) Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.	**(5.MD.5a)** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.			
	(3.MD.7b) Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real- world and mathematical problems, and represent whole- number products as rectangular areas in mathematical reasoning.	**(5.MD.5b)** Apply the formulas V = l × w × h and V = B × h for rectangular prisms to find volumes of right rectangular prisms with whole- number edge lengths in the context of solving real-world and mathematical problems.			
	(3.MD.7c) Use tiling to show in a concrete case that the area of a rectangle with whole- number side lengths a and b c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.	**(5.MD.5c)** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non- overlapping parts, applying this technique to solve real- world problems.			

	(3.MD.7d) Recognize area as additive. Find areas of rectilinear figures by decomposing them into non- overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real- world problems.		
Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.			
	(3.MD.8) Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.		

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
STATISTICS AND PROBABILITY					
Develop understanding of statistical variability.					
		(6.SP.1) Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*			
		(6.SP.2) Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (6.SP.2)			
		(6.SP.3) Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.			

Summarize and describe distributions.			Summarize, represent, and interpret data on a single count or measurable variable.	
	(6.SP.4) [Also see S.ID.1] Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	**(6.SP.5)** Summarize numerical data sets in relation to their context, such as by: reporting the number of observations; describing the nature of the attribute under investigation, including how it was measured and its units of measurement; giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered; and relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	**(S.ID.1) [Also see 6.SP.4 and 8.SP.1]** Represent data with plots on the real number line (dot plots, histograms, and box plots).	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
				(S.ID.3) [Also see 7.SP.4] Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points.	
Use random sampling to draw inferences about a population.					
			(7.SP.1) Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.		
			(7.SP.2) Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*		

Draw informal comparative inferences about two populations.		
	(7.SP.3) Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
			(7.SP.4) [Also see S.ID.3] Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in one chapter of a science book are generally longer or shorter than the words in another chapter of a lower level science book.*		
Investigate chance processes and develop, use, and evaluate probability models.					
			(7.SP.5) Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.		
			(7.SP.6) Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*		

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
			(7.SP.7) Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.		
			(7.SP.7a) Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.		
			(7.SP.7b) Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?		
			(7.SP.8b) Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.		

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
Investigate patterns of association in bivariate data.					
			(8.SP.1) [Also see S.ID.1]. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.		
			(8.SP.2) Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.		
			(8.SP.3) [Also see S.ID.7]. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.		

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5	LEVEL 6
			(8.SP.4) [Also see S.ID.5] Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they like to cook and whether they participate actively in a sport. Is there evidence that those who like to cook also tend to play sports?*		
Summarize, represent, and interpret data on two categorical and quantitative variables.					
				(S.ID.5) [Also see 8.SP.4] Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	
Interpret linear models.					
					(S.ID.7) [Also see 8.SP.3] Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
					(S.ID.9) Distinguish between correlation and causation.

Appendix F: CASAS Score Ranges for NRS and OACCRS Levels

NRS Level	OACCRS Level	CASAS Math Goals Scale Scores
Beginning ABE Literacy (ABE Level 1-all subjects)	A	193 and below
Beginning Basic Education (ABE Level 2—all subjects)	B	194-203
Low Intermediate Basic Education (ABE Level 3—all subjects)	C	204-214
Middle Intermediate Basic Education (ABE Level 4--Mathematics) *	Part of D	215-225
High Intermediate Basic Education (ABE Level 5-Mathematics)	Remainder of D	226-235
Adult Secondary Education (ABE Mathematics Level 6)	E	236 and above

Appendix G: Sample Questions to Connect the Math Content and Practice Standards

Topic	Indicators That Students Can Apply the Practice	Instructor Questions to Foster Connections
Make sense of problems and persevere in solving them	• Interpret and make meaning of the problem to find a starting point. Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask , “Does this make sense?” Use various approaches to solution	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about...? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you’ve used to this point. What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin?

Reason abstractly and quantitatively	• Make sense of quantities and their relationships. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem • Attend to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? What is the relationship of the quantities? • How is____related to______? • What is the relationship between__ and____? • What does________mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use...? • Could we have used another operation or property to solve this task? Why or why not?
Construct viable arguments and critique the reasoning of others	• Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/ revise the argument. • Compare two arguments and determine correct or flawed logic.	• What mathematical evidence would support your solution? • How can we be sure that...? • How could you prove that...? Will it still work if...? • What were you considering when...? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?)

		• Did you try a method that did not work? Why didn't it work? • Would it ever work? Why or why not? • What is the same and what is different about...? • How could you demonstrate a counter-example?
Model with mathematics	☐ Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). ☐ Apply the mathematics they know to solve everyday problems. ☐ Are able to simplify a complex problem and identify important quantities to look at relationships. ☐ Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. ☐ Reflect on whether the results make sense, possibly improving/revising the model. • Ask themselves, "How can I represent this mathematically?"	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What is an equation or expression that matches the diagram, number line.., chart..., table..? • Where did you see one of the quantities in the task in your equation or expression? • How would it help to create a diagram, graph, table...? What are some ways to visually represent...? • What formula might apply in this situation?

Use appropriate tools strategically	• Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	• What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use...a graph...,number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...? • What can using a show us that_______may not? • In what situations might it be more informative or helpful to use...?
Attend to precision	• Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand the meanings of symbols used inmathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • What would be a more efficient strategy?

	• Calculate efficiently and accurately.	☐ How are you showing the meaning of the quantities? ☐ What symbols or mathematical notations are important in this problem? ☐ What mathematical language..., definitions..., properties can you use to explain...? ☐ How could you test your solution to see if it answers the problem?
Look for and make use of structure	• Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	• What observations do you make about...? What do you notice when...? • What parts of the problem might you eliminate..., simplify...? • What patterns do you find in...? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? How does this relate to...? • In what ways does this problem connect to other mathematical concepts?

Look for and express regularity in repeated reasoning	• See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and seethe structure in similar situations. • Continually evaluate the reasonableness of their intermediate results	• Explain how this strategy work in other situations? • Is this always true, sometimes true or never true? How would we prove that...? • What do you notice about...? • What is happening in this situation? What would happen if...? • Is there a mathematical rule for...? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

Appendix H: Sample Lesson Interweaving the Standards for Mathematical Content with the Standards for Mathematical Practice

The pages in this appendix are excerpted from an open education resource textbook (Growing Math Roots by Donna Parrish and Lori Lundine posted on the Creative Commons and Open Oregon Site in July, 2019.)

In the book there are several lessons in the exponent section that for the sake of brevity are not included in this handbook. The first page in the appendix is a student page while the rest are samples from the Instructor Pages.

Though the main purpose for including this appendix in the handbook is to demonstrate how the the Standards for Mathematical Practice and the Standards for Mathematical Content are inextricably linked, there are other purposes for its addition including, but not limited to the following:

- To demonstrate how a simple lesson that is often taught strictly from the viewpoint of computation can be used as a spring board for richer mathematics instruction appropriate for an adult audience

- To demonstrate how a simple lesson can be taught to learners within the same setting that are at different levels of mathematics readiness.

- To allow instructors some exposure to an Open Education Resource (OER) in hopes of expanding knowledge of the wealth of information that can be accessed and stored in such repositories. (The sample in this appendix is licensed under a CC-BY-NC-SA license. That means users may copy, distribute, display and perform the work and make derivative works and remixes based on it only if they give the author or licensor the credits, the attribution. Further users may distribute derivative works only under a license identical - "not more restrictive" - to the license that governs the original work, and users may copy, distribute, display, and perform the work and make derivative works and remixes based on it only for non-commercial purposes.)

Warming Up on Exponents

Wonderful Widget Works Job Opening

"PILE OF PENNIES"

is in the Public Domain, C00

"PILE OF MONEY"

is in the Public Domain, CC0

The Wonderful Widget Works Company has come to town. Everyone is excited about a classified ad that appeared in the newspaper regarding the hiring of a master widget maker who needs to be a real problem solver. One unique aspect of the position is that the individual hired will have a choice of salaries.

- Salary option one: $22,000 a month.
- Salary option two: 1 cent the first day, then thereafter double the previous day's salary (day 2, the salary would be 2 cents: day three, the salary would be 4 cents; and so on) until the end of the 30-day month.

Which salary option would you choose?

What is your best guess about the total salary (day 30) for option 2?

1. Make a chart to tally the salary for several days of the month for option 2. Notice any patterns that emerge.
2. For option 2, the number of coins on the second day would be 2 cents (1 penny times 2), and on the third, the salary would be will be 2 × 2 or 4 cents. On the fourth day, the salary would be 2 x 2 x 2 cents or 8 cents. Since 2 is multiplied by itself 3 times, we can write the expression for the 4th day's salary using exponents as 23. On the fifth day, the salary would be (the fourth day's salary) x 2 or 2 x 2 x 2 x 2. How would we write that using exponents? Did you think 24? What are some ways to express the amount of money on the 6th day? The 7th day? Continue until you see a pattern emerge.
3. Write an expression for the number of coins there will be on the 30th day. Use whatever tools are available to you to figure how much money that is.

This lesson is from *Growing Math Roots*, by Donna Parrish and Lori Lundine

Teaching the Lessons (Instructor Pages)

Overview of the Exponent Section

Every one of the math practices could be addressed in these lessons, depending on the particular setting. The examples given in italics are not all inclusive.

1. Make sense of the mathematics contextualized situations and persevere in obtaining solutions. (Students complete a chart to determine which salary would be better. Completing the salary chart to day 30 has many steps unless students grasp the exponential relationship. Whether the relationship is noticed or not, getting to a satisfactory solution requires "stick-to-itiveness." The homework deals with exponents in formulas – real-life applications.)

2. Reason abstractly and quantitatively, apply life experiences and knowledge of mathematical concepts, procedures, and technology to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension. (Students connect symbols to their numerical referents. They understand exponential notation as repeated multiplication of the base number.)

3. Construct viable arguments and critique the reasoning of others. (Students state which salary is more desirable and why, present their findings to a group, and determine the validity of the arguments of others.)

4. Understand, interpret, and model with concrete objects and symbolic representations (e.g., pictures, numbers, graphs, computer representations). *(Students model simple exponential expressions and model exponential growth on graphs.)* Modeling Exponents Video.

5. Use appropriate tools strategically and estimate to predict results and to check to see if results are reasonable. (Students will use calculators or spreadsheets to relieve tedious computation.)

6. Attend to precision in calculation, symbolism, and communication. (Students begin using appropriate terms, notation, and calculator functions- base, exponent, power, "squared" and "cubed", the ^ key on the calculator)

7. Look for and make use of structure. (Students identify the pattern of increase and use that pattern to find the next steps in the activities.)

8. Look for and express regularity in repeated reasoning. (Students will verbalize patterns in repeated multiplication and express repeated multiplication with exponents.)

In teaching the concepts in this section, particular attention should be paid to the three key shifts in the Adult Basic Education Math Standards.

1. Focus is on exploring the topic of the power of exponents in depth. This set of lessons deals with the topic on the basis of much more than necessary computation and procedures. Students will learn that exponential growth is quite different from linear growth, though they may not yet have the vocabulary to express that.

2. Coherence - Connection to prior learning - linking exponents to the strands of Number Sense, Algebraic Thinking, and Data and Statistics.

   Examples of building coherence to prior learning:

   a. Multiplication is a faster way to add numbers when the addends are the same.

   b. When we add five groups of 10, we use an abbreviation and a different notation, called multiplication. $10+10+10+10+10=5\times10$

   c. If multiplication is a more efficient way to represent addition problems involving the repeated addition of the same addend, do you think there might be a more efficient way to represent the repeated multiplication of the same factor, as in $10\times10\times10\times10\times10$ =? Allow students to make suggestions; some may recall this from previous lessons. $10\times10\times10\times10\times10=105$.

   d. We see that when we add five groups of 10, we write 5×10, but when we multiply five copies of 10, we write 105. So, multiplication by 5 in the context of addition corresponds exactly to the exponent 5 in the context of multiplication.

   e. Make students aware of the correspondence between addition and multiplication because what they know about repeated addition helps them learn exponents as repeated multiplication going forward.

   f. The little 5 we write is called an exponent and is written as a superscript.

Instructor Pages for Specific Lessons in the Exponents Section

Section 1

The salary task connects repeated calculations with an expression involving exponents to create a shorthand notation that can be used to answer the questions being asked (MP.8). The instructor could pose this salary choice task with small or large groups and lead them through creating a table to demonstrate the repeated calculations necessary to solve the problem. Once the pattern has been discussed, the instructor could have students begin to discuss the exponential growth demonstrated by the table, solutions to the questions posed in the task, and guide the students in using the repeated calculations to write an expression that could be used to solve efficiently any question that could be asked about the exponential growth.

Solution: To find the general pattern, we notice that to find the number of coins in a row, we doubled the number of coins in the previous row. If we continue the pattern, at the end of day 4 there will be 2 x 4=8 coins, and at the end of day 5 there will be 2 x 8=16 coins. If we look at day 4 again, we have 16=2 x 8 coins. The 8 coins were obtained because 8=2 x 4 coins and the 4 coins came from 4=2x2 coins. On day 4 we could also say we have 8 = 2x4 = 2x(2x2) = 23 coins. We can rewrite every number of coins in the table in terms of powers of 2.

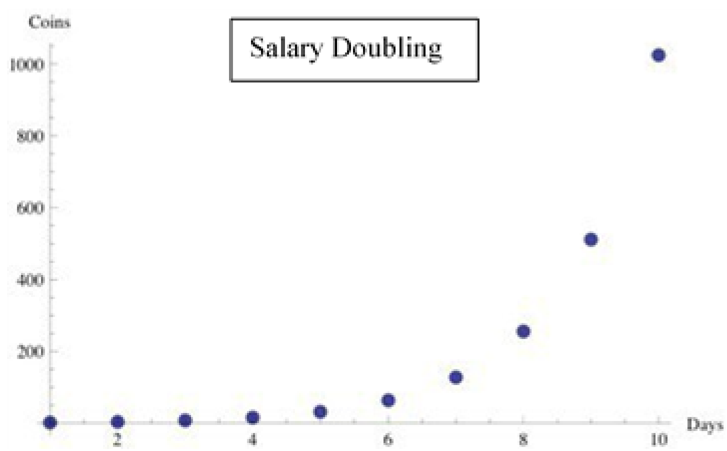
This would be a good time to introducing working with variables. A formula might be p = 2d-1 where p is the pay for the day and d is the number of the day.

Having found a way to easily express the number of coins there will be on a particular day, we don't need to fill in every row in the table; instead we can use a calculator or a spreadsheet to find that on day 30, the lucky person hired would be paid 536870912 pennies which is, of course, $5,368,709.12 and he/she would have earned almost $11 million for the month. Using the appropriate tools is another of our Math Practices.

Since this number ($11 million) is a lot bigger than $22,000, we can conclude that it is to the worker's advantage to choose the pennies doubled salary.

The data for this activity is on the next page. Note there is a column for money earned each day as well as the total salary. There are many interesting patterns in the data. What can your learners find?

Day	Number of pennies	Exponent expression	Total salary earned
1	1	2^0	$ 0.01
2	2	2^1	$ 0.03
3	4	2^2	$ 0.07
4	8	2^3	$ 0.15
5	16	2^4	$ 0.31
6	32	2^5	$ 0.63
7	64	2^6	$ 1.27
8	128	2^7	$ 2.55
9	256	2^8	$ 5.11
10	512	2^9	$ 10.23
11	1024	2^10	$ 20.47
12	2048	2^11	$ 40.95
13	4096	2^12	$ 81.91
14	8192	2^13	$ 163.83
15	16384	2^14	$ 327.67
16	32768	2^15	$ 655.35
17	65536	2^16	$ 1,310.71
18	131072	2^17	$ 2,621.43
19	262144	2^18	$ 5,242.87
20	524288	2^19	$ 10,485.75
21	1048576	2^20	$ 20,971.51
22	2097152	2^21	$ 41,943.03
23	4194304	2^22	$ 83,886.07
24	8388608	2^23	$ 167,772.15
25	16777216	2^24	$ 335,544.31
26	33554432	2^25	$ 671,088.63
27	67108864	2^26	$ 1,342,177.27
28	134217728	2^27	$ 2,684,354.55
29	268435456	2^28	$ 5,368,709.11
30	536870912	2^29	$ 10,737,418.23



Excel Spreadsheet for Doubling One Cent

Section 2

This lesson deals with exponent vocabulary and computation. In terms of fluency, whole numbers, decimal fractions, and common fractions are raised to a positive power. The instructor would be well advised to allow calculator use and may choose to teach learners how to use the exponent (^) key on the calculator.

The focus is on writing expressions in standard notation, expanded notation, and in exponential notation and moving fluently from one form to another.

Definitions of a number to the first power and a number to the zero power are developed, as is the "shortcut" of expressing powers of 10 by annexing zeroes.

Section 3

This part of the lesson deals with both whole number and variable multiplication and division expressions that contain exponents. For division and for the sake of coherence, it is good practice to develop the concept of 1 in fraction terms $\frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \cdots = \frac{aa}{aa}$

The following rules are developed:

$$xx^{aa} \cdot xx^{bb} = xx^{aa+bb}$$

$$\frac{xx}{xx^{bb}} = xx^{aa-bb} \ , \mathrm{x} \neq 0$$

The instructor might caution students about allowing the x (divisor) to be equal to zero. One way to address that issue is to have students use the calculator to divide by zero. The calculator will probably indicate "error." Mathies say division by zero is undefined. "Why is that?" the coherence building instructor might ask. Remember that multiplication and division are closely related. If we know 4 x 2 = 8, we know two division relationships: 8 ÷ 2 = 4 and 8 ÷4 = 2. We can verify that 8 ÷ 2 = 4 by multiplying 2 x 4 and getting 8. We can verify 8 ÷4 = 2 by multiplying 4 x 2 and getting 8.

Now consider 8 ÷ 0: If we say the quotient is 8, that means 8 x 0 = 8 (and of course that is incorrect.) If we say the quotient is 0, that means 0 x 8 = 8 (and of course that is incorrect.) So just what is 8 ÷ 0? It is undefined!

Section 4

It is suggested that Section 4 be used for independent practice (i.e., homework). It will help learners if they can graph those relationships using a Desmos Graphing calculator, Excel, or some similar tool.

The paper folding activity piggybacks on the Wonderful Widget Works activity done in class and deals with both exponential growth and exponential decay.
The second part of section 4 deals with plugging into equations/formulas and solving using simple computation with exponents.

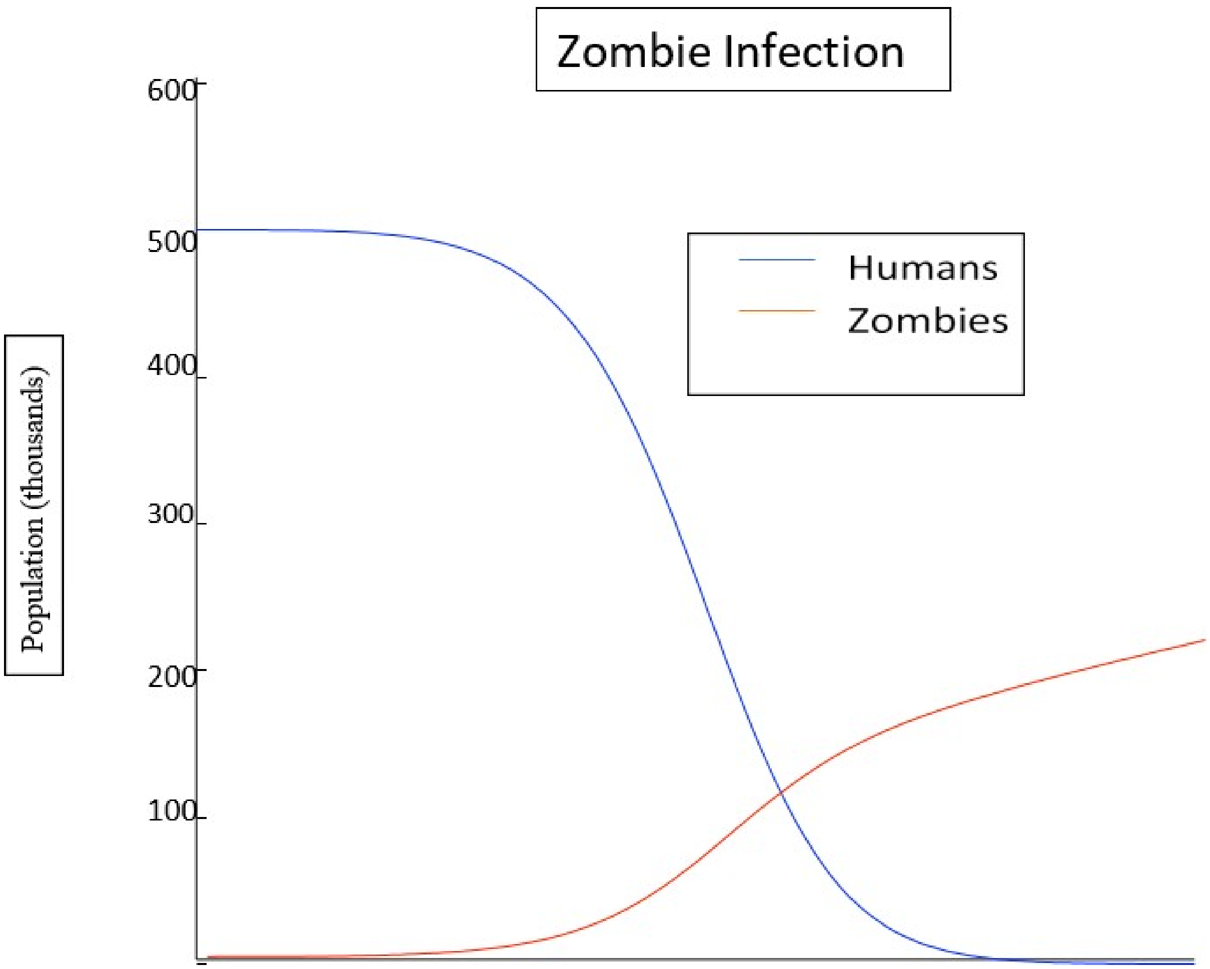
Section 5: Math Solves Real-Life Problems

The flow chart given allows an excellent way to integrate science and math teaching. Briefly stated, possible.

An excellent resource for alternate activities modeling growth can be found on plus.maths.org

Section 6: Extra for Experts

The Zombie Problem deals with both exponential growth and exponential decay. It is quite a bit more complicated than the Wonderful Widget Works problem and the Paper Folding Assignment. The pages following the activity have extensive notes.



This graph results from the "Extra for Experts" section at the end of Exponents chapter of the textbook. It is included here to indicate that though the lessons start with a very simple beginning, they can be expanded to much higher levels of mathematical competency.

Appendix I: Resources

The Standards

Pimentel, S. (2013, April). College and Career Readiness Standards for Adult Education (United States, U.S. Department of Education, Office of Vocational and Adult Education). Retrieved June 28, 2019, from https://lincs.ed.gov/publications/pdf/CCRStandardsAdultEd.pdf

National Professional Development

Professional Development Units for CCR Standards in Mathematics: Foundational Units. (2019, June 12). Retrieved June 28, 2019, from https://lincs.ed.gov/state-resources/federal-initiatives/college-career-readiness/math

Professional Development Units for CCR Standards in Mathematics: Advanced Units. (2019, June 12). Retrieved June 28, 2019, from https://lincs.ed.gov/state-resources/federal-initiatives/college-career-readiness/math#advanced

Links to Other States' And Organizations' General Resources

Resources for CCR Standards-Based Instruction (SBI). (n.d.). Retrieved June 28, 2019, from http://kyae.ky.gov/educators/ccrsbi/index.html

College and Career Readiness Standards for Adult Education. (n.d.). Retrieved June 28, 2019, from https://www.sabes.org/CCRStandards; *Massachusetts/SABES links to CCRSAE resources. Includes brief (20-minute) Introductory Videos on Aspects of the CCRSAE and the Key Instructional Shifts*

CCR Standards Resources. (n.d.). Retrieved June 28, 2019, from https://atlasabe.org/resources/ccr-standards/; *Minnesota ELA and Mathematics CCRS resources*

Journal of Research and Practice for Adult Literacy, Secondary, and Basic Education. (2017). *Journal of Research and Practice for Adult Literacy, Secondary, and Basic Education*, *6*(1). Retrieved June 28, 2019, from; https://static1.squarespace.com/static/55a158b4e4b0796a90f7c371/t/58cc9a836b8f5b67ee4b5969/1489803921184/2017+Spring+COABE+Journal.pdf; *Standards and Professional Development: Practitioner Perspective (Ohio) Includes info about identifying priority benchmarks and "retrofitting" lesson plans.*

I-BEST Collaborative Planning for Learning Outcomes. (n.d.). Retrieved June 28, 2019, from https://www.sbctc.edu/colleges-staff/programs-services/i-best/collaborative-planning.aspx
Use of Standards for IET

Connecting College and Career Readiness to the Standards, GED®, Higher Education, Training, the Workplace and Everyday Life Success (Working paper). (n.d.). Retrieved June 28, 2019, from http://kyae.ky.gov/educators/resources/rla/ConnectingWIOACollegeandCareerReadinessStandardsandtheGED.pdf; Relevance to GED

Teaching the GED® Test. (n.d.). Retrieved June 28, 2019, from https://ged.com/educators_admins/teaching/; Relevance to GED

Alignment to GED
An Independent Study of the Alignment between Items on the 2016 GED Exam and the College and Career Readiness Standards for Adult Education. Submitted to: Martin Kehe Vice President of Assessment Services GED Testing Service, Submitted by: Standards, Assessment, and Accountability Services Program at WestEd March 29, 2016

Lesson Plan Building and Sharing Tools. (summer 2017). *Journal of Research and Practice for Adult Literacy, Secondary, and Basic Education*, *6*(2), 73-75. Retrieved June 28, 2019, from; https://www.sabes.org/sites/sabes.org/files/resources/LP%20Tools%20David%20Rosen%20Review.pdf; Lesson Plan Building and Sharing Tools (in WebScan by David Rosen)

United States, U.S. Department of Education. (n.d.). College & Career Readiness Standards in-Action (Vol. 2, Workshop Material Mathematics). Retrieved June 28, 2019, from; https://lincs.ed.gov/publications/pdf/ccr/Math_Unit_2_Materials/Math_Unit_2.pdf.; Major work of the Levels

CASAS Greater Opportunities for Adult Learning Success (GOALS) for Math [PDF]. (2017). CASAS. http://www.casas.org/docs/default-source/institute/si-2017/a10-d2-new-reading-and-math-series.pdf?sfvrsn=4

Specific Resources for Mathematics

Grade 4 Mathematics. (n.d.). Retrieved July 1, 2019, from https://www.engageny.org/resource/grade- 4-mathematics-module-5-topic-d-overview
Engage NY

Math Curriculum. (n.d.). Retrieved from https://openupresources.org/math-curriculum/
Illustrative Mathematics Utah Education Network Open-Up Resources

MyOpenMath. (n.d.). Retrieved July 1, 2019, from https://www.myopenmath.com/

Open Middle®. (n.d.). Retrieved July 1, 2019, from https://www.openmiddle.com/
Challenging math problems worth solving

Desmos. (n.d.). Retrieved July 1, 2019, from https://www.desmos.com/
Explore math with desmos.

Open Education Resources Commons. (n.d.). Retrieved July 1, 2019, from https://www.oercommons.org/

Lessons. (n.d.). Retrieved July 1, 2019, from https://robertkaplinsky.com/lessons/
Robert Kaplinsky

When Math Happens. (2019, January 04). Retrieved July 1, 2019, from https://whenmathhappens.com/3-act-math/

Virtual Nerd. (n.d.). Retrieved July 1, 2019, from https://www.virtualnerd.com/

Illuminations: Lessons. (n.d.). Retrieved from https://illuminations.nctm.org/Lessons-Activities.aspx; The National Council of Teachers of Mathematics

HIGHER EDUCATION COORDINATING COMMISSION